CONSTRANT AND INVERSE KINEMATIC ANALYSIS OF 3-PRS PARALLEL MANIPULATOR

YASHAVANT PATEL1*, P M GEORGE2

1 DEPARTMENT OF MECHANICAL ENGINEERING, A D PATEL INSTITUTE OF TECHNOLOGY-388121
NEW VALLABH VIDYANAGAR, GUJARAT, INDIA
yash523@rediffmail.com

2 DEPARTMENT OF MECHANICAL ENGINEERING, BIRLA VISHVAKARMA MAHAVIDYALAYA-388120
VALLABH VIDYANAGAR, GUJARAT, INDIA
pmgeorge02@yahoo.com

Abstract

Parallel manipulators are one family of devices based on closed loop architecture, which is an emerging field in robotics. Closed kinematic structures of parallel manipulators have inherent characteristics of higher structural stiffness, less accumulation of joint errors and enhanced payload capacity. Many potential capabilities of such manipulators over serial one have gained their usage in various fields of applications like precise manufacturing, medical surgery, space technology and many more. The present work addresses analytical generic form of inverse kinematic solution of 3-PRS configuration. In this paper, axially symmetric 3-PRS parallel manipulator configuration with 3-DOF is considered for precise manufacturing applications. There are three identical limbs with only one active joint in each limb support a moving platform and make it three degrees of freedom (DOFs) fully parallel configuration. Mobility analysis is carried out. The equations for position and orientation constraints are also derived for the configuration. The inverse kinematic problem (IKP) is solved using n-independent variable for n-degrees of freedom mechanism. The obtained results are validated for assumed structural parameters with direct kinematics solutions. It is observed that there is a unique solution for a specified pose of an end-effector within workspace due to fully parallel nature of 3-PRS configuration.

Keywords: Parallel manipulator, Inverse kinematics, Constraint equations

1. Introduction

Parallel manipulator applications in field of precise manufacturing are noteworthy in recent years. There are several other fields of applications of such configurations and found in many literature due to its inherent characteristics like higher payload capacity, non-accumulation of joint errors and higher structural rigidity. These manipulators configurations are mainly governed by selection of types, number of joints and sequence of joint arrangement. Normally, forward kinematics is essential for synthesis of a new robotic configuration. Inverse kinematics is imperative for its real applications during shop floor requirements.

Forward and inverse kinematic, dexterity characteristics is investigated and reachable workspace is generated from point clouds in 3D space for three degrees of freedom 3-PRC (Prismatic-Revolute-Cylindrical) parallel manipulator by Yangmin Li and Qingsong Xu (2006). More recently, direct kinematics closed form solution of a 4PUS + 1PS parallel manipulator using dialytic elimination method to solve uni-variate eight degree polynomial and inverse kinematic solutions are also presented for said configuration by G. Abbasnejad et al. (2012). The state of any generic body in a space can be described generally by combining translational and rotational movements. Manipulator kinematics can be studied from two points of view by Ceccarelli (2004). Simulation of 3-RPR, 3-UPS and 3-RPS is carried out to determine the torque requirement at time of machining for single and two links linear actuation simultaneously by Arockia Selvakumaret al. (2010). Kinematically new structure of 3-RRRS parallel manipulator with 6 DOF is investigated using inverse and closed direct kinematic solutions. Nonsingular workspace is also determined for this mechanism by Alon wolf and Daniel Glozman (2011). In this paper, 3-PRS multi-loop
parallel manipulator with three degrees of freedom with ground mounted actuators is considered for its
kinematic investigation.

2 3-PRS configuration using parallel manipulator architecture:

A spatial 3-DOF parallel manipulator is connected to a bottom base platform with three ‘limbs’. Each limb of symmetric parallel configuration consists of one active (prismatic) joint and two passive (revolute and spherical) joints. Hence, the 3-PRS parallel configuration is fully-parallel mechanism as shown in fig. 1. A rotary base is not considered as a joint for kinematic investigation of a manipulator presently but the same is active for work space generation.

![Figure 1 Tripod with 3-PRS configuration with rotary base](image)

Forward kinematics using closed form solutions and workspace generation of 3-PRS configuration is presented by Y D Patel and P M George (2013). There are three identical sub-chains. Each one has three degrees of freedom. Therefore, the number of degrees of freedom for 3-PRS parallel manipulator is computed as,

\[ F_c = \sum_{i=1}^{n} F_i - 6(n - 1) \]

