Dynamic Stability of High Speed Micromilling Based on Modal Analysis for Determining the Tool-tip Dynamics

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Abstract

Micromilling process is used for fabrication and manufacturing of miniaturized components. Very high spindle rotational speeds are required in micromilling to reduce the chip load and to counter the low flexural rigidity of the micro-tool while machining hard materials. Apart from the high rotational speed (>100,000 rpm) which can excite dynamic instability, the dynamic force variation in micromachining can also occur due to micro-machine tool system limitations (limited tool stiffness and misalignments), micro scale cutting mechanics (critical chip thickness and size effect) and material inhomogeneity. The dynamic instability can induce surface/form errors and can result in catastrophic tool failure. This paper is focused on developing a two-degree of freedom model of the micromilling process for predicting chatter via stability lobe diagram. The dynamics of the cutting tool has been predicted by finite element analysis. After prediction of tool tip dynamics, the cutting coefficient has been determined experimentally. Finally, the dynamic stability has been predicted after considering the regenerative effect. Experimental verification of predicted stability shows the good agreement between analytical and experimental chatter free depth of cut and cutting speed. Hence, modal analysis by FEM can efficiently be used for determination of the tool tip dynamics. The predicted stability lobe diagram can be used for selection of chatter free combination of depth of cut and cutting speed prior to machining of Ti6Al4V.

Keywords: Micromilling, Chatter modeling, Modal analysis of micro end mill, Stability lobes

1. Introduction

The miniature components comprising of variety of materials can be manufactured by micromilling process. The application of these miniature components includes biomedical, optics, electronics, aerospace and sensor industries [1, 2]. Complex 3D fabrication with miniature parts consisting of variety of engineering material and alloy can be carried out by micromilling machining [3, 4]. Due to small tool diameters, the flexural rigidity of the tool is very low which cannot sustain the high chip load. Hence high rotational speed is employed to reduce the chip load. However, high speed induces additional challenges with respect to dynamic stability. Even a small fluctuation in the cutting forces and run-out can render the process unstable. In addition, the edge radius of micro end mill used in micromilling operation is comparable to the uncut chip thickness which affects the cutting mechanics leading to dynamic force variation which can give rise to chatter. This chatter can damage the machined surface and results in accelerated wear or catastrophic failure of the miniature end mill [5].

Chatter occurs due to the regenerative phenomena in the chip thickness because of the waviness in the machined surface. The waviness is due to the phase difference between the present and previous tooth pass and take place due to the variation in the cutting forces. Further, due to low uncut chip thickness in micromilling operation, no chip forms below the critical chip thickness in micromilling [6]. At such low uncut chip thickness, only ploughing and plastic deformation of the workpiece takes place. This critical chip thickness criteria and elastic recovery may result in intermittent chip formation and can induce instability. Also, the low feed/rev results in smaller uncut chip thickness where the size effect plays a role due to which the specific cutting energy increases with a decrease in uncut chip thickness. Furthermore, the misalignment and run-out coupled with very high rotational speeds can also affect the chip load per flute which can excite process instability [7]. As noted above, micromilling is especially prone to dynamic instability, and if the cutting tool and the workpiece are not stiff enough to sustain the variation in the cutting forces, chatter will be induced.

It may be noted that for accurate prediction of chatter stability, the accurate assessment of the cutting coefficients and tool tip dynamics are imperative [4]. The FRF at tool tip dynamics can be calculated directly
by the impact testing as done for the macro milling but it is not applicable for micro mills [2]. Typical natural frequency of the micro-mills may reach beyond 100 KHz where the impact test cannot be performed because of the limited excitation frequency range 10 KHz of impact hammer based modal test [1]. Schmitz et al. [8] carried out the receptance coupling substructure analysis for analytical prediction of the tool point response by dividing the tool into two parts. The first part consisted of a holder/spindle and the second part consists of the end mill and a portion of the tool shank. The frequency response function (FRF) of the first part is determined by the experimental modal test and the FRF for second part is determined by finite model analysis after considering the free-free beam with appropriate cross-sectional profile. Rahnama et al. [4] predicted the tool tip dynamics by receptance coupling method by indirectly coupling the machine tool dynamics with arbitrary tool dynamics. They divided cutting tool into two different substructures: the first substructure comprised of the lower portion of the micro end mill tool while the second substructure consisted of the remainder of the system. The dynamics of the tool tip was obtained from the finite element analysis by considering the tool tip as a Timoshenko beam and the second substructure was experimentally determined via modal analysis. The dynamics of the two substructures were coupled by taking the rigid joint between the two sub-structures to identify the tool tip dynamics of micro end mill. Jin and Altintas [1] used a piezo actuator to excite the micro tool to obtain the tool dynamics. They measured the vibration at both shank and flute tip by a laser Doppler vibrometer. Afazov et al. [9] obtained the tool dynamics by substructure receptance coupling method by using the peak picking and steepest descent fitting methods. They compared the stability lobes for modal dynamics parameters obtained by the both these methods.

Cutting coefficient prediction is necessary for accurate prediction of stability region. Here, cutting coefficient can be computed by the cutting force which involves the two main material deformation mechanisms: shearing and ploughing. Mechanistic model assumes that shearing force is proportional to uncut chip area and ploughing force is proportional to the contact length between the cutting edge and the machine surface [4]. Hence, separation of cutting zone and ploughing zone is required to determine the critical tip thickness. After getting the critical chip thickness, the cutting coefficients can be calculated. In the present work, a two-degree of freedom model of micromilling process has been used to predict the process stability. The tool tip dynamics has been obtained in form of frequency response function by finite element based modal analysis. A two-degree of freedom model of micromilling has been developed in Section 2. Experiments have been conducted to determine the cutting coefficients. The tool tip dynamics via FEM modal analysis has been presented in Section 3. Finally, stability lobe diagram in frequency domain has been created and validated experimentally during the micromilling of Ti6Al4V.

2. Micromilling process model:

Micromilling process can be modeled as a two-DOF massspringdamper vibrating system in two orthogonal directions as shown in Fig. 1. The tangential and radial forces at the tool-tip are given as

$$ F_{tx} = K_t a h(\vartheta), \quad F_{rf} = K_r F_{tx} $$

where $K_t$ and $K_r$ are the tangential cutting coefficient and constant respectively. Instantaneous chip thickness $h(\vartheta)$ is the dynamic chip thickness due to regeneration effect and $a$ is the axial depth of cut.

Fig. 1 Micromilling process model

The characteristics equation of the milling process is based on the approach given by Altintas and Budak [10]. The characteristics equation of the closed loop dynamic milling system is given by:

$$ \text{det}(I - \frac{1}{2} K_t a (1 - e^{-i\omega_c T}) [A_0][\Phi(i\omega_c)]) = 0 \quad (1) $$

where $K_t$ is the tangential cutting coefficient, $a$ is the axial depth of cut, $\omega_c$ is the chatter frequency, $\omega_c T$ is the phase difference between the vibration at successive tooth periods $T$, $A_0$ is the time invariant but immersion dependent directional cutting coefficient matrix, which is expressed as:

$$ [A_0] = \frac{N}{2\pi} \begin{bmatrix} a_{xx} & a_{xy} \\ a_{yx} & a_{yy} \end{bmatrix} (2) $$

where, $a_{xx} = \frac{1}{a_{xx}} \{\cos2\vartheta - 2K_s \vartheta + K_s \sin2\vartheta\}^{a_{xx}} \quad (3)$
expression for chatter free depth of cut comes to be

\[ \alpha_{xy} = \frac{1}{2} [-\sin \theta - \Delta \theta + K_r \cos \theta] \theta^2 \]

\[ \alpha_{yx} = \frac{1}{2} [-\sin \theta + \Delta \theta + K_r \cos \theta] \theta^2 \]

\[ \alpha_{yy} = \frac{1}{2} [-\cos \theta - 2K_r \phi - K_r \sin \theta] \theta^2 \]

where, \( \theta \) is the immersion angle of the flute, measured clockwise from the y axis, \( \theta_{st} \) and \( \theta_{ex} \) are the entry and exit angle of teeth relative to the workpiece, \( N \) is the total number of flute of the cutting tool. \( \phi \) is the phase shift extracted by frequency extraction method in ABAQUS®. After extraction of frequency, dynamic modal analysis by applying a harmonic sinusoidal load at tool tip is carried out. The harmonic sinusoidal load has been applied as a function of frequency. Note that, the harmonic sinusoidal load depends on the extracted mode shape and the Eigen frequency of the structure.

The equation of motion for the Eigen mode \( \alpha \) is given by,

\[ \ddot{q} + c_\alpha \dot{q} + \omega^2 q = \frac{1}{m} (f_{1\alpha} + i f_{2\alpha}) \exp(i\Omega t) \]

where, \( q \), \( c \), and \( \omega \) are the amplitude, damping and undamped natural frequency, respectively, for Eigen mode \( \alpha \). \( m \) is the generalized mass of mode. The term \((f_{1\alpha} + i f_{2\alpha})\exp(i\Omega t)\) is the harmonic excitation given to the system. \( F_0 \sin \omega t \) is the amplitude of the force. The real and imaginary parts of the transfer function at tool tip are shown in Fig. 3 and Fig. 4, respectively.

\[ \Phi(i\omega_c) = \begin{bmatrix} \Phi_{xx}(i\omega) \\ \Phi_{xy}(i\omega) \\ \Phi_{yx}(i\omega) \\ \Phi_{yy}(i\omega) \end{bmatrix} \]

where, \( \Phi_{xx}(i\omega) \) and \( \Phi_{xy}(i\omega) \) are the direct transfer function in x and y direction, and \( \Phi_{yx}(i\omega) \) and \( \Phi_{yy}(i\omega) \) are the cross transfer function at the tool tip. These transfer function at the tool tip is obtained by FEM based modal analysis in the present work. These transfer function at tool tip is expressed as

\[ \Phi_{xx}(i\omega) = \sum_{i=1}^{n} K_{x,i}(\omega^2 - 2\xi_{x,i}\omega_{n,x} - \omega^2) \]

\[ \Phi_{yy}(i\omega) = \sum_{i=1}^{n} K_{y,i}(\omega^2 - 2\xi_{y,i}\omega_{n,y} - \omega^2) \]

where, \( \omega_{n,x,y,i} \), \( K_{x,y,i} \), and \( \xi_{x,y,i} \) are the natural frequency, stiffness and damping factor respectively of the micro end mill for mode 1 to mode \( \alpha \).

The eigenvalue is defined as:

\[ \lambda = \frac{N}{4\pi} \alpha K_r (1 - e^{-i\alpha T}) \]

Now \( k = \frac{s}{\lambda} \), \( \sin \omega_{n,x} - \cos \omega_{n,y} = \tan \psi \), where \( \psi \) is phase shift of the eigenvalues. Tooth period \( T \) is given by \( T = \frac{e^{2k\pi}}{\omega_c} \) and \( e = \pi - 2\psi \). Hence the spindle speed is given by;

\[ \omega_s = \frac{60}{N T} \]

After substituting the value of \( k \) into eq. (10), the expression for chatter free depth of cut comes to be

\[ a_{lim} = -\frac{2\pi \omega_{n,R}}{NK_t} (1 + k^2) \]

Hence, stability lobe diagram as a function of chatter free depth of cut and spindle speed can be created.

2. Tool tip dynamics

Tool tip dynamics is required to determine the natural frequency, damping ratio, and stiffness of the cutting tool. Further, tool tip dynamics is determined from the frequency response function at the tool tip. In the present work, FRF at the tool tip is obtained by FEM modal analysis. Here, the tool is assumed to be solid beam element of non-uniform diameter. The solid model of tool is shown in Fig. 2.

Fig. 2 Solid model of micro end mill

The shank diameter of the tool is 3 mm while the length of tool is kept at 24 mm, which is the overhang length. The tool tip is assumed as equivalent cylinder with diameter as 68% of the micro end mill diameter. The properties of the tungsten carbide tool is taken as density=14.300 kg/m³, young’s modulus=580 GPa and the Poisson’s ratio=0.28 and damping ratio of 1% [4]. The beam is taken as cantilever beam. \( \omega_0 \) is the natural frequency is extracted by frequency extraction method in ABAQUS®. After extraction of frequency, dynamic modal analysis by applying a harmonic sinusoidal load at tool tip is carried out. The harmonic sinusoidal load has been applied as a function of frequency. Note that, the harmonic sinusoidal load depends on the extracted mode shape and the Eigen frequency of the structure.

Fig. 3 Real part of transfer function

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It is driven by electric motor having a variable frequency drive. The 3-axis micromachining centre has stacked x-y stages having a ball screw driven by a DC brushless servo motor. The x-y stages have a positioning resolution of 0.5 µm and an accuracy of ±1 µm. The z-stage is a pneumatically counterbalanced linear motor stage having a positioning resolution of 5 nm. The micromachining center is placed on a vibration isolation table. The set-up is shown in Fig. 5. Two-flute uncoated tungsten carbide cutting tools of 500 µm in diameter have been used in the experiments. Slot milling operations have been carried out on Ti6Al4V for different feed/rev and spindle speeds. To determine the cutting coefficients, cutting forces have been measured using a three directional Kistler dynamometer (Minidyne 9256 C2) connected to a data acquisition (NI DAQ) system.

