Axial-vector form factors in the chiral quark constituent model

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Outline

Naive Quark Model

- **Internal Structure:** The knowledge of internal structure of baryons in terms of quark and gluon degrees of freedom in QCD provides a basis for understanding more complex, strongly interacting matter.
- At high energies, (α_s is small), QCD can be used perturbatively.
- At low energies, (α_s becomes large), one has to use other methods such as effective Lagrangian models.
- Knowledge has been rather limited because of confinement and it is still a big challenge to perform the calculations from the first principles of QCD.
- **Naive Quark Model** is able to provide a intuitive picture and successfully accounts for many of the low-energy properties of the hadrons in terms of the valence quarks.

"Proton Spin Problem": Driving Question

- 1988 European Muon Collaboration (Valence quarks carry 30% of proton spin)
- Naive Quark Model contradicts this results (Based on Pure valence description: proton = 2u + d)
  "Proton spin crisis"
- Confirmed by the measurements of polarized structure functions of proton in the deep inelastic scattering (DIS) experiments by SMC, E142-3 and HERMES experiments.
- Provides evidence that the valence quarks of proton carry only a small fraction of its spin suggesting that they should be surrounded by an indistinct sea of quark-antiquark pairs.

Flavor Structure

- Several interesting facts revealed regarding the flavor distribution functions.
- 1991 NMC result: Asymmetric nucleon sea ($\bar{d} > \bar{u}$)
  Confirmed by E866 and HERMES
  Gottfried Sum Rule: $I_G = \frac{1}{3} + \frac{2}{3}\int_0^1 [\bar{u}(x) - \bar{d}(x)]dx = 0.254 \pm 0.026$.
- Confirmed by the Drell-Yan experiments measuring a large quark sea asymmetry ratio $\bar{d}/\bar{u}$.
- The conventional expectation that the quark sea perhaps can be obtained through the perturbative production of the quark-antiquark pairs by gluons produces nearly equal numbers of $\bar{u}$ and $\bar{d}$.
- Study of the quark sea is intrinsically a nonperturbative phenomena. Explained only through the generation of **"quark sea".**

Non-perturbative regime

- Recently, a wide variety of accurately measured data have been accumulated for
  static properties of hadrons: masses, electromagnetic moments, charge radii etc.
  low energy dynamical properties: scattering lengths and decay rates etc.
- These lie in the nonperturbative range of QCD.
- The direct calculations of these quantities form the first principle of QCD are extremely difficult: they require nonperturbative methods.
- Techniques such as lattice gauge theory, QCD sum rules, and a wide variety of models have been developed to study this extremely interesting energy regime.

χCQM

- χCQM initiated by Weinberg and developed by Manohar and Georgi to explain the successes of NQM.
- "Quark sea" generation $q_{\pm} \rightarrow GB^0 + q^{'}_{\mp} \rightarrow (q\bar{q}^{'}) + q^{'}_{\mp}$.
- The chiral constituent quark model probes the regime between the confinement scale and the chiral symmetry breaking scale, therefore providing vital clues for the nonperturbative structure of QCD.
  $SU(3)_L \times SU(3)_R$ spontaneously broken to $SU(3)_{L+R}$ symmetry $\Lambda_{\chi SB}$.
- “Justifies” the idea of constituent quarks.
- A detailed investigation attempts to throw considerable light on the effective degrees of freedom of QCD in the nonperturbative regime.
- Many of the theoretical as well as experimental issues concerning the problem are of interest to the nuclear physicists also, hence indicating the broader interest of the physics community.

Successes of χCQM

- χCQM improves the predictions in several cases including the spin polarization functions, strangeness content of the nucleon, weak hyperon β decay parameters and flavor distribution functions.
- χCQM successfully explains magnetic moments of octet and decuplet baryons including their transitions, violation of Coleman Glashow sum rule, charge radii, quadrupole moment and magnetic moments of $\frac{1}{2}^-$ octet baryon resonances, magnetic moments of $\frac{1}{2}^-$ and $\frac{3}{2}^-$ Λ resonances.
- The model is successfully extended to predict the important role played by the small intrinsic charm content in the nucleon spin in the SU(4) χCQM and to calculate the magnetic moment and charge radii of spin $\frac{1}{2}^+$ and spin $\frac{3}{2}^+$ charm baryons including their radiative decays.