(1)

where, \( F_i = \) Degrees of freedom of \( i^{th} \) joint in a limb

3. Constraint equations

Let \( D = [d_x, d_y, d_z, \phi_x, \phi_y, \phi_z] \) be the vector representation in Cartesian coordinates (constrained and unconstrained) to describe pose of moving platform. The vector representation from origin of fixed coordinate frame at top of rotary base to the intersection of axis of \( i^{th} \) actuator with top of this base is

\[ o_{b_i} = b_i \]

\[ b_1 = \left[ \frac{-p}{2}, \frac{p}{2\sqrt{3}} \right]^T \]

\[ b_2 = \left[ \frac{p}{2}, \frac{p}{2\sqrt{3}} \right]^T \]

\[ b_3 = \left[ 0, \frac{p}{\sqrt{3}} \right]^T \]

(2)

The distance of centre of top of moving platform to centre of spherical joint is \( \bar{O}_1 \bar{S}_1 \) when centre of spherical joints are lying in plane parallel to base platform is represented in column vector as,

\[ a_{1S_1} = \left[ \frac{-q}{2} \right]^T \]

\[ a_{1S_2} = \left[ \frac{q}{2} \right]^T \]

\[ a_{1S_3} = \left[ 0, \frac{q}{\sqrt{3}} \right]^T \]

(3)

Consider \( \hat{n}, \hat{d}, \hat{o}, \hat{a} \) are three unit vectors defined along \( \hat{n}, \hat{d}, \hat{o}, \hat{a} \) axis of the moving platform. The orthogonal rotation matrix for frame orientation for moving platform in terms of direction cosines can be expressed as,

\[ ^oR_{\hat{a}} = \begin{bmatrix} n_x & n_y & n_z \\ O_x & O_y & O_z \end{bmatrix} \]

(4)

The position vector \( \bar{S}_i \) from the origin of fixed frame to centre of \( i^{th} \) spherical joint can be expressed by,

\[ \bar{S}_i = \bar{O} \bar{O}_i + \bar{O}_1 \bar{S}_1 \]

\[ = \hat{d} + \bar{X}_i \]

(5)

where,

\[ \bar{X}_i = ^oR_{\hat{a}} \bar{O}_1 \bar{S}_i \]

(6)

Using equations (3) and (4) and substitute into (5),

\[ \bar{S}_1 = \begin{bmatrix} d_x - \frac{q}{2} n_x - \frac{q}{2\sqrt{3}} O_x \\ d_y - \frac{q}{2} n_y - \frac{q}{2\sqrt{3}} O_y \\ d_z - \frac{q}{2} n_z - \frac{q}{2\sqrt{3}} O_z \end{bmatrix} \]

(7a)
Considering various constraints imposed by different joints during the actuation of an \(i^{th}\) actuator. The centre of a spherical joint represented by \(S_i\) and tool tip can generate a planar curve defined by the plane consists of \(i^{th}\) actuated screw joint axis and the axis of link \(R_iS_i\) as shown in fig.2. The resulting three configuration constraint equations for configuration are,

\[
\begin{align*}
S_{3x} &= 0 \\
S_{2x} &= -\sqrt{3}S_{2y} \\
S_{1x} &= \sqrt{3}S_{1y}
\end{align*}
\]  

(8)

Traced curves

![Figure 2 Traced curves for centre of spherical joint and tool tip for \(i^{th}\) actuation](image)

The obtained results in all above cases satisfy the configuration constraints eq. (8). Using components of \(S_i\) from equation (7) and equation (8) yields,

\[
d_x + \frac{q}{\sqrt{3}}O_x = 0
\]  

(9)

After simplifying the above equation,

\[
\begin{align*}
\frac{q}{\sqrt{3}}O_x &= n_y \\
\frac{q}{\sqrt{3}}O_x &= n_y
\end{align*}
\]  

(12a)

Using equation (10) and (11),

\[
\begin{align*}
-qn_x - 2\sqrt{3}d_y + qO_y &= 0 \\
d_y &= \frac{q}{2\sqrt{3}}(O_y - n_x)
\end{align*}
\]  

(12b)

Using equation (9),

\[
\begin{align*}
d_x &= -\frac{q}{\sqrt{3}}O_x
\end{align*}
\]  

(12c)

Hence, the eq. (12a–c) are the constraints equations for orientation of the proposed manipulator moving platform. The resulting position and orientation constraint for three degrees of freedom 3-PRS parallel mechanism are expressed by equations (8) and (12) respectively. 

**Case study:**

Distance between two recirculating ball screw axis (p): 750 mm, Connecting link length (U): 482 mm, Centre to centre distance between spherical joints (q): 300 mm. Offset distance between prismatic and revolute axis (b): 41 mm. Linear actuation for limb-1 (\(\Delta T_1\)): 20 mm, Initial reference for linear actuation is located 160 mm above the base platform for all limbs as shown in fig.1 for analysis purpose. Actuation of all prismatic actuators is assumed with different velocity. Three screws are linearly actuated by \(\Delta T_1=20\) mm, \(\Delta T_2=60\) mm and \(\Delta T_3=40\) mm from reference.

