Modified Natural Inflation: a small single field model with large tensor to scalar ratio

Pankaj Saha

Department of Physics
Indian Institute of Technology Guwahati
Guwahati-781039

DAE Talk

December 9, 2014

Outline

Introduction

Equations of Motion and Important Parameters in Inflation

The equations for an expanding Universe containing a homogeneous scalar field are

$$H^2 = \frac{8\pi}{3m_{Pl}^2}\left[V(\phi) + \frac{1}{2}\dot{\phi}^2\right] \rightarrow \text{The Friedmann equation} \quad (1)$$

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi) \rightarrow \text{The Klien-Gordon Equation} \quad (2)$$

where prime indicates derivative with respect to ϕ. Here we have ignored the curvature term k, since we know that by definition it will quickly become negligible once inflation starts.

There are few important parameters related to inflation.

1. Slow roll parameters(ϵ and η)

$$\epsilon = \frac{M_{Pl}^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \qquad |\eta| = \frac{M_{Pl}^2}{8\pi}\left(\frac{V''(\phi)}{V(\phi)}\right) \qquad (3)$$

During inflation $\epsilon, \eta \ll 1$. Inflation ends when either of these inequalities violates.

2. The number of e-folds which measures the amount of inflation.

$$ln\left(\frac{a_2}{a_1}\right) = N(\phi_{ini} \to \phi_{end}) = \int_t^t Hdt = -\frac{8\pi}{M_p^2}\int_{\phi_{ini}}^{\phi_{end}} \frac{V(\phi)}{V'(\phi)} \qquad (4)$$

Introduction

Equations of Motion and Important Parameters in Inflation

There are few more...

1. CMB Power Spectrum

$$P_{\mathcal{R}} = \frac{3\sqrt{6}}{64\phi^2}\frac{H^2}{M_p^2\epsilon} \quad (5)$$

4. Deviations from scale invariance are characterized by the scalar spectral index:

$$n_s = 1-2\eta+6\epsilon = 0.9603\pm0.0073 \rightarrow \text{Planck 2013 results.XXII,arXiv:1303.5082[as} \quad (6)$$

3

$$n_T = -2\epsilon \quad (7)$$

3. And, the "tensor-to-scalar" ratio

$$r = -\frac{32\sqrt{6}}{9}n_T = 16\epsilon = 0.20^{+0.07}_{-0.05} \rightarrow \text{[BICEP2 arXiv:1403.3985[astro-ph.CO]]} \quad (8)$$

The Modified Natural Inflation

The Natural Inflation[1]

- Theoretically well motivated as it is naturally flat due to shift symmetries, and in the simplest version takes the form $V(\phi) = \Lambda^4[1 \pm cos(N\phi/f)]$
- A tensor-to-scalar ratio $r > 0.1$ as seen by BICEP2 requires the width of any inflationary potential to be comparable to the scale of grand unification and the width to be comparable to the Planck scale
- The cosine Natural Inflation model agrees with all cosmic microwave background measurements as long as $f > m_{Pl}$ (where $m_{Pl} = 1.22 * 10^{19}$ and $\Lambda \sim m_{GUT} \sim 10^{16} GeV$.

[1] K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. Lett. 65, 3233 (1990)

Some variants of the Natural Inflation paradigm and there observational consistency[2]

- axion monodromy with potential $V \propto \phi^2/3$ is inconsistent with the BICEP2 limits at the 95% confidence level, and low-scale inflation is strongly ruled out.
- Linear potentials $V \propto \phi$ are inconsistent with the BICEP2 limit at the 95% confidence level, but are marginally consistent with a joint Planck/BICEP2 limit at 95%.
- The pseudo-Nambu Goldstone model proposed by Kinney and Mahanthappa as a concrete realization of low-scale inflation. While the low-scale limit of the model is inconsistent with the data, the large-field limit of the model is marginally consistent with BICEP2.
- All of the models considered predict negligible running of the scalar spectral index,and would be ruled out by a detection of running.

