A Modified Hybrid Particle Swarm Optimization Approach for Unit Commitment

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Abstract- **This paper presents a new solution to thermal unit-commitment (UC) problem based on a modified hybrid particle swarm optimization (MHPSO). Hybrid real and binary PSO is coupled with the proposed heuristic based constraint satisfaction strategy that makes the solutions/particles feasible for PSO. The velocity equation of particle is also modified to prevent particle stagnation. Unit commitment priority is used to enhance the performance of binary PSO. The proposed algorithm is tested for 10, 20, 40 and 60 unit systems and the results are reported for 10 different runs. Statistical results and their comparison show a good performance of MHPSO over other existing optimization methods.**

I. INTRODUCTION

Unit commitment (UC) is a non-linear programming problem which determines the start-up and shut-down schedule for generating units over a period of time to satisfy the forecasted demand at minimum cost. At the same time, the solution has to meet constraints on reserve and individual units [1]. The UC problem needs to determine two types of basic decision variables - the unit operating status (on/off), which is binary variable and the economic load dispatch (ELD), which is a real variable denoting the power generated by the committed units in each hour. Therefore, UC problem combines two constrained optimization problems - optimal commitment and economic generation dispatch of the units during the scheduling period. Solving UC problem involves exploring a higher dimensional, non-convex search space. Hence, it is considered as one of the most complex combinational optimization problems in power system economics [2].

A bibliographical survey on UC reveals that various numerical optimization techniques such as priority list [3,4], dynamic programming [5-10], branch-and-bound algorithm [11] and Lagrangian relaxation [12, 13] have been applied to solve the classic UC problem. Although these methods are simple and fast, they usually suffer from premature convergence problems and deliver sub-optimal solutions. Since the last two decades, Genetic Algorithms (GAs) [14-17], simulated annealing (SA) [18], evolutionary programming [19], ant colony optimization [20], tabu search [21] and particle swarm optimization [22-25] have been employed to overcome the shortcomings of traditional optimization techniques.

PSO is a social behavior based optimization method in which each particle dynamically adjusts its velocity and position according to its own experience and flying experience from its neighbor [26]. PSO has many similarities with other evolutionary computation techniques such as Genetic Algorithms (GA). One of the main advantages of PSO is that it has a better information sharing and conveying mechanism. However, in ordinary PSO, particles tend to become idle after a number of iterations, resulting in inability to jump out from local optima. Binary PSO is a simple modification of the original version of PSO [27]. BPSO has been combined with PSO in solving UC problem in [22, 24, 28]. However, only small-scale systems are reported in these works.

This paper proposes a modified hybrid particle swarm optimization (MHPSO) through a combination of modified real PSO [29] and modified binary PSO to enhance their performance. Heuristic based constraint satisfaction strategy is developed that provides the feasible solutions/particles to PSO. Scenario of good feasible solution is often required in power system engineering when the system is large. Hence, MHPSO is tested for 10, 20, 40, and 60 unit systems and the results are compared with the existing optimization techniques.

The remaining paper is organized as: UC problem formulation is given in section II. A description of MHPSO is presented in section III. Section IV details the simulations and results using the proposed method. The paper concludes with some remarks in section V.

II. PROBLEM FORMULATION

A. Objectives

The objectives of the UC problem is to minimize the total operating cost subjected to a set of system and unit constraints over the scheduling horizon. The fuel cost for the unit i at any given time interval is assumed as a quadratic function of the generator power output, p_i at that time

$$FC_i(p_i) = a_i \times p_i^2 + b_i \times p_i + c_i \tag{1}$$

where a_i, b_i and c_i are the unit cost coefficients.

The generator start-up cost SC_i for unit i is defined as follows

$$SC_i = \begin{cases} h\cos t_i \text{ if } T_i^{off} \le X_i^{off} \le H_i^{off} \\ c\cos t_i \text{ if } X_i^{off} > H_i^{off} \end{cases} \tag{2}$$

$$H_i^{off} = T_i^{off} + cshour_i \tag{3}$$

where $hcost_i$ and $ccost_i$ are hot start cost and cold start cost respectively, T_i^{off} is minimum down time of unit i, X_i^{off} is the duration of unit i being continuously off and $cshour_i$ is the cold start hour of unit i.

