On the Flexible Applied Boundary and Support Conditions of Compliant Mechanisms using Customized Evolutionary Algorithm

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Abstract. In structure topology optimization, the applied boundary and support conditions are often fixed in a-priori. These conditions can affect the behavior and the properties of single-piece elastic structures known as compliant mechanisms. In this paper, the same aspect is explored for path generating compliant mechanisms by considering them as design variables and their values are evolved using customized NSGA-II algorithm. Three examples are solved and the innovative facts among the applied boundary and support conditions are presented. The elastic structures are also presented in this paper.

1 Introduction

Structural topology optimization is a fast growing field that is finding numerous applications in automotive, aerospace and mechanical design processes. It optimizes the material distribution or layout within a given design-domain under the applied boundary and support conditions [1].

Quite often, it has been observed in the literature of structural topology optimization that the applied boundary and support conditions are fixed a-priori. Sometimes, these conditions are known or the design constraints and variables limit them. However, the applied boundary and support conditions can affect the behavior and optimal properties of structures. It has been shown elsewhere [2] that the optimum set of support and loading positions generated the improved compliant mechanisms in their objective values. It can also influence the final shape of elastic structures [3]. However, various design principals and facts can be discovered on the basis of design goals and variables [4]. In this paper, an attempt is made to explore the innovative facts by considering the applied and boundary conditions as design variables for three examples of path generating compliant mechanisms. Unique facts of these conditions for compliant mechanisms are explored that can be beneficial to the designers. The elastic structures of three examples of path generating complaint mechanisms are also presented. In the remaining part of the paper, section 2 described the methodology followed in this paper. The experimental results are discussed and optimum elastic structures are presented in section 3. The paper is concluded in section 4.

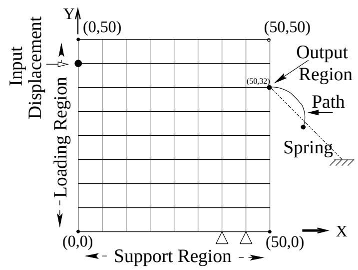

Fig. 1. A design-domain.

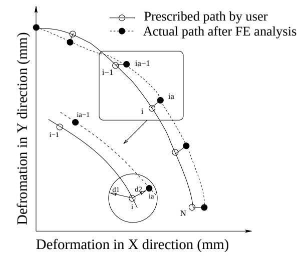

Fig. 2. Prescribed and actual paths.

2 Methodology

2.1 Formulation

The formulation is designed for path generating compliant mechanism (PGCMs) that trace-out the prescribed path by undergo through elastic deformation. In this paper, the design-domain for PGCMs is categorized into three regions called support, loading and output regions (cf. Fig. 1). The elastic structures are supported in support region whereas the load is applied at the loading region. The elastic structures trace-out the prescribed path at the output region. As Fig. 2 shows, the constraints are imposed at precision points of prescribed path so that the actual path generates the similar path ($d_2 \leq d_1$) [5].

In this paper, the compliant mechanisms are designed using two bi-objective sets. In both sets, the primary objective is to minimize the weight of elastic structures. The another objective for first bi-objective set is the minimization of supplied input energy [5] that is calculated with respect to the stress and the strain developed during the large deformation of the elastic structures. For second bi-objective set, the maximization of geometrical diversity [2] is chosen that is calculated by comparing the dissimilarity in the bits of binary strings of the *reference design* and the elastic structure of GA population. In this paper, the compliant mechanism evolved by single-objective optimization is chosen as a *reference design*. The single and bi-objective sets and the constraints are given in appendix A.

2.2 Customized Evolutionary Algorithm

Among the multi-objective evolutionary algorithms, NSGA-II [6] is the fastest and has shown to have a good convergence property to the global 'Pareto-optimal' front for various two objective test and engineering problems [7]. Thus, NSGA-II is used as a global search and optimizer in this paper. However, there is a need to modify the existing NSGA-II for structure topology optimization. A local search method is also used which acts as a post-processing method to refine the non-dominated compliant mechanisms evolved by the modified NSGA-II. The flow chart of the customized NSGA-II algorithm is shown in Fig. 3.