[2] K. Freese, W. H. Kinney, arXiv:1403.5277 [astro-ph.CO]

The Modified Natural Inflation

The Natural Inflation

The Lyth Bound[3]

$$\delta\phi \equiv |\phi_{in} - \phi_{end}| \gtrsim N M_{Pl} \left(\frac{r}{8}\right)^{\frac{1}{2}}$$

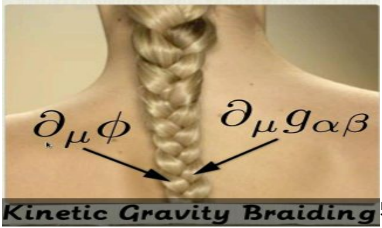
- Super Planckian field excursion for detectable gravitational wave
- A Physical theory with super-planckian scale is not suitable from effective field theory frame work

[3] D. H. Lyth, Phys. Rev. Lett. 78, 1861 (1997) hep-ph/9606387

The Modified Natural Inflation

Kinetic Gravity Braiding

- A large class of scalar-tensor modles with interactions containing the second derivatives of the scalar field but not leading to additionnal degrees of freedom
- ϕ kinetically mixes / braids[4] with the metric
- Manifestly stable (no ghosts and no gradient instabilities)



[5]

[4] C. Deffayet, O. Pujolas, I. Sawicki nad A. Vikman: JCAP 1010:026, 2010, arXiv:1008.0048 [hep-th],

[5] pic courtsey:A Vikman,"Kinetic Gravity from Braiding Imperfect Dark Energy“

Works on Galileon Inflation

- A class of inflation model, was proposed, G inflation[6], which has a Galileon-like nonlinear derivative interaction of the form $G(\phi, (\nabla\phi)^2)\Box\phi$
- The most striking property of this generic Lagrangian is that it gives rise to derivative in time no higher than two both in the gravitational and scalar-field equations.
- G-inflation can generate (almost) scale-invariant density perturbations, together with a large amplitude of primordial gravitational waves

[6] Kobayashi, Yamaguchi and Yokoyama PRL **105**, 23102 (2010)

The scalar-field Lagrangian is of the form

$$\mathcal{L}_\phi = K(\phi, X) - G(\phi, X)\Box\Phi \tag{9}$$

assuming that ϕ is minimally coupled to gravity, the total action is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R + \mathcal{L}_\phi\right] \tag{10}$$

The energy-momentum tensor $T_{\mu\nu}$ reads

$$T_{\mu\nu} = K_X\nabla_\mu\Phi\nabla_\nu\phi + Kg_{\mu\nu} - 2\nabla_{(\mu}G\nabla_{\nu)}\phi + g_{\mu\nu}\nabla_\lambda G\nabla^\lambda\phi - G_X\Box\phi\nabla_\mu\phi\nabla_\nu\phi \tag{11}$$

Taking the FRLW ansatz, the energy-momentum tensor (5) has the form T_μ^ν $=$ diag $(-\rho, p, p, p)$ with

$$\rho = 2K_X X - K + 3G_X H\dot{\phi}^3 - 2G_\phi X \tag{12}$$

$$p = K - 2(G_\phi + G_X\ddot{\phi}X) \tag{13}$$

Here, ρ has an explicit dependence on Hubble rate H.

The gravitational field equations are thus given by

$$3M_{Pl}^2 H^2 = \rho \qquad -M_{Pl}^2(3H^2 + 2\dot{H}) = p \tag{14}$$

and the scalar-field equation reads

$$K_X(\ddot{\phi} + 3H\dot{\phi} + 2K_{XX}X\ddot{\phi} + 2K_{X\phi}X - K_\phi) - 2(G_\phi - G_{X\Phi}X)(\ddot{\phi} + 3H\dot{\phi}) + 6G_X[\dot{H}X + 3H^2X] - 4G_{X\phi}X\ddot{\phi} - 2G_{\phi\phi}X + 6HG_{XX}X\dot{X} = 0 \tag{15}$$

Works on Galileon Inflation

Inspired by the above model, a KGB model was proposed[7]

$$K(\phi, X) = X - V(\phi) \qquad\qquad G(\phi, X) = M(\phi)X$$

- In these models, the value of $n_s \simeq 0.96$ is not consistent with the high value of r predicted by BICEP2.
- To get a high value of r, the field excursion should be super-planckian.

