Phase transition of Gross Neveu model with Borici Creutz fermions

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Outline

Introduction

- Simulation with dynamical fermions in a lattice is always a challenging task.
- The famous no-go theorem:Lattice fermion actions with,
  - locality
  - chiral symmetry
  - hermiticity

  must produce massless fermions in multiples of two in continuum limit.
- There exist lot of fermion prescriptions to avoid fermion doubling caused by the naive fermions.
- Every model has its own advantges and also individual shortcomings.

- Lattice fermions and shortcomings
  - Wilson fermion: No chiral symmetry
  - Staggered fermion: Doublers not remove totally and rooting needed
  - Domain wall and Overlap fermion: Complicated simulation algorithms
- Another possible way is lattice action with 2 massless species, the minimum number is required by the no-go theorem, called minimal-doubling fermions.
- There are three types of minimally doubled actions,
  - Karsten-Wilczek
  - Borici-Creutz
  - Twisted-ordering types.
- These all possess one exact chiral symmetry but lack discrete symmetries.

- Wilson type fermions are written as,

$$aD_{fm}(p) = i\gamma_\mu \sin p_\mu a + M_f(p)$$

where $M_f(p)$ is flavoured mass terms. [1]

- Now if we write,

$$aD_{fm}(p) = i\gamma_\mu \sin p_\mu a + i\Gamma M_f(p)$$

Where Γ stands for $\gamma_1, \gamma_2, \gamma_3$ *and* γ_4 or their combinations

- Now this type of term preserves the chiral symmetry but breaks the hypercubic symmetry.

[1] M.Creutz et. all arxiv :1011.0761

Borici Creutz fermions in 4D

Introduction

- Borici Creutz action in 4 dimensional space is written as,

$$
\begin{aligned}
S_{BC} \quad = \quad & \sum_n [\frac{1}{2} \sum_\mu \bar{\psi}_n \gamma_\mu (\psi_{n+\mu} - \psi_{n-\mu}) \\
& -\frac{ir}{2} \sum_\mu \bar{\psi}_n (\Gamma - \gamma_\mu)(2\psi_n - \psi_{n+\mu} - \psi_{n-\mu}) \\
& + ic_3 \bar{\psi}_n \Gamma \psi_n + m \bar{\psi}_n \psi_n]
\end{aligned}
$$

where $\Gamma = \frac{1}{2}(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4)$ and $\{\Gamma, \gamma_\mu\} = 1$.

- We write this action using the hopping and onsite operators as,

$$S_{BC} = \sum_n [\sum_\mu (\bar{\psi}_n P_\mu^+ \psi_{n+\mu} - \bar{\psi}_n P_\mu^- \psi_{n-\mu}) + \bar{\psi}_n \hat{M} \psi_n]$$

where the hopping operators are defined as $P_\mu^+ = \frac{\gamma_\mu}{2}(1 - ir) + \frac{ir\Gamma}{2}$, $P_\mu^- = \frac{\gamma_\mu}{2}(1 + ir) - \frac{ir\Gamma}{2}$ and the onsite operator $\hat{M} = m + i(c_3 - 2r)\Gamma$.

Contd...

- In the strong coupling limit the effective action is,

$$S_{eff} \sim \sum_n [\sum_\mu Tr(M(n)(P^+_\mu)^T M(n+\hat{\mu})(P^-_\mu)^T) + Tr(\hat{M}M(n)) - \mathrm{Tr}(\log M(n))$$

  where $M(n) = \bar{\psi}(n)\psi(n)/N_c$ and the trace is over spinor indices.

- The condensate which is the vacuum expectation value of M(n), has both σ and π_Γ condensates,

$$\langle M(n) \rangle = M_0 = \sigma I_4 + i\Gamma\pi_\Gamma .$$

- After putting this into previous equation we get the effective action as,

$$S_{eff} = N_c[4\sigma^2(1+r^2) + 2\pi_\Gamma{}^2(1+r^2) + 4m\sigma - 4(c_3 - 2r)\pi_\Gamma - 2\log(\sigma^2 + \pi_\Gamma{}^2)]$$

Phase Diagram in 4D

Gap equations

- From the saddle point solutions the gap equations are,

$$2\sigma(1+r^2)+m-\frac{\sigma}{\sigma^2+{\pi_\Gamma}^2}=0,$$
$$\pi_\Gamma(1+r^2)-(c_3-2r)-\frac{\pi_\Gamma}{\sigma^2+{\pi_\Gamma}^2}=0.$$

- These equantions can be solved analytically for $m=0$. Setting $\sigma\to 0$, we get the chiral boundaries for massless Borici-Creutz fermions at,

$$c_3-2r=\pm\sqrt{\frac{1+r^2}{2}}.$$

- For $r=1$ the chiral boundaries are at $\bar{c}_3=c_3-2=\pm 1$.
- We get two solutions for the condensates for $m=0$ and $r=1$ as

$$\sigma=0,\pi_\Gamma=\frac{1}{4}\left(\bar{c}_3\pm\sqrt{8+{\bar{c}_3}^2}\right)$$

$$\textit{and } \sigma = \frac{\sqrt{1-\bar{c}_3{}^2}}{2}, \pi_\Gamma = -\frac{\bar{c}_3}{2}.$$