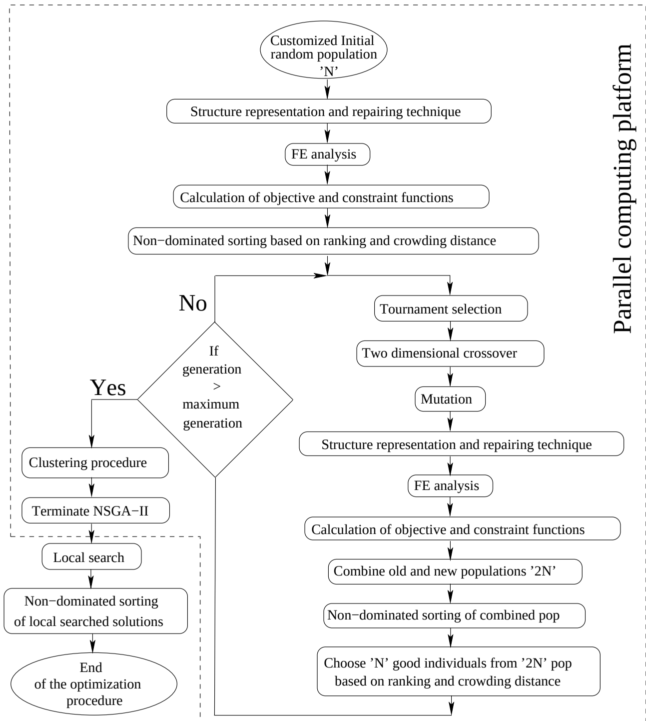

Fig. 3. A flow chart of customized NSGA-II algorithm

Start with NSGA-II parameters which are given in Table 1. A binary string of 12 bits is used to evolve the applied boundary and support conditions in their respective regions of the given design domain (cf. Fig. 1). To calculate the values, 12 bits are divided into three groups of five, three and four bits respectively as given in Table 2. The decoded value of first five bits indicates the location of an element from the origin where the elastic structure is to be supported. The decoded value of subsequent three bits helps to determine the loading position, that is, a node where the input load is applied. The decoded value of last four bits are used to evaluate the magnitude of input displacement which can vary from 1 mm to 16 mm at step of 1 mm.

Population	240	Generation	100
Crossover probability	0.95	Mutation probability	1/string length
String length for a structure	625	String length for applied boundary & support conditions	12

Table 1. GA parameters.

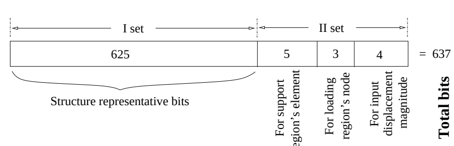

Table 2. A binary string of a GA population member.

For structure representation, a binary string of 625 bits is used to represent the material distribution for the elastic structure. A binary string is copied to two dimensional array followed by the material assignment as shown in Fig. 4. The bit value ‘1’ signifies that material is present whereas, ‘0’ represents the void. This scheme divides a design domain of structure into 25×25 ($= 625$) grids in x and y directions, respectively.

In this paper, the domain specific initial population strategy is used which has shown its advantage over random initialization of material in the design domain [3]. The initial population strategy is described by showing the material connectivity between the support and loading regions in Fig. 5. The intermediate points (between 1 to 5) are randomly generating within the design domain. In Fig. 5, four points (P1, P2, P3 and P4) are generated and they connect the support (S1) and the loading (L1) positions by straight lines. Thereafter, a material is assigned to those elements where these straight lines pass as shown in Fig. 5. Similarly, a set of piece-wise linear line segments between the support and output regions and another set between the loading and output regions are explained. Here, the element positions of support and loading regions are calculated after decoding the binary string of 12 bits (cf. Table 2). The location of output region is fixed in this study because this point will trace-out the user-defined path. This initial population strategy ensures the geometrically feasible structures in the initial population.

As two bi-objective sets are used to capture the facts between applied boundary and support conditions, the two crossover operators are also used in this paper. For each example of PGCMs, NSGA-II is coupled with both operators individually and the conditions are evolved on different optimization platforms. The first two-dimensional crossover has shown its successful application in shape optimization [8, 9] and compliant mechanisms [5, 2, 3]. It works on exchanging the rows or column (refer Fig. 6) with equal probability. The size and location of common patch are found randomly and it is swapped between the two parents. Another crossover operator is a domain specific crossover that divides the given two-dimensional design domain into four sub-regions. Points P1, P2 and P3 of Fig. 8 are chosen randomly on their respective edges and are joined by straight lines. With an equal probability, two sub-regions out of four are swapped between the two parents. For the crossover of 12 bit binary string, a standard

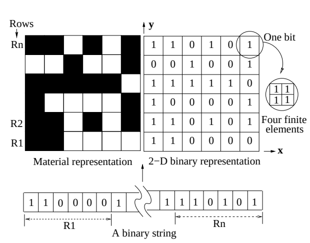

Fig. 4. A representation of structure using binary string.