Axial-Vector Form Factors

- Form factors parameterized from the isovector axial-vector current operator are important in hadron physics as they provide a deep insight in understanding the internal structure. The electromagnetic Dirac and Pauli form factors are well know over a wide region of momentum transfer squared Q^2.
- The measured first moment related to the combinations of the axial-vector coupling constants which are combinations of the spin polarizations, Δu, Δd and Δs

$$\Gamma_1^p(Q^2) = \int_0^1 g_1^p(x, Q^2)dx = \frac{C_s(Q^2)}{9} g_A^0 + \frac{C_{ns}(Q^2)}{12} g_A^3 + \frac{C_{ns}(Q^2)}{36} g_A^8,$$

C_s and C_{ns}: flavor singlet and non-singlet Wilson coefficients.
g_A^0 corresponds to the flavor singlet component related to the total quark spin content, g_A^3 corresponds to the flavor non-singlet components usually obtained from the neutron $\beta-$decay and g_A^8 is obtained from the semi-leptonic weak decays of hyperons.

- Axial-vector current $A^{\mu,a}$ $(\bar{\mathbf{q}}\gamma^\mu\gamma_5\frac{\lambda^a}{2}\mathbf{q})$ defined through the matrix elements

$$\langle B(p')|A^{\mu,a}|B(p)\rangle = \bar{u}(p')\left[\gamma^\mu\gamma_5 G_A^i(Q^2) + \frac{q^\mu}{2M_B}\gamma_5 G_P^i(Q^2)\right]u(p),$$

  - M_B: Baryon mass
  - $u(p)$ $(\bar{u}(p'))$: Dirac spinors of the initial (final) baryon states
  - $Q^2 = -q^2$: four momenta transfer $(q \equiv p - p')$
  - λ^a $(a = 1, 2, ..8)$: Gell-Mann matrices describing the flavor structure of SU(3)
  - The matrices having diagonal representation flavor singlet current $(a = 0)$, isovector current $(a = 3)$ and hypercharge axial current $(a = 8)$ are taken
  - $G_A^i(Q^2)$ and $G_P^i(Q^2)$ $(i = 0, 3, 8)$ are the axial and induced pseudoscalar form factors.
- The singlet and non-singlet combinations of the spin structure

$$\begin{aligned}
g^0_{A,B} &= \langle B|u^+u^- + d^+d^- + s^+s^-|B\rangle = \Delta u_B + \Delta d_B + \Delta s_B\,, \\
g^3_{A,B} &= \langle B|u^+u^- - d^+d^-|B\rangle = \Delta u_B - \Delta d_B\,, \\
g^8_{A,B} &= \langle B|u^+u^- + d^+d^- + 2s^+s^-|B\rangle = \Delta u_B + \Delta d_B - 2\Delta s_B\,.
\end{aligned}$$

- Recently, SAMPLE at MIT-Bates, G0 at JLab, PVA4 at MAMI and HAPPEX at JLab have provided considerable insight on the role played by strange quarks in the charge, current and spin structure.
- The nucleon axial coupling constant g_A^3, determined precisely from nuclear $\beta-$decay and fundamental parameter to test the chiral symmetry breaking effects, corresponds to the value of the axial form factor at zero-momentum transfer ($Q^2 = -q^2 = 0$).
- Our information about the other low lying octet baryon axial-vector form factors from experiment is also rather limited because it is difficult to measure the hyperon properties experimentally due to their short lifetimes.
- Experiments involving elastic scattering of neutrinos and antineutrinos and the pion electro-production on the proton have explored Q^2 dependence of axial form factors in the past. Recently, there has been considerable refinement in the higher-energy Minerνa experiment at Fermilab.

Purpose

- We have determined the axial-vector form factors of the low lying octet baryons in the chiral constituent quark model (χCQM).
- We begin by computing the static properties of the axial-vector current. The singlet (g_A^0) and non-singlet (g_A^3 and g_A^8) axial-vector coupling constants expressed as combinations of the spin polarizations at zero momentum transfer have been investigated for the case of N, Σ, Ξ and Λ baryons.
- Further, it would be significant to analyse the Q^2 dependence of the axial-vector form factors ($G_A^0(Q^2)$, $G_A^3(Q^2)$ and $G_A^8(Q^2)$) as well as their explicit flavor contributions ($G_A^u(Q^2)$, $G_A^d(Q^2)$ and $G_A^s(Q^2)$) by using a conventional dipole form of parametrization.
- The calculations are extended to predict the total strange singlet and non-singlet contents ($G_s^0(Q^2)$, $G_s^3(Q^2)$ and $G_s^8(Q^2)$) of the nucleon and determine the strange quark contribution to the nucleon spin (Δs).