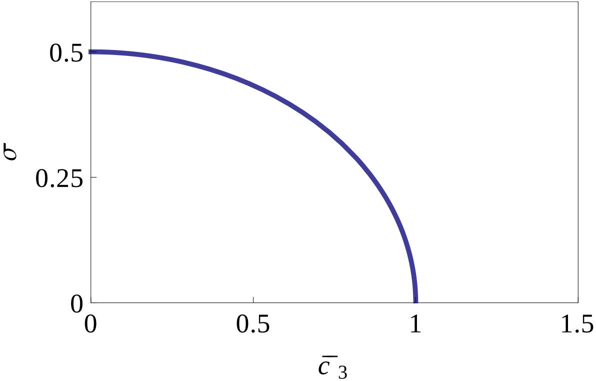

Figure : $\bar{c}_3$ vs σ for Borici-Creutz fermions when m=0 and r=1.

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Gross Neveu Model in 2 dimensions

Multipicity of the free Dirac operator

- The Borici-Creutz action has already been defined previously. In 2D, $\Gamma = \frac{1}{2}(\gamma_1 + \gamma_2)$, $\{\Gamma, \gamma_\mu\} = 1$, and $\Gamma^2 = \frac{1}{2}$.[(2×2) gamma matrices]
- The free Dirac operator in momentum space is written as,

$$D_{BC}(p) = \sum_\mu [i\gamma_\mu \sin p_\mu + i(\Gamma - \gamma_\mu) cos(p_\mu)] + i(c_3 - 2)\Gamma.$$

  - For $c_3 = 0$ and $c_3 = 4$ only one zero of the dirac operator but dispersion becomes unphysical.
  - For $0 < c_3 < 0.59$ and $3.41 < c_3 < 4$ the Dirac operator has only two zeros i.e this is the region of minimal doubling.
  - And for the rest of the region i.e $0.59 < c_3 < 3.41$ the Dirac operator has four zeros. Out of those zeros, we get correct continuum limit of the Dirac operator only when $p_1 = p_2$.

The four fermi interaction I

- The free action (with $r = 1$) is

$$
\begin{aligned}
S_{BC} &= \sum_n [\frac{1}{2} \sum_\mu \bar{\psi}_n \gamma_\mu (\psi_{n+\mu} - \psi_{n-\mu}) \\
&\quad - \frac{i}{2} \sum_\mu \bar{\psi}_n (\Gamma - \gamma_\mu)(2\psi_n - \psi_{n+\mu} - \psi_{n-\mu}) \\
&\quad + i(c_3 - 2)\bar{\psi}_n \Gamma \psi_n + m\bar{\psi}_n \psi_n
\end{aligned}
$$

- After including the four fermi interactions,

$$
S_{BCGN} = \sum_n [S_{BC} - \frac{g^2}{2N}[(\bar{\psi}_n \psi_n)^2 + (\bar{\psi}_n i\Gamma \psi_n)^2]],
$$

The four fermi interaction II

- To linearize the four fermion interactions, we introduce two real auxiliary fields σ and π_Γ:

$$\begin{aligned}\sigma(n) &= m - \frac{g^2}{N}(\bar{\psi}_n \psi_n) \\ \pi_\Gamma(n) &= c_3 - 2 - \frac{g^2}{N}(\bar{\psi}_n i\Gamma \psi_n).\end{aligned}$$

- The action becomes,

$$\begin{aligned}S_{BC} &= \sum_n [\frac{1}{2}\sum_\mu \bar{\psi}_n \gamma_\mu (\psi_{n+\mu} - \psi_{n-\mu}) \\ &+ \frac{i}{2}\sum_\mu \bar{\psi}_n (\Gamma - \gamma_\mu)(\psi_{n+\mu} + \psi_{n-\mu}) \\ &+ \frac{N}{2g^2}[(\sigma(n) - m)^2 + (\pi_\Gamma(n) - c_3 + 2)^2] + \bar{\psi}_n[\sigma(n) + i\Gamma\pi_\Gamma(n)]]\psi_n.\end{aligned}$$

- Then the gap equations are obtained as

$$\frac{(\sigma - m)}{g^2} = \int \frac{d^2k}{(2\pi)^2} \frac{2\sigma}{(\sigma^2 + \frac{\pi_\Gamma^2}{2} + \pi_\Gamma(C+D) + C^2 + D^2)},$$

$$\frac{(\pi_\Gamma - c_3 + 2)}{g^2} = \int \frac{d^2k}{(2\pi)^2} \frac{\pi_\Gamma + (C+D)}{(\sigma^2 + \frac{\pi_\Gamma^2}{2} + \pi_\Gamma(C+D) + C^2 + D^2)};$$

where,

$$C = \sin(k_1) - \frac{1}{2}(\cos(k_1) - \cos(k_2)),$$

$$D = \sin(k_2) - \frac{1}{2}(\cos(k_2) - \cos(k_1)).$$

- For exact chiral structure m=0 and at the chiral boundary σ=0,
- the mass of σ is zero on the critical line which indicates a second order phase transition.