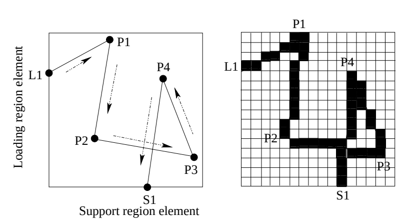

Fig. 5. Connectivity between support and loading regions.

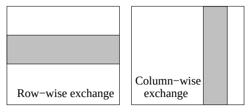

Fig. 6. Row/Column-wise crossover operator

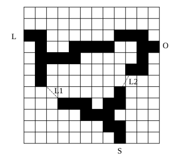

Fig. 7. Disconnected topology: a hypothetical case.

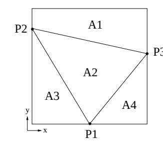

Fig. 8. Domain specific crossover operator

single point crossover operator is used. In this paper, the mutation of each bit of binary string representing the structure is done with a probability of 1/string length.

Because of the crossover and mutation operators, the new solutions can suffer from disconnected topology problem. As shown in Fig. 7, the support region (**S**) is not connected to the loading (**L**) and the output (**O**) regions. In this disconnected scenario, the individual distances are calculated from the centroid of each grid of material of **S** to the centroid of each grid of material of **L** and **O**. Then, the straight lines (**L1, L2**) are drawn from the centroid of those two grids which show minimum distances between **S-L** and **S-O**. In this way, the connectivity among **S**, **L** and **O** regions of a structure is checked.

The point singularity between the two material element's can arise due to the developed initial population strategy, GA operators and after the connectivity technique. An ad-hoc repairing technique motivated from the image processing concept [10] is employed in this paper. In Fig. 9, the material at positions 2, 4, 6 and 8 can create point singularity. If position 2 creates point connectivity, then an extra material can be filled at 1 or 3 with equal probability. In this way, the point singularity for each element of material is eliminated. Due to the mutation operator, any floating material element can appear which is not connected to the seed elements. In this case, this isolated element is changed to void by assigning value '0'.

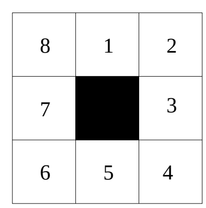

Fig. 9. Eight neighborhood connectivity.

After above steps, the elastic structures are now undergo for finite element analysis (FEA). In this study, one grid of a structure is further discretized into four finite elements with same Boolean variable value as shown in Fig. 4. In the present process, the structure is discretized with 4×625 ($= 2500$) 4-node rectangular finite elements and analyzed through a non-linear large deformation FE analysis using *ANSYS* package. However, the GA operations are performed on 625 bits representing the same structure.

The function evaluations and FE simulations are performed on the parallel computing platform. Master-slave architecture is used in the present paper in which the population members are evaluated on the slave processors and rest of the operations are done on the master processor. A MPI based Linux cluster with 24 processors is used in the present study.

When the non-dominated solutions are evolved by the customized NSGA-II, these solutions are refined by local search method. The weighted-sum approach is used in this paper to reduce the multi-objective problem into single-objective. Weights are calculated according to the positions of non-dominated solutions evolved by NSGA-II in the objective space (refer Eqn. 1) [7].

$$\overline{w}_j^x = \frac{(f_{j_{max}}^x - f_j^x) \setminus (f_{j_{max}}^x - f_{j_{min}}^x)}{\sum_{k=0}^{M} (f_{k_{max}}^x - f_k^x) \setminus (f_{k_{max}}^x - f_{k_{min}}^x)}, \tag{1}$$

where $\overline{w}_j^x$ is the corresponding weight to the j^{th} objective function, f_j^x is j^{th} objective function, $f_{j_{min}}^x$ and $f_{j_{max}}^x$ are minimum and maximum values of j^{th} objective function of non-dominated front, M is the number of non-dominated solutions.

In the local search method, the weighted sum of scaled fitness of a selected representative solution is evaluated. Thereafter, the two-dimensional array of solution is checked for the grids having a material. For each material's grid, there are maximum of eight possible neighborhoods as shown in Fig. 9. One by one, all neighboring bits including its own bit, are mutated. The new elastic structure is now extracted on which FEA is performed for objective function and constraints values. The new elastic structure will discard if it is infeasible. If it is feasible, then the changes are accepted when the weighted sum of scaled fitness of new elastic structure is better. When the scaled fitness of elastic structure before checking the material's grid is same as after mutating all bits having material and their neighborhood, the local search method is terminated. In the same way, all representative solutions are mutated.