- The quark spin polarization can be defined as $\Delta q = q^{+} - q^{-}$, where $q^{\pm}$ can be calculated from the spin structure of a baryon

$$\hat{B} \equiv \langle B|\mathcal{N}|B\rangle = \langle B|q^{+}q^{-}|B\rangle\,.$$

- $|B\rangle$ is the baryon wave function and the number operator measuring the sum of the quark numbers with spin up or down

$$q^{+}q^{-} = n_{u^{+}}u^{+} + n_{u^{-}}u^{-} + n_{d^{+}}d^{+} + n_{d^{-}}d^{-} + n_{s^{+}}s^{+} + n_{s^{-}}s^{-}\,.$$

- The contributions of the quark sea comes from the fluctuation process and for every constituent quark

$$q^{\pm} \rightarrow \sum P_q q^{\pm} + |\psi(q^{\pm})|^2,$$

where the transition probability of the emission of a GB from any of the q quark ($\sum P_q$) and the transition probability of the $q^{\pm}$ quark ($|\psi(q^{\pm})|^2$) can be calculated from the Lagrangian.

$$\sum P_u = a\left(\frac{9+\beta^2+2\zeta^2}{6}+\alpha^2\right), |\psi(u^{\pm})|^2 = \frac{a}{6}(3+\beta^2+2\zeta^2)u^{\mp}+ad^{\mp}+a\alpha^2 s^{\mp},$$

$$\sum P_d = a\left(\frac{9+\beta^2+2\zeta^2}{6}+\alpha^2\right), |\psi(d^{\pm})|^2 = au^{\mp}+\frac{a}{6}(3+\beta^2+2\zeta^2)d^{\mp}+a\alpha^2 s^{\mp},$$

$$\sum P_s = a\left(\frac{2\beta^2+\zeta^2}{3}+2\alpha^2\right), |\psi(s^{\pm})|^2 = a\alpha^2 u^{\mp}+a\alpha^2 d^{\mp}+\frac{a}{3}(2\beta^2+\zeta^2)s^{\mp}.$$

Quark Spin Polarizations and the Axial Coupling Constants

Table: The χCQM results for the quark spin polarizations and the axial coupling constants for the N, Σ, Ξ and Λ octet baryons.

Quantity	N	Σ	Ξ	Λ
Δu_B	0.904	0.881	−0.329	0.002
Δd_B	−0.362	−0.137	0.00	0.002
Δs_B	−0.023	−0.252	1.109	0.805
$g^0_{A,B}$	0.519	0.492	0.780	0.809
$g^3_{A,B}$	1.266	1.018	−0.329	0.00
$g^8_{A,B}$	0.588	1.248	−2.547	−1.606

- For the case of N,

$$\Delta u_N^{\rm expt} = 0.85 \pm 0.05, \quad \Delta d_N^{\rm expt} = -0.41 \pm 0.05, \quad \Delta s_N^{\rm expt} = -0.07 \pm 0.05\,,$$

$$g_{A,N}^{0\ \rm expt} = 0.30 \pm 0.06, \quad g_{A,N}^{3\ \rm expt} = 1.267 \pm 0.0025, \quad g_{A,N}^{8\ \rm expt} = 0.588 \pm 0.033\,,$$