With an assumption that the shut-down cost is zero, the total operating cost for the scheduling period T is the sum of fuel costs and start-up costs for N units:

$$TC = \sum_{i=1}^{N}\sum_{t=1}^{T}[FC_i(p_i) + SC_i \times (1 - u_i(t-1))] \times u_i(t) \qquad (4)$$

where $u_i(t)$ is on/off status of unit *i* at hour *t*.

B. *Constraints*

1) Power balance constraint:
   The total generated power at each hour *t* must be equal to the load demand of the corresponding hour, $D(t)$

$$\sum_{i=1}^{N} p_i(t) \times u_i(t) = D(t) \qquad (5)$$

2) Spinning reserve constraint:
   The power system has to maintain a certain megawatt capacity as spinning reserve, $R(t)$ (10% of the load demand) for reliability

$$\sum_{i=1}^{N} P_i^{\max} \times u_i(t) = D(t) + R(t) \qquad (6)$$

3) Generation limit constraint:
   The power output of each generating unit must be limited within a specified range.

$$P_i^{\min} \le p_i(t) \le P_i^{\max} \qquad (7)$$

4) Minimum up/down time constraint:
   Once a unit is committed/de-committed, it should be kept stable for a minimum period of time before the transition. The constraint is illustrated as below

$$T_i^{on} \le X_i^{on}(t)$$
$$T_i^{off} \le X_i^{off}(t) \qquad (8)$$

   where T_i^{on} and T_i^{off} are the minimum up and minimum down time of unit *i* respectively while $X_i^{on}(t)$ and $X_i^{off}(t)$ are the duration of unit *i* of being continuously on and off respectively at *t*.

III. PARTICLE SWARM OPTIMIZATION

A. *Hybrid PSO (HPSO)*

Particle swarm optimization (PSO) was first proposed by Kennedy and Eberhart [26] as an alternative to GAs. PSO is population-based stochastic optimization technique inspired from the social behavior of organisms such as bird flocking or fish schooling. The individuals in a PSO have their own positions (X) and velocities (V). These individuals are denoted as particles. Each particle remembers its own best positions found so far in the exploration. This position is called personal best (*pbest*). Among these *pbest*s, the particle which has the best fitness value is called as global best (*gbest*). During the flight, the particles are attracted stochastically toward their own *pbest* and *gbest* achieved so far. Therefore, the particles are manipulated according to the following equations:

$$V_i^{k+1} = w \times V_i^k + c_1 \times rand() \times (pbest - X_i^k) + c_2 \times rand() \times (gbest - X_i^k) \qquad (9)$$

$$X_i^{k+1} = X_i^k + V_i^{k+1} \qquad (10)$$

where *w* is known as the inertia weight, which controls the exploration properties of the algorithm as described in [26, 30]. The parameters c_1 and c_2 are two positive constants which keep the balance between local and global behavior of the particles and *rand* is uniformly distributed random number in the range [0, 1].

Binary particle swarm optimization (BPSO) is a simple modification to the original version of PSO [27]. In BPSO, the position *X*, *pbest* and *gbest* are binary numbers while velocity V_i is real number and limited by $\pm V_{max}$. The velocity V_i will determine a probability of threshold using logistic functions below

$$s(V_i) = \frac{1}{1 + \exp(-V_i)} \qquad (11)$$

A random number between [0, 1] is generated and *X* is set 1 if the random number is smaller than value from the sigmoid function (12), which is illustrated as follows

If $random < s(V_i)$, then $X_i = 1$; (12)

