APPLICATION OF GREY RELATIONAL ANALYSIS FOR GEOMETRICAL CHARACTERISTICS IN ABRASIVE WATER JET MILLED CHANNELS

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Abstract

In this study, the effects of process parameters on the geometrical characteristics and the topography were investigated in milling of SS304 material using Abrasive Water Jet machining technique. In addition, optimal control of the input variables for achieving the quality of the channel was determined by using Grey relational analysis. The input process parameters include traverse speed, abrasive flow rate; abrasive size and standoff distance each at three levels are considered for experimentation. Grey relational analysis was employed to minimize and maximize the response parameters as per the requirement. Parameters like surface roughness, taper, impact force, vibration are minimized; depth is maximized and the width of cut is kept optimum which is the inside diameter of the focusing tube. Based on the grey coefficients and grades of the experimental data, a traverse speed of 3000 mm/min, a diameter of 0.125 mm of abrasive particle at 0.49 kg/min abrasive flow rate and a standoff distance of 4mm gives an optimum machining conditions for the required output.

Keywords: grey relational analysis, depth, taper, width, surface roughness

1 Introduction

Abrasive water jet machining (AWJM) is a unique unconventional machining process and a state of the art technology in the current manufacturing industry possessing wide variety of applications and advantages for machining hard to cut materials. One of the major advantages of the process is minimum heat affected zone which allows the structural characteristics of the specimen to remain unaltered even after machining using AWJM. However, the process of AWJM is complex phenomenon influenced by a large set of process parameters. The fundamental mechanism of material removal in AWJM is erosion as reported by Finnie [1960], which is based on material removal due to impact of a highly energized jet of high pressure water mixed with abrasive particles onto the surface. In this process, the entire material removal mechanism is divided into two modes, cutting wear mode which happens at higher jet impact angles and deformation wear mode occurring at shallow angles as specified by Bitter [1963a, 1963b], Guo [1994] and Arola et al., [1996] characterized the geometry of a machined specimen by AWJM are top and bottom width of cut, taper, initial damage zone consisting of depth, taper, etc. Literature reveals that the geometrical parameters are affected by majority of the process parameters while machining ceramics, Hocheng and Chang [1994], glass, Geskin et al., [1995], metal matrix composites, Hashish [1991], etc. The process of AWJM finds numerous applications in cutting (part separation) where the process has been well understood and published in the last few decades.

Milling applications with AWJM technique is gaining importance in processing hard materials in industry. The literature on AWJ milling is adequately published as on date and a paradigm shift is observed in partial material removal instead of cutting using this process. AWJ milling will stand as a base for the creation of 3D shape generation. The role of process parameters on the geometrical characteristics of the channel on SS304 material by Gupta et al., [2013], elaborates the influence of input parameters on the response values. Initial experiments by Gupta et al., [2013] on the creation of milled pockets on different materials have shown the deviations in the jet is influenced by traverse speed and the depth is highly influenced by the resistance offered by the work piece, governed by the material hardness.

The present work is an attempt to study the geometrical structure of the milled channels and to
optimize the process parameters for a given set of target parameters for obtaining the required geometrical characteristics of the channel.

2 Experimental details

In the present investigations, experiments are conducted to fabricate micro-channels on SS304, which is commonly used material in many of the manufacturing industries. The setup consisting of the test material mounted on a force dynamometer is placed on an abrasive water jet machining center (OMAX 2626, USA) for experimentation. The machine is equipped with a 3-stage plunger type pump which can generate pressures up to 345 MPa. A magnetic type accelerometer is placed on the mixing chamber assembly to capture the signals of the jet vibrations. The impact force influence the depth of the channel and the jet vibration affects the width and taper of the cut.

The ranges of input parameters at different levels with corresponding values are mentioned in Table 1. Considering each parameter at 3-levels, a full factorial experimental design set is developed using MINITAB software. This has led to a total of 81 experiments (3^4, 4 parameters at 3 levels). All the trials are conducted on SS304 material to fabricate the required micro-channels. Figure 1 shows a typical plate on which micro-channels are fabricated, and the channel geometry schematic is shown in Figure 2. The cut section of the fabricated channels is shown in Figure 3. The geometrical characteristics of the channel are width, depth, taper and surface roughness. A typical input process parameters set is shown in Table 2.