3 Experimental Results

In this section, three examples of compliant mechanisms tracing (i) curvilinear path, (ii) straight line path and (iii) upward curvilinear path are solved with different objective sets using customized NSGA-II with two different crossover operators. The applied boundary and support conditions are evolved for the wide-variety of optimization frame-works and the innovative facts are discovered.

Deb and Srinivasan [11] introduced a new design methodology called "innovization" in which the new and innovative design principles are developed by means of optimization techniques. In this paper, an attempt is made to find such principals or facts that are based on the applied boundary and support conditions of various PGCMs. It can help the designers and decision makers to get more insight into the topology optimization of compliant mechanism tracing user-defined path.

The optimum set of applied boundary and support conditions of all single and bi-objective studies are given in Table 3. In this table, 'OX' is referred for the row/column crossover-wise operator based studies and similarly, 'NX' is used for the domain specific crossover operator based studies.

Table 3. Applied boundary and support conditions for different optimization frame-works.

Example: Curvilinear Path Tracing Compliant Mechanisms (CPTCM)						
Conditions	Single-objective study		I^{st} bi-objective study		II^{nd} bi-objective study	
	OX	NX	OX	NX	OX	NX
Support position	20	2	2	2	16	18
Loading position	32	24	32	40	24	32
Input displacement magnitude	7	5	7	9	5	7
Example: Straight Line Path Tracing Compliant Mechanisms (SLPTCM)						
Conditions	Single-objective study		I^{st} bi-objective study		II^{nd} bi-objective study	
	OX	NX	OX	NX	OX	NX
Support position	46	46	46	46	46	46
Loading position	40	28	20	20	20	28
Input displacement magnitude	8	5	4	4	4	5
Example: Upward Non-Linear Path Tracing Compliant Mechanisms (UNPTCM)						
Conditions	Single-objective study		I^{st} bi-objective study		II^{nd} bi-objective study	
	OX	NX	OX	NX	OX	NX
Support position	44	46	44	46	46	44
Loading position	44	48	44	44	48	44
Input displacement magnitude	5	6	5	5	6	5

Let us first identify the common support positions in Table 3. For the curvilinear path tracing compliant mechanisms (CPTCM), the support position of 2 mm is common in single-objective 'NX', I^{st} bi-objective 'OX', and I^{st} bi-objective 'NX' studies. The corresponding loading positions are at 24 mm, 32 mm and 40 mm, respectively. The required input displacement magnitudes of single-objective 'NX', I^{st} bi-objective 'OX', and I^{st} bi-objective 'NX' studies are 5 mm, 7 mm and 9 mm, respectively.

Similar information can also be unfold from the examples of straight-line path tracing compliant mechanisms (SLPTCM) and upward non-linear path tracing compliant mechanisms (UNPTCM). In case of SLPTCM, the identical support position at 46 mm is evolved for all studies. The corresponding loading positions are at 40 mm, 28 mm and 20 mm from the origin. The respective magnitudes of input displacement are 8 mm, 5 mm and 4 mm to trace the straight line prescribed path. Similarly, an example of UNPTCM shows the common support position at 44 mm for single-objective 'OX', I^{st} bi-objective 'OX' and II^{nd} bi-objective 'NX' studies and another support position at 46 mm for single-objective 'NX', I^{st} bi-objective 'NX' and II^{nd} bi-objective 'OX' studies. When the topologies are supported at 44 mm, then they are loaded at 44 mm and required 5 mm of input displacement magnitude. On the other hand, the elastic structures that are supported at 46 mm, require 6 mm and 5 mm of input displacement magnitudes for the loading positions at 48 mm and 44 mm respectively to trace the upward non-linear path.

For identical support positioned compliant mechanisms, we observe that the magnitude of input displacement required to trace the prescribed path increases as the load is applied away from the origin. This common feature is independent of the nature of compliant mechanisms tracing variety of paths.