Else $X_i = 0$

B. *Modified HPSO*

One major disadvantage of PSO is the tendency of particles to become idle after some iterations, and losing local and global search capabilities as a result. If the particle's position and its *pbest* are very close to *gbest* and its velocity is close to zero (for all dimensions), then the particle will only be flying within a quite small space. As a result, it suffers from the loss of ability to improve the *gbest*. The loss of local searching capability occurs when flying within the limited space has no perceptible effect on fitness. To solve this drawback, a modified PSO is introduced in [29] by adding additional part in the velocity updating equation. They have a third promising value (the first and second promising values are *gbest* and *pbest* respectively). The new expression for velocity of the *j*th dimension of the *i*th particle in the swarm is

$$v_{ij} = w \times v_{ij} + \sum_{r=1}^{\zeta} c_r \times rand() \times (\Pr m_{ij}^{(r)} - p_{ij}) \qquad (13)$$

where ζ is the number of promising values (which is 2 for the standard PSO and 3 in this method). The third one is introduced as follows

$$\Pr m_{ij}^{(3)} = \begin{cases} \Pr m_{ij}^{(1)} + random(\left|\Pr m_{ij}^{(2)} - \Pr m_{ij}^{(1)}\right|) \text{ if } \Pr m_{ij}^{(2)} \ne \Pr m_{ij}^{(1)} \\ P_j^{\min} + random(P_j^{\max} - P_j^{\min});\ \text{Otherwise.} \end{cases} \qquad (14)$$

where $\Pr m_{ij}^{(r)}$ is the *j*th dimension of *r*th promising value of particle *i* in the swam. The position is updated as

$$p_{ij} = p_{ij} + \frac{\sum_{i=1}^{\zeta} \left| Error_i \right| + 1}{\sqrt{Iter}} \times \overline{u_{ij}} \quad , \tag{15}$$

where *Error* is the amount of violation from the nearest upper or lower limit, and $\overline{u_{ij}}$ is defined as

$$\overline{u_{ij}} = \frac{v_{ij}}{\left\| V_i \right\|} = \frac{v_{ij}}{\sqrt{\sum_{j=1}^{N} v_{ij}^2}} \tag{16}$$

where $V_i = (v_{i1}, v_{i2}, ..., v_{iN})$, and $\left\| V_i \right\|$ is the norm of V_i. To make sure all particles are feasible, we propose a heuristic based constraint satisfaction strategy (discussed in Section III.C), in that case, *Error* = 0. Hence, (15) can be written as

$$p_{ij} = p_{ij} + \frac{1}{\sqrt{Iter}} \times \overline{u_{ij}} \tag{17}$$

Binary PSO has been employed to determine the status of the units in [22, 24, 28]. However, those papers only consider small-scale systems with maximum 10 units. Since the standard binary PSO did not yield good results for larger-scale systems, a modified binary PSO is obtained by modifying Eqn. (12) as

If $random < s(V_i)/c$, then $X_i = 1$; (18)

Else $X_i = 0$.

The value of *c* is determined based on the unit priority. If the unit is generally not preferred to be committed, then *c* is set high so that the probability that unit is turned on is lower.

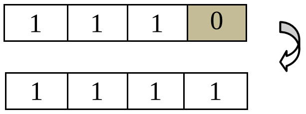
C. Heuristic constraint Satisfaction Strategy

There are two main ways to deal with the constraints in the optimization problems. First, repairing methods are used to repair all the violations occurred. Secondly, penalty function methods are employed to transform the constrained problem into an unconstrained problem. However, the main disadvantage of penalty function methods is, when the problem is highly constrained, the search space reduces and algorithm may spend a lot of time to find the feasible solutions. Therefore in this paper, a rule-based repairing method is proposed which is as follows:

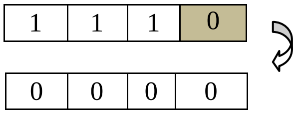
1. Spinning reserve constraint
   The computation method is illustrated as below:

```
At each hour t,
Step 1: If t < T, go to step 2. Otherwise, go to step 7.
Step 2: Create the array of de-committed units at t.
Step 3: Calculate the sum of maximum power generated by all committed units at t.
Step 4: If sum < 1.1 D(t), go to step 5. Otherwise go to step 6.
Step 5: Based on the priority list, the most preferred unit in the array is turned on first. Extract that unit out of the array.
    Step 5.1: Update the sum of maximum power generated.
    Step 5.2: Go back to step 3.
Step 6: t = t+1, go back to step 1.
Step 7: Exit.
```

