Topology Optimization of Low Frequency Structure with Application to Vibration Energy Harvester

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ABSTRACT

In this paper, topology optimization for minimizing first natural frequency is presented. This problem has application in the design of an energy harvester for harvesting omnipresent low frequency seismic noise. The harvested energy can be power source for the wireless sensor network nodes. The optimization problem is posed and solved using deterministic optimization method. Two examples of beam like structures with different boundary conditions are solved and the optimal topologies are presented. In all the examples mass gets concentrated away from the fixed support(s) which is connected by a flexible structure.

Keywords: Energy harvester, Frequency minimization, topology optimization, Wireless sensor networks.

1. INTRODUCTION

With advent of microelectronics and wireless communication technologies, wireless sensor networks (WSNs) became prominence in sensing/accessing the data at remote locations. The application of WSNs ranges from manufacturing industry to structural health monitoring and surveillance [1, 2]. The advantage with the WSNs is that no wired connection is needed unlike other conventional sensors. Although the WSNs overcome the complications that are arising with wired connections, the energy supply becomes bottleneck for the sensors when they are operated for longer duration, i.e., for decades. Furthermore, there are limitations in using conventional electrochemical power sources for WSNs due to their short lifespan.

The problem of continuous energy supply for the sensors in WSNs can be addressed looking at renewable sources such as solar, wind, geothermal energies and seismic vibrations. Although solar, wind and geothermal energies are replacing conventional massive energy sources they cannot provide uninterrupted energy due to highly uncertain fluctuations. On the other hand, the seismic noise cannot provide massive energy but uninterrupted small quantity of energy can be obtained. Fortunately, WSNs demand small quantity of energy and thus seismic noise based vibration energy harvester can be potential solution to the continuous energy supply.

The frequency of omnipresent seismic noise is in the range of 0.17 Hz to 0.5 Hz [3]. Therefore, it is advantageous to design a structure with natural frequency is close to that of seismic noise, in order to extract maximum energy. We know from elementary theory of vibrations that the frequency of the structure is proportional to the square root of stiffness and inversely proportional to the square root of mass [4]. However, in structural design if the mass is removed from the domain of structure then stiffness and mass simultaneously decreases. The decrement in stiffness favors the objective of lowering the frequency while the decrement in mass creates unfavorable condition. Therefore, to resolve the contradicting nature of mass and stiffness to lower the frequency, the topology optimization problem is posed to minimize the structural frequency.

The topology optimization originally introduced to address the problem of maximize stiffness of structures [5]. Later, the topology optimization technique has been extended to the design of compliant [5, 6] and also to the frequency maximization [7]. The frequency maximization suffers with numerical artefact called localized mode [8]. The remedies for the localized mode are proposed by penalizing stiffness of low density elements [8]. In case of frequency minimization, the localize mode would not create a numerical issue as low stiffness aids lowering the frequency. We note that the maximizing frequency is motivated by the civil engineering and automobile problems. Whereas minimizing frequency is motivated by the energy harvester. The problem is solved using both deterministic method and genetic algorithm.

2. TOPOLOGY OPTIMIZATION FOR MINIMIZING THE FREQUENCY

In this section the problem formulation for minimize the structural frequency and sensitivity calculation are presented in perspective of topology optimization and FEA.

Let ω_i be i^{th} natural frequency of structure. Let λ_i be square of i^{th} natural frequency. Then the problem is defined for minimize the i^{th} natural frequency as follows

$$\text{Objective function} \quad \min_{\rho_e} \{\lambda_{min} = \min_{i} \lambda_i\}$$

$$s.t.: (K - \lambda_i M)\phi_i = 0, \qquad i = 1, \dots \dots \dots . N_{dof,} \tag{1}$$

$$\textstyle\sum_{e=1}^{N} v_e \rho_e \leq V \tag{2}$$

$$0 < \rho_{min} \leq \rho_e \leq 1, \quad e = 1, \dots \dots \dots \dots N \tag{3}$$

In above "Eqs (1), (2) and (3)" N denotes the total number of elements in the FE discretization. The design variable 'ρ_e', $e = 1, \dots \dots \dots \dots N$ represents the densities of the elements. An "Eq. (3)" specifies lower and upper limits ρ_{min} and 1 for ρ_e. To avoid singularity of the stiffness and mass matrix ρ_{min} is taken to be a small value for example $\rho_{min} = 1e - 6$. We also note that ρ_{min} should be chosen such that the performance of optimal design should not have considerable affect. In "Eq. (2)" the symbol V is the given volume of the design domain and v_e is the elemental volume.

Now, Sensitivity calculation of objective function:

The natural frequency f_i of the designated mode is related to the eigenvalue λ_i as shown in the following equation:

$$f_i = \frac{\sqrt{\lambda_i}}{2\pi}. \tag{4}$$

Taking the differentiation of frequency with respect to the design variable $\boldsymbol{\rho}$, we get

$$\frac{\partial f_i}{\partial \boldsymbol{\rho}} = \frac{1}{4\pi\sqrt{\lambda_i}} \frac{\partial \lambda_i}{\partial \boldsymbol{\rho}}. \tag{5}$$

Clearly, it is sufficient to find the differentiation of i^{th} eigenvalue with respect to design variable. We now use the state equation to obtain the sensitivities of i^{th} eigenvalue. The state equation for the natural frequencies of a linearly elastic structure is given by

$$[K(\boldsymbol{\rho}) - \lambda_i(\boldsymbol{\rho})M(\boldsymbol{\rho})]\varphi_i(\boldsymbol{\rho}) = 0, \tag{6}$$

Where, K is the global stiffness matrix, M is the global mass matrix, λ_i is i^{th} natural frequency and ϕ_i is the corresponding eigenvector.