[7] K. Kamada, et al., Phys. Rev. D 83 (2011) 083515 ,D. Maity, Phys.Lett.B 720 (2013) 389-392

Motivation of Our Work[8]

- One of our guiding principles to construct a KGB model is the constant shift symmetry of the axion
- We have chosen the form of the KGB term in such a way that it predicts the required value of $n_s \simeq 0.96$ and a large tensor to scalar ratio r > 0.1
- We find sub-Planckian field excursion for the axion field $\Delta\phi \simeq f$ for the sufficient number of e-folding $N \gtrsim 50$

[8] D. Maity, P. Saha: arXiv:1407.7692[hep-th], (Accepted in Phys.Rev.D)

Work

We use the following Lagrangian where in addition to the usual canonical term we also have higher derivative **KGB** terms

$$\mathcal{L} = \frac{M_{Pl}^2 R}{2} - X - M(\phi)X\Box\phi - \Lambda^4\left(1 - cos\left(\frac{\phi}{f}\right)\right) \tag{16}$$

Where $X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ and $\Box = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu)$ and $\{f, \Lambda\}$ are the width and hight of the potential

With the FRLW background ansatz for the spacetime

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2) \tag{17}$$

We get the following Einstein's equation for the scale factor a

$$H^2 = -H\dot{\phi}^3 M(\phi) - \frac{X}{3} + \frac{2}{3}X^2 M'(\phi) + \frac{\Lambda^4}{3}\left(1 - cos\left(\frac{\phi}{f}\right)\right) \tag{18}$$

and variation with respect to the scalar field yields the scalar field equation

$$\frac{1}{a^3}\frac{d}{dt}\left[a^3\left(1-3HM\dot{\phi}-2M'X\right)\dot{\phi}\right]=\partial^{\mu}\phi\partial_{\mu}(M'X)-\frac{\Lambda^4}{f}sin\left(\frac{\phi}{f}\right) \tag{19}$$

Where, H $=\frac{\dot{a}}{a}$ is the Hubble constant
Using Slow-Roll codition $\epsilon H^2 < 1$, the scalar field equation takes the following form

$$3H\dot{\phi}\left(1-3M(\phi)H\dot{\phi}\right)+\frac{\lambda^4}{f}sin\left(\frac{\phi}{f}\right)=0 \tag{20}$$

Since usual axion inflation does not solve the probles mentioned. Our obvious choice would be inflation driven by the KGB term. i.e., we need the following condition to be satisfied: $|M(\phi)V'(\phi)| \gg 1$ leading to the following inequality:

$$\tau = M(\phi)V'(\phi) = \frac{M(\phi)\Lambda^4}{f} sin\left(\frac{\phi}{f}\right) \gg 1 \tag{21}$$

The Slow-roll parameters are:

$$\epsilon = \frac{M_p^2}{2f^2\sqrt{\tau}} \frac{\sin\left(\frac{\phi}{f}\right)^2}{\left(1 - \cos\left(\frac{\phi}{f}\right)\right)^2} \qquad \eta = \frac{M_p^2}{2f^2\sqrt{\tau}} \frac{\cos\left(\frac{\phi}{f}\right)}{\left(1 - \cos\left(\frac{\phi}{f}\right)\right)}$$

$$\alpha = \frac{M_p}{2} \frac{M'}{M} \left(\frac{4\epsilon^2}{\tau}\right)^{\frac{1}{4}} \qquad \beta = \frac{M_p^2}{36} \frac{M''}{M} \left(\frac{4\epsilon^2}{\tau}\right)^{\frac{1}{2}}$$