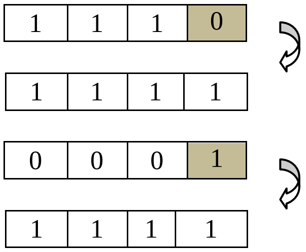
2. Up/down time constraints
   There are two possible approaches to fix the up/down violations: flip the status of the unit at that particular time



or, keep flipping the status of that unit backward from the previous hour till the up/down constraints is satisfied



However, since this UC problem has multiple constraints, repairing the schedule to satisfy one constraint may, in turn, violate others. In this case, if a unit is turned off to satisfy the up/down time constraints, it may violate the spinning reserve constraint which has been solved in part 1. Therefore, in this paper, it is assumed that if there is any up/down time violation, the unit is only allowed to be committed and depending on the case, the first or second approach is chosen. The idea is illustrated as below



3. Equality constraint
   The algorithm is presented below:

```
At each hour t,
Step 1: If t < T, go to step 2. Otherwise, go to step 7.
Step 2: Put all of the committed units in one array. Calculate the sum of power generated by all committed units.
Step 3: Calculate $gap = D(t) - sum$.
    If gap > $10^{-6}$, go to step 4.
    If gap < 0 and abs(gap) > $10^{-6}$, go to step 5.
    Otherwise, go to step 6.
Step 4: If gap > $10^{-6}$
```

Step 4.1: Increase the power generated by the most preferred unit *i* in the array by *gap*. Extract that unit out of the array.

If the power generated by that unit excess P_i^{max}, then set $gap = p_i(t) - P_i^{max}$ and $p_i(t) = P_i^{max}$. Go to step 4.

If power generated is within P_i^{min} and P_i^{max}, set $gap = 0$. Go to step 6.

Step 5: If $gap < 0$ and $abs(gap) > 10^{-6}$,

Step 5.1: Reduce the power generated by the least preferred unit *i* in the array by *gap*. Extract that unit out of the array.

If the power generated by that unit is lower than P_i^{min}, then set $gap = p_i(t) - P_i^{min}$ and $p_i(t) = P_i^{min}$. Go to step 5.

If power generated is within P_i^{min} and P_i^{max}, set gap = 0. Go to step 6.

Step 6: Go to the next hour. Back to step 1.

Step 7: Exit

IV. RESULTS AND DISCUSSION

The proposed method (MHPSO) was first tested for the 10 unit system for which the forecasted load demand is given in Table I. Unit characteristics and cost coefficients are given in Table II. The scheduling period is fixed as T = 24 hours. MHPSO was also tested for 20, 40 and 60 unit systems. The unit characteristic and cost coefficients for these systems are duplicated from Table II. The load demands are multiplied by 2, 4 and 6 respectively for each system.

Table I. LOAD DEMAND

Hour	Load(MW)	Hour	Load(MW)	Hour	Load(MW
1	700	9	1300	17	1000
2	750	10	1400	18	1100
3	850	11	1450	19	1200
4	950	12	1500	20	1400
5	1000	13	1400	21	1300
6	1100	14	1300	22	1100
7	1150	15	1200	23	900
8	1200	16	1050	24	800

The setting for MHPSO for different unit system is kept fixed which is as follows:

- Population size = 100;
- Maximum Iterations = 1200
- Maximum and minimum velocity of MPSO $V_i^{\max} = -V_i^{\min} = 0.1 \times (P_i^{\max} - P_i^{\min})$
- Maximum and minimum velocity of MBPSO $V_{bin\_\min} = 2.0$ and $V_{bin\_\max} = -2.0$
- Acceleration constant $c_1 = 1.5$ and $c_2 = 2.5$
- Inertia weight $w = 1.0$

It is shown elsewhere [30] that PSO can perform better by linearly decreasing inertia weight from 0.9 to 0.4. However, this strategy cannot perform well for every case and can yield poor results for UC problem [22]. Therefore, we keep inertia weight constant for all simulations. Note that the velocity is reset after 600 iterations as an additional measure to avoid particle stagnation.