- The NQM, which is quite successful in explaining a good deal of low energy data, has the following predictions

$$\Delta u_N = 1.33, \quad \Delta d_N = -0.33, \quad \Delta s_N = 0\,,$$

$$g_{A,N}^{0} = 1, \quad g_{A,N}^{3} = 1.66, \quad g_{A,N}^{8} = 1\,.$$

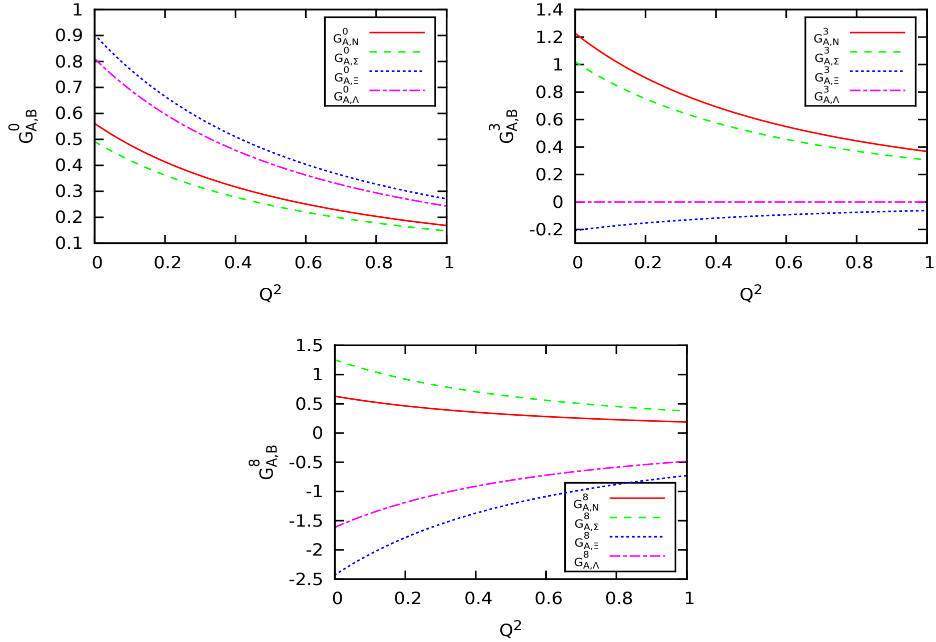
- This disagreement was broadly characterized as “proton spin crisis”. The results of χCQM for the case of Δu_N, Δd_N, Δs_N, $g_{A,N}^3$ and $g_{A,N}^8$ are more or less in agreement with data.
- This justifies the success of χCQM and strengthens our conclusion regarding the qualitative and quantitative role of the “quark sea” in right direction.

Q^2 dependence

- The Q^2 dependence of the axial-vector form factors has been experimentally investigated from the quasi elastic neutrino scattering and from the pion electroproduction.
- The dipole form of parametrization conventionally used is

$$G^i_{A,B}(Q^2) = \frac{g^i_{A,B}(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2},$$

  where $g^0_A(0)$, $g^3_A(0)$ and $g^8_A(0)$ are the isovector axial-vector coupling constants at zero momentum transfer.
- Global average extracted from neutrino scattering experiments gives axial mass $M_A = (1.026 \pm 0.021)$GeV. The most recent value obtained by the MiniBooNE Collaboration $M_A = 1.10^{+0.13}_{-0.15}$GeV.



$G^0_{A,B}$
Q^2
$G^0_{A,N}$
$G^0_{A,\Sigma}$
$G^0_{A,\Xi}$
$G^0_{A,\Lambda}$
$G^3_{A,B}$
$G^3_{A,N}$
$G^3_{A,\Sigma}$
$G^3_{A,\Xi}$
$G^3_{A,\Lambda}$
$G^8_{A,B}$
$G^8_{A,N}$
$G^8_{A,\Sigma}$
$G^8_{A,\Xi}$
$G^8_{A,\Lambda}$

- The sensitivity of the singlet and non-singlet form factors for different baryons varies as