<table>
<thead>
<tr>
<th>Process parameter</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traverse speed (mm/min)</td>
<td>3000</td>
<td>3500</td>
<td>4000</td>
</tr>
<tr>
<td>Abrasive flow rate (kg/min)</td>
<td>0.27</td>
<td>0.38</td>
<td>0.49</td>
</tr>
<tr>
<td>Standoff distance (mm)</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Abrasive mesh size (#) (mm)</td>
<td>80 (0.177)</td>
<td>120 (0.125)</td>
<td>160 (0.110)</td>
</tr>
</tbody>
</table>

2.1 Measurements

The width of the channel measured is the top width while depth considered is the maximum distance measured top surface to the bottom point of the channel. The channel image is imported in standard CAD software and the taper is measured. The surface roughness of the channel is measured using RUGOSURF surface analyzer instrument. The sample surface roughness measured with the specified instrument is shown in Figure 4. A typical experimental response data is given in Table 3.

<table>
<thead>
<tr>
<th>Exp. No</th>
<th>Width (µ)</th>
<th>Depth (µ)</th>
<th>Taper (Deg)</th>
<th>Ra (µ)</th>
<th>F (N)</th>
<th>Vib. (Acc., g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>785</td>
<td>225.91</td>
<td>58.90</td>
<td>3.61</td>
<td>10.33</td>
<td>0.000552</td>
</tr>
<tr>
<td>2</td>
<td>753</td>
<td>183.55</td>
<td>40.22</td>
<td>1.26</td>
<td>11.60</td>
<td>0.000382</td>
</tr>
</tbody>
</table>
3 Grey Relational Analysis Methodology

Grey theory is a simple and accurate method for multiple attribute decision problems. Grey is a color which is a blend of black and white colors; black is represented as a lack of information while white is full of information. Thus, the system having incomplete information is known as Grey system. In Grey analysis, the information from the Grey system is dynamically compared to each factor quantitatively. This approach is based on the level of similarity and variability among all factors to establish their relation. Such kind of interaction is mainly through the connection among machining parameters and the operating conditions that are known.

In grey analysis, optimization is carried out in different steps. Firstly, the data is pre-processed where the original sequence is transferred to a comparable sequence. This is required because the range, unit and objective of the data sequence (for a response) vary between each other. Data normalization in the range between zero and one is performed using the method of linear data pre-processing. Based on the response characteristics, it is carried in 3 different types as follows:

(a) Higher the better (HB)

\[ x_i^{(k)} = \frac{x_i^{(0)} - \min x_i^{(0)}(k)}{\max x_i^{(0)}(k) - \min x_i^{(0)}(k)} \]

(b) Lower the better (LB)

\[ x_i^{(k)} = \frac{\max x_i^{(0)}(k) - x_i^{(0)}(k)}{\max x_i^{(0)}(k) - \min x_i^{(0)}(k)} \]

(c) Desired value

\[ x_i^{(k)} = 1 - \frac{|x_i^{(0)}(k) - x_i^{(0)}|}{\max x_i^{(0)}(k) - x_i^{(0)}} \]

where \( x_i^{(k)} \) is the generating value, \( \min x_i^{(0)}(k) \) is the minimum value of \( x_i^{(0)}(k) \) and \( \max x_i^{(0)}(k) \) is the maximum value of \( x_i^{(0)}(k) \) and \( x_i^{(k)} \) is the desired value.

The grey coefficients from the normalized experimental data are calculated as per the standard procedure. The grade of the grey analysis is then calculated by averaging the grey relational coefficients corresponding to each performance characteristics. The term Grey relational grade, \( \Gamma \) is used to show the connection among original and comparative series. The procedure for calculating grey relational grade is given by Huang and Liao (2003). For a Grey relational space \( (X, \Gamma) \) with \( N \) collection of Grey relational factors, the comparative series, \( x_i^{(k)} \) and reference series, \( x_0^{(k)} \) are

\[ x_0^{(k)} = x_0(1), x_0(2), x_0(3), x_0(4), \ldots, x_0(n) \]
\[ x_i^{(k)} = x_i(1), x_i(2), x_i(3), x_i(4), \ldots, x_i(n) \cdot X \]

where \( f = 1, 2, \ldots, n \)

