Discrete Mathematics (CS202/MA221) **IIT**
Problem set #1 **Guwahati, India**
September 13, 2011

---

1. Which of the following sets are enumerable?

    1. Set of all finite graphs where V is a subset of $\mathbb{N}$.
    2. Set of all functions from $\mathbb{N}$ to $\{0, 1\}$
    3. Set of all C programs.
    4. Set of all finite walks on strongly connected graph.
    5. Set of all infinite walks on a strongly connected graph.
    6. Set of all walks on a directed graph without cycles.
    7. Set of all real numbers x such that x^2 is rational.
    8. Set of all real numbers in $[0, 1]$ with finitely many non zero digits in their decimal representation.
    9. Set of all real numbers in $[0, 1]$ with finitely many 1 in their decimal representation.

2. Give bijections between the following sets or explain why there cant be any.

    1. Set of all prime numbers and set of all composite numbers.
    2. Set of all irrational numbers and set of all complex number of the form $a + ib$, $a, b \in \mathbb{Z}$
    3. Set of all binary sequences and set of all ternary sequences.
    4. Set of all binary sequences and $\mathcal{P}^{(\mathcal{P}^{\mathbb{N}})}$
    5. Set of all arithmetic progressions on integers and $\mathbb{R}$
    6. $\mathbb{R}^2$ and $[0, 1] \times [0, 1]$

3. We write $A \sim B$ if there is a bijection between A and B. Show that $\sim$ is an equivalence relation.

4. True or False

    1. $A \subseteq B$ and B is countable then A is countable.
    2. A real number x is said to be "algebraic" if x is a root of a polynomial with integer coefficients. The set of algebraic numbers is uncountable.
    3. Set of all infinite sequences on $\mathbb{N}$ is countable.
    4. Let S denote the set of all convergent geometric series whose sum is a rational number. S is countable.