**Table 1 Spherical joint coordinates for assumed structural parameters**

<table>
<thead>
<tr>
<th></th>
<th>(S_1)</th>
<th>(S_2)</th>
<th>(S_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_{1x})</td>
<td>(-148.73)</td>
<td>(148.574)</td>
<td>(0)</td>
</tr>
<tr>
<td>(S_{1y})</td>
<td>(-85.871)</td>
<td>(-85.779)</td>
<td>(173.979)</td>
</tr>
<tr>
<td>(S_{1z})</td>
<td>(568.725)</td>
<td>(528.631)</td>
<td>(549.867)</td>
</tr>
</tbody>
</table>

Tool position and its orientation should be synchronized to avoid overcut or undercut during
maching or to astray from path while performing any operation on component.

The orientation matrix for end effectors can be expressed as using Y-X-Z Euler angles about fixed reference frame [69,70].

\[ ^0 R_{01} = R_y(\theta)R_z(\psi)R_x(\phi) \]

\[ = \begin{bmatrix}
  \c theta + s theta sphi & s theta + c theta sphi & c theta cphi \\
  s theta cpsi & c theta cpsi & s phi \\
  c theta cpsi & c theta cpsi & -s psi
\end{bmatrix} \] (13)

Note that the trigonometric functions are abbreviated by first letter of the trigonometric functions as \( c \theta = \cos \theta, s \theta = \sin \theta \) and so on. \( R_y(\theta) \) represents rotation matrix about the y axis by an angle \( \theta \). Rotation about z-axis is aligned with axis of symmetry of parallel manipulator architecture, hence the rotation matrix \( R_z(\phi) \) is considered at last. It can be noted from the third column of matrix \([s \theta cpsi, -s \psi, c \theta cphi]\) function of tilt angles \( \theta \) and \( \psi \) only.

A strange phenomenon appears in general 3-RPS, which is a motion in the constrained DOF. 3-RPS mechanism has three DOF with one translational and two rotational DOF about x and y axis of moving platform. When platform rotates about the x-axis, its centre point also has an unwanted displacement along x axis, which is called parasitic motion. This is a simple case of roller rolling on ground. Actually, body is not translating or rotating about a particular axis. But this motion belongs to category of special rotation, in which moving axodes rolls along a fixed axodes without sliding. Every point in the rolling body forms a different locus. From this point of view, so it is called parasitic motion as highlighted by Qin Chuan Li, Jacques Marie Hervé. Similarly, same phenomenon is also observed in 3-PRS parallel manipulator. Thus, three motions (one translational and two rotations) are at the expense of the three parasitic motions which are translations in x and y axes and a rotation about the z-axis. Using matrix as presented in (13) and constraint equations (12),

\[ O_x = n_y \]
\[ s \theta cpsi + c \theta sphi = cpsi cphi \]
\[ tan(\phi) = \frac{s \theta cphi}{c \theta + cphi} \] (14a)

\[ d_y = -\frac{q}{2\sqrt{3}}(O_y - n_x) \]
\[ d_y = \frac{1}{4\sqrt{3}}(cpsi cphi - c \theta cphi) \]
\[ d_x = -\frac{q}{\sqrt{3}}O_x \] (14b)

Three constraint equations (14) that will give the constrained variables of moving platform \((d_x, d_y, \phi)\) as functions of the unconstrained variables \((d_x, \theta, \psi)\) must be obtained. It is also observed that above constrained equations are independent of variable \(d_z\). At home position, the value of \( \phi \) is equal to zero. \( \phi \) parasitic motion is function of \( \theta, \psi \) only, while \( d_x, d_y \) parasitic motion is function of \( q, \theta, \psi \)

4. Inverse Kinematics

Placement of tool frame at required position and orientation in space within workspace and computation of joint parameters to achieve required tool frame in workspace is known as inverse kinematics problem formulation. There can be multiple solutions in such case. Sometimes, the resulting solutions may violate existence of configurations. Hence, constraints required to be impose for a viable solution to meet the requirement.

\[ n \cdot (S - O_t) = 0 \] (15)

The equation of sphere passing through three centre point of spherical joints, radius \( R_m \) of moving platform and centre of sphere is \( O_t(x_0, y_0, z_0) \) is expressed as,

\[ (S_{ix} - x_0)^2 + (S_{iy} - y_0)^2 + (S_{iz} - z_0)^2 - R_m^2 = 0 \] where, \( i = 1, 2, 3 \) (16)

Normal to moving platform is determined using equation (15) for a given tool tip coordinates (tip),
In matrix form, the spherical joint centre coordinates and constraint equations as shown in fig. 4.