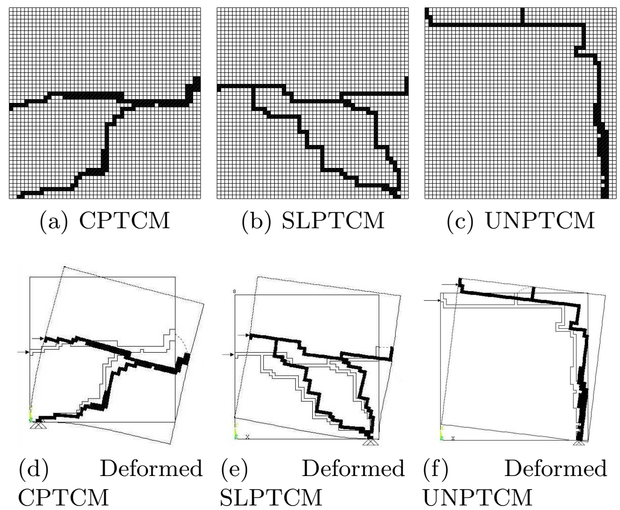

Fig. 10. Topologies.

Another interesting information can be drawn out when we observe the common loading positions of each study mentioned in Table 3. In the example of CPTCM, the first identical loading position is at 32 mm for which the elastic structures are supported at 20 mm, 2 mm and 18 mm from the origin and require 7 mm of input displacement. The elastic structures with second common loading position of 24 mm are supported at 2 mm and 16 mm and require 5 mm of input displacement. The example SLPTCM indicates 28 mm and 20 mm common loading positions for which all compliant mechanisms are supported at same position and require 5 mm and 4 mm values of input displacement, respectively. Similarly in the example of UNPTCM, the topologies which are loaded at 44 mm, are supported at 44 mm and 46 mm from the origin and require 5 mm of input displacement whereas, the topologies with common loading position of 48 mm are supported at 46 mm and require 6 mm of input displacement. The observation reveals that if the loading position of compliant mechanisms is same, then these mechanisms require same magnitude of input displacement to trace the prescribed path irrespective of the different support positions.

The topologies of single-objective optimization using NSGA-II with domain-specific crossover operator are shown in Fig. 10 for three examples of PGCMs. According to the nature of generating prescribed paths, the topologies and shapes of CPTCM, SLPTCM and UNPTCM are evolved. Although the applied boundary and support conditions of CPTCM and UNPTCM and their prescribed paths are different but they have same topology. If we look at the support positions of above three examples, CPTCM is supported on the bottom left-hand side (cf. Fig. 10(d)) and SLPTCM is on the bottom right-hand side (refer Fig. 10(e)). It is because the elastic structure supported in left-hand side generates higher downward curvilinear paths compared to right-hand side supported PGCMs. In case of UNPTCM, the output region is positioned at the middle of top edge of the given design domain. In this scenario, the output point can only trace the upward non-linear path when the support position lies on the right hand side of the output point (refer Fig. 10(f)). Such behaviors of the elastic structures are expected and the evolved support conditions also abide the same principal [2, 3].

Another important fact of considering the applied boundary and support conditions as design variables can observe when these conditions are unknown. The designer does not have to define these conditions a-priori. Moreover, the optimum set of evolved conditions can explore the possibilities of non-optimum applied boundary conditions that might be considered in the previous practice of designers [3].

4 Conclusions

This paper has explored the innovative facts of the applied boundary and support conditions for variety of PGCMs, irrespective of the different optimization frameworks. Moreover, the optimum sets of these conditions were evolved without any priori information. The possibility of non-optimum conditions was also explored which might be considered in the previous practices. Beside all facts, the evolved support positions by the customized NSGA-II for three examples of PGCMs abided the expected rule of elastic deformation of structures. These unfold facts and information can be beneficial to the designers to get deeper insight into the problem. In the future work, the concept of flexible applied boundary and support conditions can be used for variety of structure topology optimization problems.

5 Acknowledgement

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A PGCM Formulation

Single-objective optimization:
Minimize: Weight of structure
Ist Bi-objective set:
Minimize: Weight of structure (primary objective),
Minimize: Supplied Input energy (secondary objective),
IInd Bi-objective set:
Minimize: Weight of structure (primary objective),
Maximize: Geometrical Diversity of structure (helper objective),
Optimization problems are subjected to:
$1 - \frac{\sqrt{(x_{ia}-x_i)^2+(y_{ia}-y_i)^2}}{\eta\times\sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}} \geq 0, \quad i = 1, 2, ..., N$
$\sigma_{flexural} - \sigma \geq 0,$
where $\eta = 15\%$ is the permissible deviation and N is number of precision points representing the prescribed path. $\sigma_{flexural}$ and σ are flexural yield strength of material and maximum stress developed in the elastic structure, respectively. x_i and y_i are the coordinates of precision points whereas the coordinates of x_{ia} and y_{ia} are the corresponding points on the actual path traced by elastic structure. Note that the points on actual path are found from the non-linear finite element analysis of elastics structures based on equal load steps.