### Table 1. Tool tip dynamics

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural frequency (Hz)</th>
<th>Damping ratio (ζ)</th>
<th>Stiffness (N/µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3930</td>
<td>0.071</td>
<td>2.07</td>
</tr>
<tr>
<td>2</td>
<td>4135</td>
<td>0.077</td>
<td>0.32</td>
</tr>
</tbody>
</table>

The tool tip dynamics obtained by the FEM modal analysis shows a deviation of 2% and 24% in the natural frequency prediction for 1st and 2nd mode, respectively, by the values obtained via receptance coupling by Malekian et al. [11]. This difference can be attributed to the fact that the stiffness of the spindle and the machine structure has been ignored. There is deviation of 15% in natural frequency obtained by the Rayleigh-Ritz minimum potential energy method [13]. Based on the comparisons with results reported in the literature, it can be inferred that the tool tip dynamics obtained by FEM modal analysis is reasonably accurate.

### 3. Cutting coefficient determination

The cutting coefficient determination requires the series of slot test at varying feed and constant depth of cut. For each experiment, the X and Y direction cutting forces have been measured and then root mean square value of the force data at different feed is used to compute tangential and radial cutting coefficient. The experimental setup and the details are presented in detail in this section.

#### 3.1 Experimental setup

Experiments for the calculation of the cutting coefficients have been carried out on a high speed micromachining centre developed in-house. The high-speed spindle has a ceramic bearing with a maximum speed of 140000 rpm and average torque of ~4.3 N-cm.
The liner curve fitted plot between RMS value of force and feed at 80,000 rpm in x and y direction is given by Fig. 6 and 7 respectively. Slope of linear curve fitted plot is compared with the semi analytical average force to obtain the cutting coefficient. The cutting coefficients are obtained by below average force relation.

\[ K_{tc} = \frac{\bar{F}_x}{N.a}, \quad K_{rc} = \frac{\bar{F}_y}{N.a} \]

where \( \bar{F}_x \) and \( \bar{F}_y \) are the average RMS value of the cutting forces in x and y direction respectively. The cutting coefficients are shown in Table 2.

Table 2. Cutting coefficients

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>( K_c ) (N/mm(^2))</th>
<th>( K_{rc} ) (N/mm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti6Al4V</td>
<td>2880</td>
<td>1280</td>
</tr>
</tbody>
</table>

4. Stability prediction and discussion

Stability prediction has been done to identify the processing conditions for chatter free machining operation. Chatter free machining parameters is obtained at spindle speed and depth of cut combinations lying in the stable zone. Stability lobe is obtained from the numerical simulation of the analytical model discussed in the above modeling step. Predicted stability lobe is the verified experimentally by carrying the experiment at different combination of speed and depth of cut. Fig. 8 shows a frequency spectrum of the force signal at 90000 spindle speed and 90 µm depth of cut, where chatter has occurred. It can be seen that chatter frequency (~4405 Hz) is higher than that of the 2\(^{nd}\) mode natural frequency. Also, from frequency spectrum at 70000 spindle speed and 50 µm depth of cut in Fig. 9, the amplitude at 1200 Hz is higher than that of the tool passing frequency (~2333 Hz) and its harmonics, where the forced vibration/instability has occurred but it may not be chatter as frequency is below the natural frequency of the micro end mill.

The frequency spectrum of force signal at 100000 spindle speed and 30 µm depth of cut is shown in Fig. 10. The entire amplitude peak is at either tooth passing frequency or its harmonics hence, machining is stable at this process parameter. Frequency spectrum of force signal at 60000 rpm and 50 µm depth of cut is shown in Fig. 11, chatter is observed at frequency (~4600 Hz), higher than the natural frequency of the micro end mill.
Fig. 11 FFT at 80000 rpm, 40 µm depth of cut and at feed of 3 µm/flute

The stability lobe diagram is shown in Fig. 12. Stability lobe diagram shows that there is reasonably good agreement between experimental and simulation lobe at low rotational speeds. However, at higher rotational speeds higher deviation from the predicted lobe is observed. This can be attributed to increased uncertainty in cutting coefficients which affects the stability predictions[13].

4. Conclusions

In the present work, chatter stability analysis has been carried out by FEM modal analysis of the micro end mill. Main contribution of this paper is the determination of tool tip dynamics by modal analysis. The modal analysis, gives the good agreement between the simulation and experiential lobe following conclusions can be drawn from the present work:

- Modal analysis of micro end mill gives the tool tip dynamics fairly accurately within deviation of 5% of the values predicted by the receptance coupling method.
- There is good agreement between the simulation and experimental lobes, but at high speeds, there is slight deviation from the predicted values. This can be attributed to increased uncertainty in cutting coefficients at high speeds.
- Finite element based modal analysis can be used as an alternative to the experimental modal analysis for determining the tool-tip dynamics for identifying stable process parameters.

References