\[
\begin{bmatrix}
S_{1x} & S_{2x} & S_{3x} \\
S_{1y} & S_{2y} & S_{3y} \\
S_{1z} & S_{2z} & S_{3z}
\end{bmatrix}
\]

\[1\]

The elements of above matrix can be obtained using forward kinematic formulation,

\[
\begin{align*}
S_{1x} &= -\frac{q}{2} + \frac{\sqrt{3}}{2} b + \frac{\sqrt{3}}{2} U\cos\theta_1 \\
S_{1y} &= -\frac{q}{2} + \frac{\sqrt{3}}{2} b + \frac{\sqrt{3}}{2} U\cos\theta_1 \\
S_{1z} &= T_1 + U\sin\theta_1 \\
S_{2x} &= \frac{q}{2} - \frac{\sqrt{3}}{2} b - \frac{\sqrt{3}}{2} U\cos\theta_1 \\
S_{2y} &= \frac{q}{2} - \frac{\sqrt{3}}{2} b - \frac{\sqrt{3}}{2} U\cos\theta_1 \\
S_{2z} &= T_2 + U\sin\theta_2 \\
S_{3x} &= 0 \\
S_{3y} &= \frac{q}{\sqrt{3}} - b - U\cos\theta_3 \\
S_{3z} &= T_3 + U\sin\theta_3
\end{align*}
\]

The tilt angles of connecting links are determined from above three equations and can be cross verified for at least \(\theta_1\) and \(\theta_2\).

\[
\begin{align*}
\cos\theta_1 &= \frac{1}{U} \left(2y_1 + \frac{q}{\sqrt{3}} - b\right) \\
\cos\theta_2 &= \frac{1}{U} \left(2y_2 + \frac{q}{\sqrt{3}} - b\right) \\
\cos\theta_3 &= \frac{1}{U} \left(-y_3 + \frac{q}{\sqrt{3}} - b\right)
\end{align*}
\]

(18)

The required actuation for known spherical joints centre points coordinates can be computed using equations (18) and (19) as discussed earlier. The above formulation is applicable for all cases and validated for same cases as under. Moreover, the above results can be cross verified using,

\[
\begin{align*}
OO_1 &= \frac{1}{3} \sum_{i=1}^{3} (z_i + T_i) \\
&= \frac{1}{3} \left((z_1 + T_1) + (z_2 + T_2) + (z_3 + T_3)\right)
\end{align*}
\]

(20)

Case study: Same structural parameters are considered as previous case.

\[
\overline{OO_1} = (MCP_x, MCP_y, MCP_z) = (0.01047, -0.005926, 581.9676)
\]

Tip coordinates: \((t_{ip_x}, t_{ip_y}, t_{ip_z})\)

\[
\begin{align*}
\overline{t_i} &= (-1.45864, -2.5257, 756.943) \\
\overline{\beta} &= -1.4691 \hat{i} - 2.5198 \hat{j} + 174.9760 \hat{k} \\
|\overline{\beta}| &= 175
\end{align*}
\]

Normalized \(\overline{\beta} = -0.0084 \hat{i} - 0.0144 \hat{j} + 0.9999 \hat{k}\)

Analytically, three spherical joints centre point coordinates are obtained using equations (8, 15, 16).

\[
\begin{align*}
s_1 &= (-149.979, -86.5904, 579.4608) \\
s_2 &= (150.0104, -86.6086, 581.9793) \\
s_3 &= (0, 173.1812, 584.461)
\end{align*}
\]

Table 2: Joint parameters for assumed structural parameters

| \(\theta_1\) | \(62.9989^\circ\) | Required displacement using linear actuation: \(T_1 = 10\) mm |
| \(\theta_2\) | \(63.0037^\circ\) | Required displacement using linear actuation: \(T_2 = 7.5\) mm |
| \(\theta_3\) | \(62.9989^\circ\) | Required displacement using linear actuation: \(T_3 = 5\) mm |
5. Conclusion

Constraint equations of 3-PRS parallel manipulator configuration is derived. The inverse kinematic problem (IKP) is solved using n-independent variable for n-degrees of freedom mechanism. Inverse kinematic solution is obtained as an intersection of sphere and plane passing through centre point of spherical joints coordinates $S_i(x_i, y_i, z_i)$, and using position constraints as shown in fig. 4. Inverse kinematic problem solution is also presented in generic form and required prismatic joints actuation is validated for the given pose of forward kinematic solution. It is observed that a unique solution for a specified pose of an end-effector within workspace as configuration is a fully parallel manipulator. The present work is useful for trajectory planning of complex geometry on inclined plane surfaces within work volume of this parallel configuration.

References:


