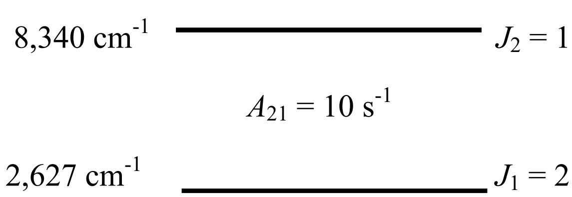
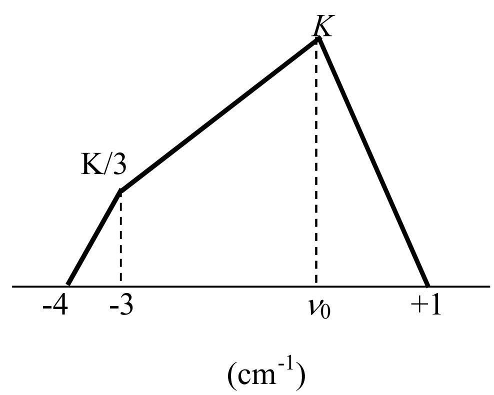
Indian Institute of Technology Guwahati
Department of Physics
PH306/Lasers & Ultrafast Optics/2022-23/Tutorial-1/AKSharma
Due date: 28.01.23

1. A two-level atom is placed within a cavity and is allowed to come into equilibrium with blackbody radiation of temperature T.
   (a) Show that the condition for the rate of stimulated emission from the upper level being equal to the rate of spontaneous emission is $k_B T = \hbar\omega_{21}/\ln 2$ where $\hbar\omega_{21}$ is the energy spacing of the two levels.
   (b) Under this condition, what is the ratio of the populations per state in the upper and lower level? Explain your answer in terms of the spontaneous and stimulated transition rates between the two levels.
2. Suppose the two levels in problem 1 are now subjected to radiation of energy density $\rho(\omega)$. Show that the total rate of radiative transitions from the upper level can be written as

$$R_2 = N_2 A_{21}\left[\frac{\pi^2 c^3}{\hbar\omega_{21}^3}\rho(\omega_{21}) + 1\right].$$

3. The spontaneous emission profile from a certain transition can be approximated by the shape shown below. Calculate the stimulated emission cross-section.





4. For the transition $2P \rightarrow 1S$ in the hydrogen atom, evaluate $\left|\int \psi_{1S}^{*}\vec{r}\psi_{2P}d\tau\right|^2$. Hence calculate the corresponding lifetime for spontaneous emission.
5. The Einstein coefficient A can be estimated using a simple harmonic oscillator model as follows. Consider the equation of motion of an electron e oscillating about a positive charge (nucleus) given by $\ddot{\mathbf{x}} + \omega_0^2\mathbf{x} = 0$, where ω_0 is the frequency of oscillation.
   (a) Obtain the solution of this equation, $\mathbf{x}(t)$, subject to the boundary conditions $\mathbf{x}(0) = \mathbf{x}_0$ and $\dot{\mathbf{x}}(0) = \mathbf{v}_0$.

(b) Treat the given system as an oscillating dipole and use the Larmor formula (non-relativistic) to calculate the average power radiated by such a dipole.

(c) Show that average power radiated can be expressed in the form $\frac{dW}{dt} = -AW,$ where W is the oscillator energy and A is then identified as the rate of spontaneous emission. Obtain the numerical value of A for a wavelength of 500 nm. Note that, in order to arrive at a more accurate result, the expression obtained for A must be multiplied by a factor $3f$ where f is the oscillator strength of the transition.

6. (a) Express the inhomogeneously broadened lineshape function g(ν) in terms of the Doppler width $\Delta\nu_D$.

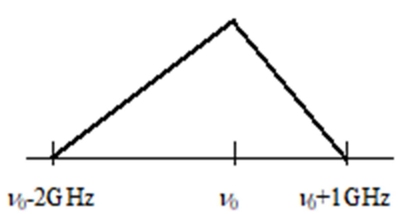
   (b) Compare the Doppler linewidths of He-Ne laser (atomic mass = 20 and transition wavelength = 632.8 nm) with that of CO_2 laser (transition wavelength = 10.6 μm) at room temperature (300 K).

7. Consider an atomic system with $J_1 = 2$ and $J_2 = 1$.

   (a) What is the ratio B_{12}/B_{21}?

   (b) What is the formula for the small signal gain coefficient for the 2 → 1 transition?

   (c) If the line shape function could be approximated by the graph shown with $A_{21} = 10^6$ s^{-1}, λ = 6401 Å, and $N_1 = N_2 = 10^{12}$ cm^{-3}, what is the small signal gain coefficient for the 2 → 1 transition at $\nu = \nu_0$?



8. The rate of spontaneous emission, A_{21}, of a homogeneously broadened laser transition at a wavelength of 10.6 μm is 0.34 s^{-1}, while linewdth is 1 GHz. The degeneracies of the upper and lower states are 41 and 43, respectively.

   (a) Calculate the stimulated emission cross-section at the line center.

   (b) Calculate the population inversion to obtain a gain coefficient of 5 m^{-1}.

9. A laser cavity consists of two mirrors with reflectivity $R_1 = 1$ and $R_2 = 0.5$ while the internal loss per pass is L_i = 1%. If the length of the active material is l = 7.5 cm and the transition cross-section is $\sigma = 2.8 \times 10^{-19}$ cm^2, calculate the threshold inversion.

10. A CW laser is operating at 700 nm, having homogeneously broadened line profile with FWHM of 100 nm. The saturated absorption coefficient is 0.1 m^{-1}. The intensity is equal to I_{sat}. Calculate the small signal gain

coefficient (unsaturated absorption coefficient), the stimulated emission cross section at the line center and the population inversion required to produce such a small signal gain coefficient.

11. The upper level of a two-level system is pumped at a constant rate R as shown. The lifetime of upper and lower laser levels are τ_2 and τ_1, respectively.

    (a) Calculate the population of upper and lower levels under steady state.

    (b) Is the system suitable for the CW laser and why?