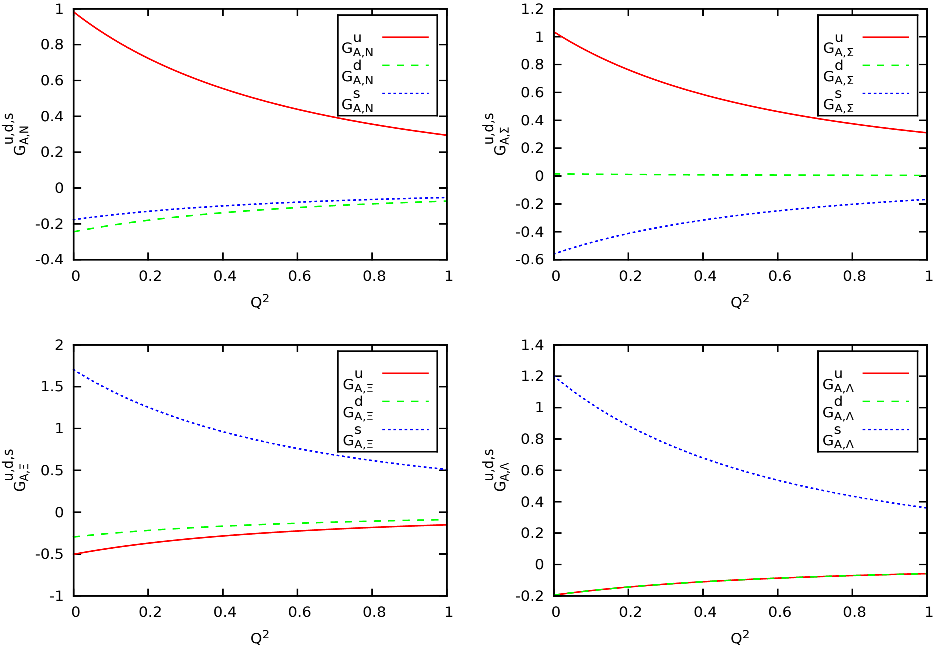
$$G^0_{A,\Xi} > G^0_{A,\Lambda} > G^0_{A,N} > G^0_{A,\Sigma},$$
$$G^3_{A,N} > G^3_{A,\Sigma} > G^3_{A,\Xi} > G^3_{A,\Lambda},$$
$$G^8_{A,\Xi} > G^8_{A,\Lambda} > G^8_{A,N} > G^8_{A,\Sigma}.$$

- The behaviour of the form factors for Ξ and Λ is similar to each other. This may possibly due to the presence of more strange quarks in the valence structure. The form factors for N and Σ, which have the dominance of u quarks in the valence structure, show similar variation with Q^2.
- $G^0_{A,B}$ form factors falls off rapidly with the increase of Q^2 for N, Σ, Ξ and Λ.
- $G^3_{A,B}$ and $G^8_{A,B}$, the N and Σ form factors fall off with increasing Q^2 whereas the Ξ and Λ form factors increase with increasing Q^2.
- $G^3_{A,\Lambda}$ is particularly interesting and has no Q^2 dependence because it has equal numbers of u, d, and s quarks in its valence structure.

Flavor Axial Vector Form Factors

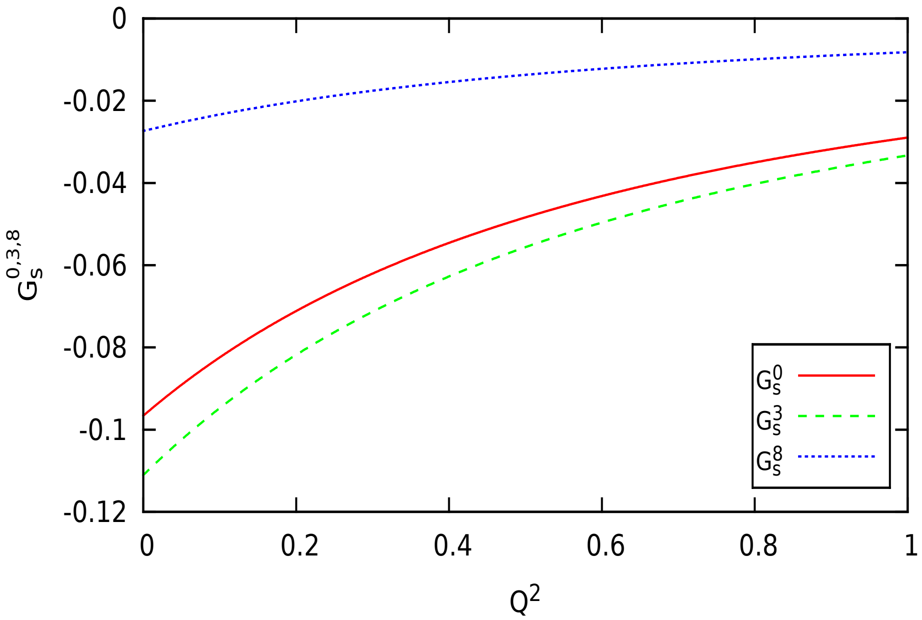
- The role of non-valence quarks in the spin structure can be studied in detail by calculating the flavor axial-vector form factors using the dipole form of parametrization.
- In terms of the singlet and non-singlet combinations of the spin structure

$$\begin{aligned} G^{u}_{A,B} &= \frac{1}{3}G^{0}_{A,B} + \frac{1}{2}G^{3}_{A,B} + \frac{1}{2\sqrt{3}}G^{8}_{A,B}\,, \\ G^{d}_{A,B} &= \frac{1}{3}G^{0}_{A,B} - \frac{1}{2}G^{3}_{A,B} + \frac{1}{2\sqrt{3}}G^{8}_{A,B}\,, \\ G^{s}_{A,B} &= \frac{1}{3}G^{0}_{A,B} - \frac{1}{\sqrt{3}}G^{8}_{A,B}\,. \end{aligned}$$



$G^{u,d,s}_{A,N}$
$G^{u}_{A,N}$
$G^{d}_{A,N}$
$G^{s}_{A,N}$
Q^2
$G^{u,d,s}_{A,\Sigma}$
$G^{u}_{A,\Sigma}$
$G^{d}_{A,\Sigma}$
$G^{s}_{A,\Sigma}$
Q^2
$G^{u,d,s}_{A,\Xi}$
$G^{u}_{A,\Xi}$
$G^{d}_{A,\Xi}$
$G^{s}_{A,\Xi}$
Q^2
$G^{u,d,s}_{A,\Lambda}$
$G^{u}_{A,\Lambda}$
$G^{d}_{A,\Lambda}$
$G^{s}_{A,\Lambda}$
Q^2

- Valence quark structure of the baryon is projected. N is dominated by u quark and clearly $G^u_{A,N}$ dominates, $G^d_{A,N}$ and $G^s_{A,N}$ have comparatively smaller contribution.
- Even though there are no s quarks in the valence structure the contribution of $G^s_{A,N}$ implies a presence of "quark sea" which is even more at zero momentum transfer.
- The valence quark distribution is spread over the entire Q^2 region and the sea contributions decrease as the value of Q^2 increases. At even higher values of Q^2, the contributions should be completely dominated by the valence quarks.
- In $G^{u,d,s}_{A,\Sigma}$ and $G^{u,d,s}_{A,\Xi}$ there is a significant contribution from the u and s quarks. The small but significant G^d_A contribution can have important implications for the role of sea quarks at low Q^2.
- The $G^s_{A,\Lambda}$ clearly dominates over $G^u_{A,\Lambda}$ and $G^d_{A,\Lambda}$ which is expected because of the u and d quarks also contribute towards $G^{u,d,s}_{A,\Lambda}$ through quark fluctuations.


0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
0
0.2
0.4
0.6
0.8
1
Q^2
$G_s^{0,3,8}$
G_s^0
G_s^3
G_s^8

- For the case of N, the strange quarks contribute to the spin polarizations of u and d quarks apart from contributing to the strange spin polarization because of the presence of the non-valence "quark sea".
- $G_s^0(Q^2)$, $G_s^3(Q^2)$ and $G_s^8(Q^2)$ can be calculated. The explicit strangeness contribution for the other octet baryons is not so significant because of the presence of strange quarks in their valence structure.
- We find that the magnitude of $G_s^0(Q^2)$ and $G_s^8(Q^2)$ fall off with the increasing value of Q^2 whereas $G_s^3(Q^2)$ has a weak Q^2 dependence. These quantities not only provide a direct method to determine the presence of a significant amount of quark sea but also impose important constraint on a model that attempts to describe the origin of the quark sea. Any refinement in the measurements of the strangeness dependent quantities would have important implications for the basic tenets of χCQM.

- χCQM is able to phenomenologically estimate and give a qualitative and quantitative description of the quantities having implications for chiral symmetry breaking and SU(3) symmetry breaking. The results are consistent with the recent available experimental results.
- Chiral symmetry breaking and SU(3) symmetry breaking play an important role in understanding the spin structure of the baryon in the nonperturbative regime of QCD where the constituent quarks and the weakly interacting Goldstone bosons constitute the appropriate degrees of freedom at the leading order.
- The future experiments to measure the axial-vector form factors will not only provide a direct method to determine the presence of appropriate amount of quark sea but also impose important constraint on the parity-violating asymmetries in different kinematical regions.

Thank You