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Result: If $f : D \to \mathbb{R}$ is differentiable at $x_0 \in D$, then f is continuous at x_0.

Examples:

1. For $n = 1, 2, 3,$ let $f_n(x) = \begin{cases} x^n \sin \frac{1}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$

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Result: If $f : D \to \mathbb{R}$ has a local maximum or local minimum at an interior point x_0 of D and if f is differentiable at x_0, then $f'(x_0) = 0$.

Rolle's theorem: If $f : [a, b] \to \mathbb{R}$ is continuous, if f is differentiable on (a, b) and if $f(a) = f(b)$, then there exists $c \in (a, b)$ such that $f'(c) = 0$.

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(a) $\sin x \geq x - \frac{x^3}{6}$ for all $x \in [0, \frac{\pi}{2}]$.

(b) If $f(x) = x^3 + x^2 - 5x + 3$ for all $x \in \mathbb{R}$, then f is one-one on $[1, 5]$ but not one-one on $\mathbb{R}$.

Intermediate value property of derivatives: Let $f : I \to \mathbb{R}$ be differentiable and let $a, b \in I$ with $a < b$. If $f'(a) < k < f'(b)$, then there exists $c \in (a, b)$ such that $f'(c) = k$.

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Example: Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable such that $f(-1) = 5$, $f(0) = 0$ and $f(1) = 10$. Then there exist $c_1, c_2 \in (-1, 1)$ such that $f'(c_1) = -3$ and $f'(c_2) = 3$.

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Example: Local maxima and local minima of f, where $f(x) = 1 - x^{2/3}$ for all $x \in \mathbb{R}$.

L'Hôpital's rules:

1. Let $f : (a, b) \to \mathbb{R}$ and $g : (a, b) \to \mathbb{R}$ be differentiable at $x_0 \in (a, b)$. Also, let $f(x_0) = g(x_0) = 0$ and $g'(x_0) \neq 0$. Then $\lim_{x \to x_0} \frac{f(x)}{g(x)} = \frac{f'(x_0)}{g'(x_0)}$.

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Example: $1 + \frac{x}{2} - \frac{x^2}{8} \leq \sqrt{1+x} \leq 1 + \frac{x}{2}$ for all $x > 0$.

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It is sufficient to consider the series $\sum_{n=0}^{\infty} a_n x^n$.

Convergence - Examples:

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The series may or may not converge for $|x| = R$.

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Examples: Taylor series expansions of e^x, $\sin x$ and $\cos x$.

Result on local maxima and local minima:

Let $x_0 \in (a, b)$ and let $n \geq 2$. Also, let $f, f', ..., f^{(n)}$ be continuous on (a, b) and
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(a) If n is even and $f^{(n)}(x_0) < 0$, then f has a local maximum at x_0.

(b) If n is even and $f^{(n)}(x_0) > 0$, then f has a local minimum at x_0.

(c) If n is odd, then f has neither a local maximum nor a local minimum at x_0.

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Let $x_0 \in (a, b)$ and let $n \geq 2$. Also, let $f, f', ..., f^{(n)}$ be continuous on (a, b) and
$f'(x_0) = f''(x_0) = \cdots = f^{(n-1)}(x_0) = 0$ but $f^{(n)}(x_0) \neq 0$.

(a) If n is even and $f^{(n)}(x_0) < 0$, then f has a local maximum at x_0.

(b) If n is even and $f^{(n)}(x_0) > 0$, then f has a local minimum at x_0.

(c) If n is odd, then f has neither a local maximum nor a local minimum at x_0.

Example Local maximum and local minimum values of f, where $f(x) = x^5 - 5x^4 + 5x^3 + 12$ for all $x \in \mathbb{R}$.