The number of e-foldings is in terms of M(ϕ), [below we defined small x as $x = (\phi/f)$]

$$\mathcal{N} = \mathcal{A}\int_{x_1}^{x_2} \frac{(1 - cosx)(s^3 M(x))}{\sqrt{sinx}} dx \tag{22}$$

The Modified Lyth Bound

$$\Delta\phi \geq (\mathcal{N} M_p) \left| \frac{\sqrt{2\epsilon_{min}}}{\sqrt[4]{\tau_{max}}} \right| = \frac{f}{\sqrt{\mathcal{A}}} \frac{\mathcal{N}}{\tau_{max}} \sqrt{\frac{9r}{36\sqrt{6}}} \tag{23}$$

where, $\tau_{max} = (s^3 M(x_{in}) sinx_{in})^{(1/4)}$ and $\mathcal{A} = \sqrt{\tau_0}(f/M_p)^2$

1 Bound on the axion field is supressed by $\mathcal{A}$
2 So, suitabley choosing the value of $\mathcal{A}$ one can make all the result consistent with observation and still get the sub-planckian $\Delta\phi$

Work

Specific model: Form of M(ϕ)

In the model of axion inflation, one of our first goal is to reduce the value of f by some mechanism.

1. Introduce multiple axion fields[9] with respective sub-planckian decay constants and the dynamics of the combined system is super-planckian
2. choose Specific form of the KGB function $M(\phi)$

[9] J. E. Kim et. al., JCAP **0501**, 005 (2005); N. Barnaby, M Peloso, Phys.Rev.Lett. **106**, 181301 (2011); E. Silverstein, A. Westphal, Phys.Rev.D **78**, 106003 (2008); P. Adshead, M. Wyman, Phys.Rev.Lett. **108**, 261302 (2012)

Work

Specific model: Form of M(ϕ)

We have consudered the following particular class of KGB function of the form

$$M(\phi) = \frac{1}{s^3} sin^p x[1 - cosxsin^2 x]^q \tag{24}$$

where p is odd interger and q is nay integer. We have considered three possible choices of p = {5,7,9}

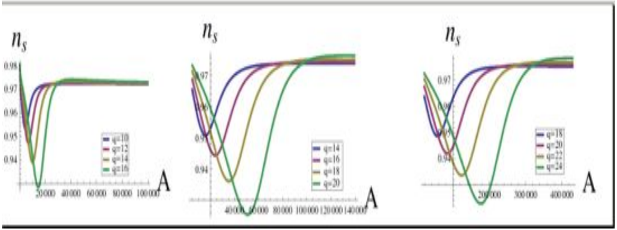


Figure : Behaviour of Spectral index n_s with respect to the derived parameter $\mathcal{A}$ for three different functional form of $M(\phi)$.

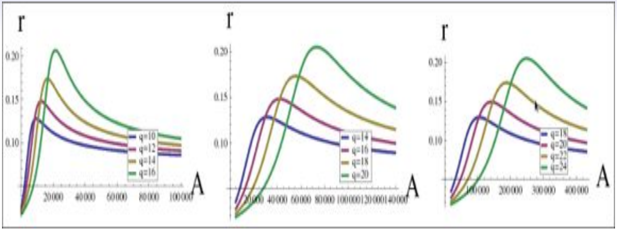


Figure : Behaviour of scalar to tensor ratio r with respect to the derived parameter $\mathcal{A}$ for three different functional form of $M(\phi)$.