Table II. UNIT CHACRACTERISTIC AND COST COEFFICIENTS

	Unit1	Unit2	Unit3	Unit4	Unit5
P_min	150	150	20	20	25
P_max	455	455	130	130	162
T_on	8	8	5	5	6
T_off	8	8	5	5	6
Is(hr)	8	8	-5	-5	-6
a	0.00048	0.00031	0.002	0.00211	0.00398
b	16.19	17.26	16.6	16.5	19.7
c	1000	970	700	680	450
d	0	0	0	0	0
HSC	4500	5000	550	560	900
CSC	9000	10000	1100	1120	1800
CSH	5	5	4	4	4

	Unit6	Unit7	Unit8	Unit9	Unit10
P_min	20	25	10	10	10
P_max	80	85	55	55	55
T_on	3	3	1	1	1
T_off	3	3	1	1	1
Is(hr)	-3	-3	-1	-1	-1
a	0.00712	0.00079	0.00413	0.00222	0.00173
b	22.26	27.74	25.92	27.27	27.79
c	370	480	660	665	670
d	0	0	0	0	0
HSC	170	260	30	30	30
CSC	340	520	60	60	60
CSH	2	2	0	0	0

In this paper, the results of proposed MHPSO for 10, 20, 40 and 60 units system are shown in Table III. For every unit system, MHPSO is run for 10 different runs and the statistical values of minimum cost are reported. For 10 unit system, the difference between best-worst and mean-median is small that leads to a smaller standard deviation. However, the standard deviation of HPSO [33] is ($926) that shows the good consistency in the optimum solutions evolved by proposed MHPSO. The optimal schedule for 10 unit case is given in

Table IV. THE OPTIMAL SOLUTION FOR A 10 UNIT SYSTEM

Hour	Unit1	Unit2	Unit3	Unit4	Unit5	Unit6	Unit7	Unit8	Unit9	Unit10	Power	Fuel cost ($)	Start-up cost
1	455	245	0	0	0	0	0	0	0	0	700	13683.2	0
2	455	295	0	0	0	0	0	0	0	0	750	14554.5	0
3	455	370	0	0	25	0	0	0	0	0	850	16809.5	900
4	455	455	0	0	40	0	0	0	0	0	950	18597.7	0
5	455	455	0	65	25	0	0	0	0	0	1000	20059.7	560
6	455	455	130	35	25	0	0	0	0	0	1100	22450.3	1100
7	455	455	130	84.9982	25.002	0	0	0	0	0	1150	23287.8	0
8	455	455	130	130	30	0	0	0	0	0	1200	24150.3	0
9	455	455	130	130	85	20	25	0	0	0	1300	27251.1	860
10	455	455	130	130	162	33	25	10	0	0	1400	30057.5	60
11	455	455	130	130	162	73	25	10	10	0	1450	31916.1	60
12	455	455	130	130	162	80	47.024	16.333	14.643	10	1500	33931.5	60
13	455	455	130	130	162	33	25	10	0	0	1400	30057.5	0
14	455	455	130	130	85	20	25	0	0	0	1300	27251.1	0
15	455	455	130	130	30	0	0	0	0	0	1200	24150.3	0
16	455	455	95	20	25	0	0	0	0	0	1050	21604.2	0
17	455	455	45	20	25	0	0	0	0	0	1000	20760.2	0
18	455	455	130	35	25	0	0	0	0	0	1100	22450.2	0
19	455	455	130	130	30	0	0	0	0	0	1200	24150.3	0
20	455	455	130	130	162	33	25	10	0	0	1400	30057.5	490
21	455	455	130	130	85	20	25	0	0	0	1300	27251.1	0
22	455	455	0	0	145	20	25	0	0	0	1100	22735.5	0
23	455	420	0	0	25	0	0	0	0	0	900	17684.7	0
24	455	345	0	0	0	0	0	0	0	0	800	15427.4	0
												560329.2	4090

Table IV. The schedule is same as reported in [22] using HPSO except for 23rd hour. In Table IV, unit 5 is ON for which unit 2 has to reduce its power to meet the demand constraint. However in [22], unit 6 was ON and unit 2 delivered more power with less production cost. The startup cost is remain same as given in [22].