The Grey relational grade can be calculated by the following equation. \( \Delta_{\min} \), \( \Delta_{\max} \) are the minimum and maximum value among all the \( \Delta_{ij} \) values respectively. \( \Delta_{ij}(k) \) is the absolute value of difference between \( x_0 \) and \( x_i \) at the \( \text{k}^{th} \) point.

\[ \Gamma_{i0} = \frac{\Delta_{\min} + \Delta_{\max}}{\Delta_i + \Delta_{\max}} \]

where,

(1) \( \Delta = \frac{\Delta_{ij}(k)}{\Delta_{\max}} \)
(2) \( x_0(k) \): reference sequence, \( x_i(k) \): comparative sequences
(3) \( \Delta_{ij}(k) = |x_0(k) - x_i(k)| \)
(4) \( \Delta_{\min} = \min_{i,j} \max_k |x_0(k) - x_i(k)| \)
(5) \( \Delta_{\max} = \max_{i,j} \min_k |x_0(k) - x_i(k)| \)

\[ \Delta_i = \sum_{k=1}^{n} \frac{\Delta_{ij}(k)}{n} \]

The grade in Grey analysis is used to evaluate the experimental data for multi-response characteristics. The highest grade corresponds to the optimal level of experimental set where optimization of any complicated multiple performance characteristics can be converted into a single Grey relational grade.
3.1 Grey Relational Analysis for channel geometry

In the present work, Grey Relational Analysis has been carried on the response parameters to characterize the geometry of the micro-channel. The experimental response data in Table 3 is considered to perform the grey analysis for the multiple attribute decision problems for evaluation. The multiple characteristics of the process considered for the analysis are surface roughness, depth, taper, width, force and vibration of the jet. Basic requirements in fabrication of micro-channels are minimum surface roughness, maximum depth, minimum taper, and width equivalent to jet diameter, minimum impact force and minimum jet vibration to achieve the best process performance. Table 4 gives the grade relational coefficients for all the response parameters and the grade specified is the average of all grey relational coefficients of the concerned run. The maximum value of the grey relational grade gives the Rank of the experimental set.

Table 4 Grey relational coefficients, Grade and Rank for the micro-channels (partial set)

<table>
<thead>
<tr>
<th>Exp. No</th>
<th>Width</th>
<th>Depth</th>
<th>Taper</th>
<th>Ra</th>
<th>F</th>
<th>Vib.</th>
<th>Grade</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.842</td>
<td>0.607</td>
<td>0.627</td>
<td>0.333</td>
<td>0.549</td>
<td>0.516</td>
<td>0.579</td>
<td>34</td>
</tr>
<tr>
<td>2</td>
<td>0.671</td>
<td>0.533</td>
<td>0.510</td>
<td>0.884</td>
<td>0.418</td>
<td>0.806</td>
<td>0.637</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>0.529</td>
<td>0.552</td>
<td>0.420</td>
<td>1.000</td>
<td>0.416</td>
<td>0.903</td>
<td>0.637</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>0.465</td>
<td>0.430</td>
<td>0.431</td>
<td>0.423</td>
<td>0.531</td>
<td>0.532</td>
<td>0.469</td>
<td>81</td>
</tr>
<tr>
<td>5</td>
<td>0.588</td>
<td>0.430</td>
<td>0.485</td>
<td>0.454</td>
<td>0.480</td>
<td>0.518</td>
<td>0.493</td>
<td>76</td>
</tr>
<tr>
<td>6</td>
<td>0.625</td>
<td>0.607</td>
<td>0.512</td>
<td>0.445</td>
<td>0.546</td>
<td>0.533</td>
<td>0.545</td>
<td>54</td>
</tr>
<tr>
<td>7</td>
<td>0.444</td>
<td>0.568</td>
<td>0.539</td>
<td>0.510</td>
<td>0.396</td>
<td>0.760</td>
<td>0.536</td>
<td>62</td>
</tr>
<tr>
<td>8</td>
<td>0.613</td>
<td>0.415</td>
<td>0.613</td>
<td>0.377</td>
<td>0.477</td>
<td>0.399</td>
<td>0.482</td>
<td>80</td>
</tr>
<tr>
<td>9</td>
<td>0.513</td>
<td>0.333</td>
<td>0.717</td>
<td>0.693</td>
<td>0.571</td>
<td>0.517</td>
<td>0.557</td>
<td>45</td>
</tr>
<tr>
<td>10</td>
<td>0.520</td>
<td>0.607</td>
<td>0.518</td>
<td>0.595</td>
<td>0.486</td>
<td>0.520</td>
<td>0.541</td>
<td>56</td>
</tr>
<tr>
<td>11</td>
<td>0.492</td>
<td>0.472</td>
<td>0.510</td>
<td>0.589</td>
<td>0.530</td>
<td>1.000</td>
<td>0.599</td>
<td>24</td>
</tr>
<tr>
<td>12</td>
<td>0.567</td>
<td>0.687</td>
<td>0.539</td>
<td>0.885</td>
<td>1.000</td>
<td>0.883</td>
<td>0.760</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>0.719</td>
<td>0.447</td>
<td>1.000</td>
<td>0.388</td>
<td>0.464</td>
<td>0.562</td>
<td>0.597</td>
<td>25</td>
</tr>
<tr>
<td>14</td>
<td>0.684</td>
<td>0.415</td>
<td>0.403</td>
<td>0.659</td>
<td>0.467</td>
<td>0.522</td>
<td>0.525</td>
<td>67</td>
</tr>
<tr>
<td>15</td>
<td>0.499</td>
<td>0.482</td>
<td>0.470</td>
<td>0.636</td>
<td>0.478</td>
<td>0.484</td>
<td>0.508</td>
<td>72</td>
</tr>
<tr>
<td>16</td>
<td>0.430</td>
<td>1.000</td>
<td>0.490</td>
<td>0.730</td>
<td>0.400</td>
<td>0.753</td>
<td>0.634</td>
<td>7</td>
</tr>
</tbody>
</table>