$$m_\sigma \propto V \frac{\delta^2 \bar{S}_{eff}}{\delta\sigma^2}\bigg|_{(c_3)_c} = 0$$

Phase Diagram in parameter space

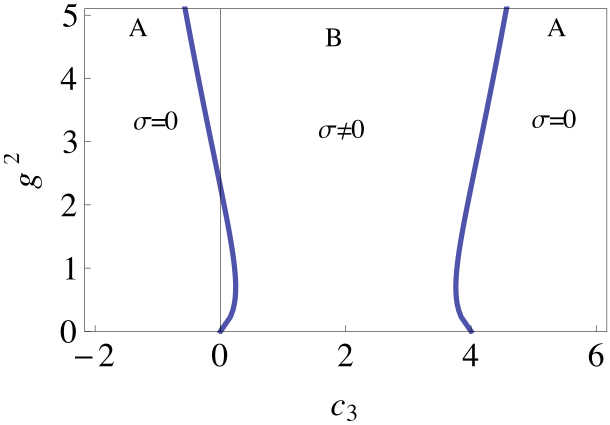

Figure : Chiral boundaries in the parametric space i.e. c_3 vs g^2 for BC fermions

HMC of the model

- For numerical simulation we take $c_3 = 0 + \epsilon$ where $\epsilon = 10^{-5}$
- The lattice version of the action is written as,

$$S = \bar{\psi}_i M_{ij} \psi_j + \frac{N}{2g^2}(\sigma^2 + \pi_\Gamma^2),$$

$$M_{ij} = D_{ij} + \frac{1}{4} \sum_{<x,\tilde{x}>} (\sigma + i\pi_\Gamma \Gamma).$$

- where the auxiliary fields are defined in the dual lattice sites $\tilde{x}$ surrounding the direct lattice site [3] x.

[3] S. J. Hands, A. Kocić, J. B. Kogut, Nucl. Phys. B**390**, 355 (1993), Ann. Phys. **224**, 29 (1993).

HMC of the Model I

- We simulate our model by hybrid monte carlo (HMC) method and evaluate the order parameter for the chiral phase transition $\langle\sigma\rangle$ as a function of coupling constant. We use single noise vector to estimate the condensate.

$$\begin{aligned} \langle\bar{\psi}\psi\rangle &= -\langle TrM^{-1}\rangle \\ \langle\sigma\rangle &= -\beta\langle\bar{\psi}\psi\rangle \\ \text{where } \beta &= \frac{1}{g^2}. \end{aligned}$$

- The configurations are generated by considering stepsize ($\triangle t$=0.1) in the leapfrog method and ten steps per trajectory in the molecular dynamics chain.
  First 500 ensembles are rejected for thermalization and data are collected from next 16000 ensembles.

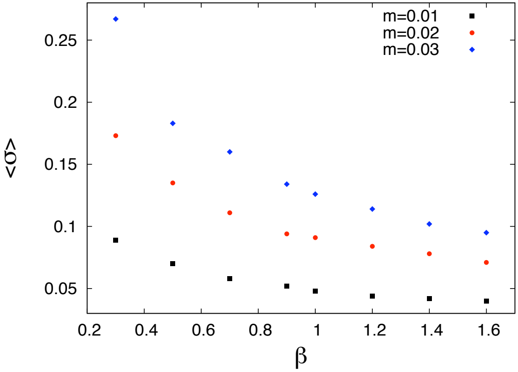

Figure : $\langle\sigma\rangle$ vs β of m=0.01,0.02 & 0.03 for Gross-Neveu model with BC fermions in a 32×32 lattice

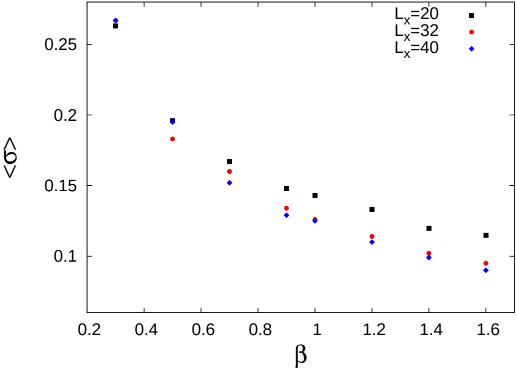

Figure : Finite volume effects of $\langle\sigma\rangle$ vs β for m=0.03 of three different lattice sizes 20×20, 32×32, and 40×40

Summary

- We have studied the Gross-Neveu model with minimally doubled fermion action which has been proposed by Creutz and Borici.
- We have analytically shown a second order phase transition boundary from symmetric to broken chiral phase.
- Then we have studied the model with HMC algorithm. The order parameter $\langle\sigma\rangle$ is plotted against $\beta = 1/g^2$ shows chiral phase transition.


Thank you