In this paper, the comparison between the proposed MHPSO and other techniques such as GA [17, 33], UCC-GA [31], DP [17], LR [17], LRGA [32] and ICGA [33] and HPSO [22] for 10 unit system is done in Table V. The proposed method evolves the better results than the methods mentioned in Table V except the best value by HPSO. If we compare the average value of production cost for all methods, then the proposed MHPSO gives the better value. The percentage error in the minimum production cost is also shown in Table V where the error is calculated with respect to the best value of production cost so far. For the proposed MHPSO, %error is 0 because the best average value for production cost is obtained by the proposed method. Average value of HPSO shows a smaller percentage error because it is close to the best average value. However, difference between the average values of production cost for rest of the methods and the proposed MHPSO is large which results in higher percentage error.

Table III also shows the statistical values for larger unit systems to investigate the performance of the proposed method. For 60 units system, the standard deviation is 0.023% of average cost. Similarly, 20 and 40 units systems show 0.002% and 0.0154% of standard deviation with respect to the average cost, respectively. This shows consistence performance of proposed method for ever larger systems.

Table III. MEAN, MEDIAN, BEST, WORST AND
STANDARD DEVIATION OF 10 RUNS
FOR DIFFERENT SIZE SYSTEMS

Number of units	Mean ($)	Median ($)	Best ($)	Worst ($)	SD($)
10	564423.9	564421.8	564419	564432.7	4.4
20	1126370.6	1126212.1	1126089.2	1126759.2	258.4
40	2253301.5	2253376.5	2252707.5	2253667.1	354,5
60	3377856.7	3377928.3	3376707.5	3378907	786.9

Table V. COMPARISION OF RESULTS FOR 10-UNIT SYSTEMS

Methods	Total cost ($)			% Error in avg.value
	Best	Average	Worst	
GA[17, 33]	565825	567367	570032	0.52
UCC-GA[31]	563977	N/A	565606	
DP[17]	565825	N/A	N/A	
LR[17]	N/A	565825	N/A	0.25
LRGA[32]	564800	N/A	N/A	
ICGA[33]	N/A	566404	N/A	0.35
HPSO[22]	563942.3	564772.3	565785.3	0.06
MHPSO	564419	564423.9	564432.7	0

In study [22], HPSO algorithm was tested for only 10 unit system. Hence, comparison between MHPSO and HPSO can be made for larger unit system. However, the average cost for 20 units, 40 units and 60 units system given by MHPSO are compared with LR [17], GA [33] and ICGA [33]. Table VI shows that the average cost generated by proposed MHSPO is better than the average cost evolved by other methods. This shows the superiority of MHPSO compared to LR, GA and ICGA methods not only for small scale system but also for the large scale systems. Because of the limitation of page limits, only the optimal schedule of 20 units is shown in Table VII.

Table VI. COMPARISION OF RESULTS FOR LARGE SCALE SYSTEMS

Number of units	Average cost ($)			
	LR[17]	GA[33]	ICGA[33]	MHPSO
10	565825	567367	566404	564423.9
20	1130660	1130291	1127244	1126370.6
40	2258503	2256590	2254123	2253301.5
60	3394066	3382913	3378108	3377856.7

V. CONCLUSION

A modified hybrid PSO method was proposed in this paper that outperformed the results of other optimization techniques for smaller to larger unit system. The concept of heuristic based constraint satisfaction strategy worked well for MHPSO method. The statistical results further supported the consistent performance of proposed MHPSO.

Since, the values of *c1* and *c2* of equation (9) are kept constant, they might not be the most appropriate values to get the best possible solution. However, it shown elsewhere that a good combination of c1 and c2 can help evolving better results and also can assist PSO for faster convergence. Hence, a possible future work can be to consider *c1* and *c2* as variables and let PSO decides the optimum combination.