Data:

$p = 5$	$\mathcal{N} = 50$					$\mathcal{N} = 60$				
q	$\mathcal{A}$	r	x_1	x_2	$\frac{\Lambda}{M_p}$	$\mathcal{A}$	r	x_1	x_2	$\frac{\Lambda}{M_p}$
10	7300	0.124	0.89	0.202	0.010	5300	0.077	1.053	0.022	0.0086
12	11500	0.147	0.84	0.185	0.011	10900	0.112	0.931	0.187	0.00997
14	16300	0.174	0.82	0.172	0.012	16900	0.140	0.884	0.171	0.0105
16	22300	0.206	0.80	0.162	0.013	124700	0.172	0.842	0.158	0.0116

$p=7$	$\mathcal{N}=50$					$\mathcal{N}=60$				
q	$\mathcal{A}$	r	x_1	x_2	$\frac{\Lambda}{M_p}$	$\mathcal{A}$	r	x_1	x_2	$\frac{\Lambda}{M_p}$
14	26000	0.125	0.868	0.219	0.011	22600	0.087	0.975	0.225	0.009
16	39400	0.148	0.835	0.204	0.011	40000	0.116	0.899	0.203	0.010
18	56000	0.173	0.814	0.192	0.012	60000	0.142	0.864	0.190	0.011
20	76000	0.204	0.803	0.183	0.012	85000	0.171	0.838	0.179	0.012

$p=9$	$\mathcal{N}=50$					$\mathcal{N}=60$				
q	$\mathcal{A}$	r	x_1	x_2	$\frac{\Lambda}{M_p}$	$\mathcal{A}$	r	x_1	x_2	$\frac{\Lambda}{M_p}$
18	92000	0.127	0.851	0.232	0.010	88000	0.095	0.927	0.234	0.0096
20	135000	0.149	0.829	0.219	0.011	140000	0.118	0.884	0.218	0.010
22	189000	0.174	0.814	0.209	0.012	206000	0.142	0.857	0.20617	0.011
24	256000	0.204	0.835	0.200	0.012	289000	0.170	0.837	0.196	0.012

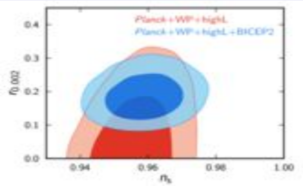


Figure : Behaviour of the spectral index n_s with respect to scalar to tensor ratio r taken from PlanckXVI[11].

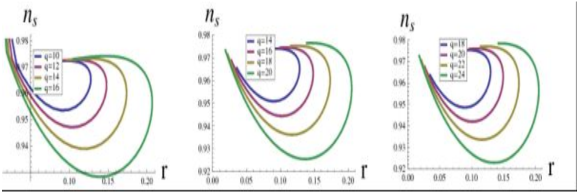


Figure : Behaviour of the spectral index n_s with respect to scalar to tensor ratio r for three different functional form of $M(\phi)$.

[10] Planck Collaboration XVI, (2013), arXiv:1303.5076
[11] Planck Collaboration XVI, (2013), arXiv:1303.5076

Summary of the plots

- The higher values of p have same qualititative behaviour. But importantly it is further lowering down the limiting value of the axion decay constant f
- We also see that for p > 5, f becomes sub-planckian consistent with reheating.
- For p $= 5$, even though we get f litle higher than M_p but $\Delta\phi$ is still sub-planckian.
- For every value of p, we choose some value of q and see how the value of $\{n_s, r\}$ depend on q.

Conclusions:

- We choose the form of $M(\phi)$ in such a way that we can reproduce all the important results of inflationary cosmology and it also consistent with low energy effective field theory.
- For our model, for the central ovserved value $n_s = 0.960$, we will have the following one particular choice of all other parameters for $r \sim 0.147$ for $\mathcal{N} = 50$:

p	$\mathcal{A}$	$\frac{f}{M_p}$	$\frac{\Delta\phi}{M_p}$	$\frac{s}{M_p}$	$\frac{\Lambda}{M_p}$
5	11500	1.26	0.825	6.20×10^{-6}	0.011
7	39400	0.90	0.568	1.96×10^{-6}	0.011
9	135000	0.71	0.433	6.84×10^{-7}	0.011

- With this value of parameters the axion field oscillates coherently after the end of inflation, which will lead lead to successful reheating of the universe.

THANK YOU