4 Results and discussion

According to the grey relational grades from Table 4, experiment no. 12 has the highest grey relational grade. Thus, experiment no. 12 gives the best performance characteristics in the present experimental set in fabricating the channel. The corresponding input parameters are the optimum set of process parameters. The response graph for the mean grey relational grades at different levels of the input parameters is shown in Figure 5. The graph indicates the optimal factorial sets like traverse speed (level 1), AFR (level 3), abrasive size (level 3) and SOD (level 2) i.e., the combination of 3000 mm/min traverse speed, 0.49 kg/min AFR with # 160 mesh abrasive size (0.125mm) at SOD of 4 mm gives the best possible response parametric set. The graph reveals that abrasive flow rate has minimum influence on the channel geometry.

Table 5 gives the average response table for the grey relational grade values. Delta is the difference between the maximum and minimum grade values of the each input parameter and the significance row gives the order of influence of the process parameter. The results obtained through grey relational analysis are in agreement with the results obtained from ANOVA analysis.

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![Figure 5 Response graph for the mean grey relational grades](image)

### Table 5 Average Grey Relational Grade for Factor and Levels

<table>
<thead>
<tr>
<th>Levels</th>
<th>v (mm/min)</th>
<th>k (g/min)</th>
<th>d (mm)</th>
<th>h (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5878</td>
<td>0.5705</td>
<td>0.5292</td>
<td>0.5565</td>
</tr>
<tr>
<td>2</td>
<td>0.5619</td>
<td>0.5700</td>
<td>0.5874</td>
<td>0.5844</td>
</tr>
<tr>
<td>3</td>
<td>0.5618</td>
<td>0.5710</td>
<td>0.5959</td>
<td>0.5706</td>
</tr>
</tbody>
</table>

Delta 0.0026 0.0010 0.0667 0.0279

Significance 3 4 1 2

### Conclusions:

The work presented in the paper is an attempt to understand the process of milling applications with abrasive water jet machining technique. The use of grey analysis aided us to obtain the optimum process parameters to characterize the channel geometry which gave us an insight of the significance of the process parameters selected in the present experimental range. The abrasive size is a crucial parameter followed by standoff distance for getting a channel of the required geometry. The average grey relational grade gives the order of the influencing parameters which are in conjunction with the statistical ANOVA results.

### References:

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