VI. ACKNOWLEGMENT

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Table VII. OPTIMAL SOLUTION FOR 20 UNITS SYSTEM

Unit/Hour	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12
Unit 1	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00
Unit 2	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00
Unit 3	339.99	439.99	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00
Unit 4	150.01	150.01	310.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00
Unit 5	0.00	0.00	0.00	0.00	0.00	0.00	130.00	130.00	130.00	130.00	130.00	130.00
Unit 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	130.00	130.00	130.00	130.00	130.00
Unit 7	0.00	0.00	0.00	0.00	0.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00
Unit 8	0.00	0.00	0.00	0.00	130.00	130.00	130.00	120.00	130.00	130.00	130.00	130.00
Unit 9	0.00	0.00	0.00	55.00	25.00	75.00	45.00	25.00	162.00	162.00	162.00	162.00
Unit 10	0.00	0.00	25.00	25.00	25.00	25.00	25.00	25.00	33.00	162.00	162.00	162.00
Unit 11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	20.00	36.09	80.00	80.00
Unit 12	0.00	0.00	0.00	0.00	0.00	20.00	20.00	20.00	20.00	25.25	54.51	80.00
Unit 13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	25.00	29.65	29.12	81.71
Unit 14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	25.00	29.97	30.38
Unit 15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	12.39	11.07
Unit 16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	10.00	11.08
Unit 17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	11.77
Unit 18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	10.00
Unit 19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00
Unit 20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00
Power	1400.00	1500.00	1700.00	1900.00	2000.00	2200.00	2300.00	2400.00	2600.00	2800.00	2900.00	3000.00
Fuel Cost	27371.85	29122.03	33114.50	37197.13	39457.24	44140.84	46427.31	48749.27	53871.89	60139.11	63882.17	67886.55
Start-up	0.00	0.00	900.00	900.00	560.00	1460.00	1100.00	1100.00	860.00	640.00	120.00	120.00

Unit/Hour	H13	H14	H15	H16	H17	H18	H19	H20	H21	H22	H23	H24	
Unit 1	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	
Unit 2	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	
Unit 3	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	
Unit 4	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	410.00	235.00	
Unit 5	130.00	130.00	130.00	130.00	69.99	130.00	130.00	130.00	130.00	130.00	0.00	0.00	
Unit 6	130.00	130.00	130.00	59.99	20.00	130.00	130.00	130.00	130.00	0.00	0.00	0.00	
Unit 7	130.00	130.00	130.00	20.00	20.00	50.00	130.00	130.00	130.00	0.00	0.00	0.00	
Unit 8	130.00	130.00	130.00	20.00	20.00	20.00	130.00	130.00	0.00	0.00	0.00	0.00	
Unit 9	162.00	162.00	35.00	25.00	25.00	25.00	35.00	162.00	162.00	160.00	25.00	0.00	
Unit 10	162.00	33.00	25.00	25.00	25.00	25.00	25.00	162.00	137.99	0.00	0.00	0.00	
Unit 11	37.14	20.00	0.00	0.00	0.00	0.00	0.00	37.77	20.00	20.00	0.00	0.00	
Unit 12	25.13	20.00	0.00	0.00	0.00	0.00	0.00	22.12	20.00	20.00	0.00	0.00	
Unit 13	27.44	25.00	0.00	0.00	0.00	0.00	0.00	29.66	25.00	25.00	0.00	0.00	
Unit 14	26.28	0.00	0.00	0.00	0.00	0.00	0.00	26.44	25.00	25.00	0.00	0.00	
Unit 15	10.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	0.00	0.00	0.00	0.00	
Unit 16	10.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	0.00	0.00	0.00	0.00	
Unit 17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
Unit 18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
Unit 19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
Unit 20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
Power	2800.00	2600.00	2400.00	2100.00	2000.00	2200.00	2400.00	2800.00	2600.00	2200.00	1800.00	1600.00	
Fuel Cost	60134.47	53871.89	48300.88	43213.28	41522.87	44901.30	48300.88	60146.95	54325.21	45286.38	34862.83	30862.34	1117089.17
Start-up	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1240.00	0.00	0.00	0.00	0.00	9000.00
												Total	1126089.17

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