Taking the differentiation of "Eq. (6)" with respect to design variable $\boldsymbol{\rho}$ and then pre-multiplying by transpose of eigenvector ϕ_i, we obtain

$$\varphi_i{}^T\left[\frac{\partial K}{\partial \boldsymbol{\rho}}-\frac{\partial \lambda_i}{\partial \boldsymbol{\rho}}M-\lambda_i\frac{\partial M}{\partial \boldsymbol{\rho}}\right]\varphi_i+\varphi_i{}^T[K-\lambda_i M]\frac{\partial \varphi_i}{\partial \boldsymbol{\rho}}=0\,. \tag{7}$$

Substitution of "Eq. (6)" in "Eq. (7)" yields

$$\varphi_i{}^T\left[\frac{\partial K}{\partial \boldsymbol{\rho}}-\frac{\partial \lambda_i}{\partial \boldsymbol{\rho}}M-\lambda_i\frac{\partial M}{\partial \boldsymbol{\rho}}\right]\varphi_i=0. \tag{8}$$

The sensitivities of eigenvalue follow from the rearrangement of "Eq. (8)", i.e.

$$\frac{\partial \lambda_i}{\partial \boldsymbol{\rho}}=\frac{1}{c_i}\varphi_i{}^T\left[\frac{\partial K}{\partial \boldsymbol{\rho}}-\lambda_i\frac{\partial M}{\partial \boldsymbol{\rho}}\right]\varphi_i\,, \tag{9}$$

Where $c_i=\varphi_i{}^T M\varphi_i$, we note that the non-trivial solution to "Eq. (6)" should be obtained in order to get ϕ_i. Let mass normalization for eigenvector i.e. $c_i=\varphi_i{}^T M\varphi_i=1$. Then "Eq. (9)" written as

$$\frac{\partial \lambda_i}{\partial \boldsymbol{\rho}}=\varphi_i{}^T\left[\frac{\partial K}{\partial \boldsymbol{\rho}}-\lambda_i\frac{\partial M}{\partial \boldsymbol{\rho}}\right]\varphi_i. \tag{10}$$

"Eq. (10)" represents the sensitivity of eigenvalue with respect to design variable. The substitution of "Eq. (10)" in "Eq. (5)" yields the following sensitivities of frequency with respect to design variables:

$$\frac{\partial f_i}{\partial \boldsymbol{\rho}}=\frac{1}{4\pi\sqrt{\lambda_i}}\,\varphi_i{}^T\left[\frac{\partial K}{\partial \boldsymbol{\rho}}-\lambda_i\frac{\partial M}{\partial \boldsymbol{\rho}}\right]\varphi_i. \tag{11}$$

3. RESULTS AND DISCUSSION

We consider beam like structure with two boundary conditions. In first example, one end of the beam is fixed so that it behaves like cantilever beam and in second example, both ends are fixed so that it behaves like fixed-fixed beam. The first natural frequency is minimized in both the examples. The size of design domain is assumed to be 6 m X 1 m rectangular domain in both the examples. It is discretized using four node rectangular element. It is also assumed to be 1 m out of plane thickness and assumed to be plane stress problem. The design domain is solved for 50% volume fraction. The initial guess for the optimization is taken as uniformly distributed mass over the whole design domain. To find out mass and stiffness matrix of each element to calculate objective function i.e. first natural frequency, the SIMP (Solid Isotropic Material with Penalization) formulation is used. The “fmincon” function (sequential quadratic programming) in MATALB optimization tool is used for the optimization or to update the design variable i.e. density of each element to minimize the first frequency based on sensitivity of objective function.

3.1 Example-1

The beam like structure with one end fixed, as shown in Fig.(1). The convergence of objective function value is plotted in Fig. (2) with respect to iteration. It can be seen that after 220 iteration, the solution is converged to the approximate optimal solution. The topology of the approximate optimal solution is shown in “Fig. (3)” in which mass is concentrated toward the free end of the beam and it is connected to the fixed end by a flexible structure. Since, the density is so low near the fixed end which cannot be observed in “Fig. (3)”. This distribution of mass and stiffness provides minimum frequency for this example**.** In “Fig. (4)” first Eigen mode shape corresponding to optimal topology is plotted and “Fig. (5)” represent the first three natural frequency variation with iteration and it can be observed that all first three natural frequency decreases with iteration. Furthermore, the difference between second and third frequencies becomes very small.

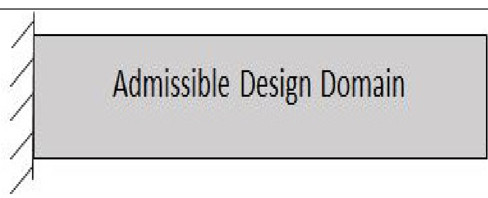


Figure 1. Cantilever beam like structure.

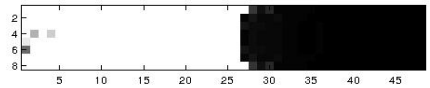


Figure 3. Topology of the approximate optimal

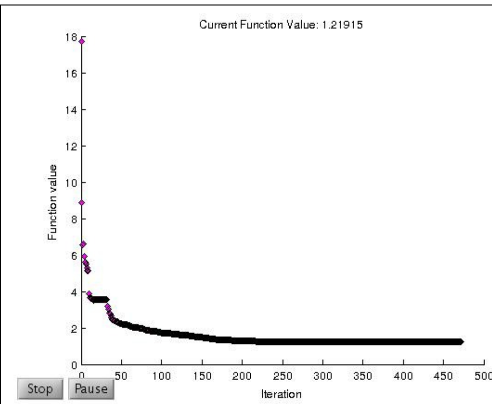
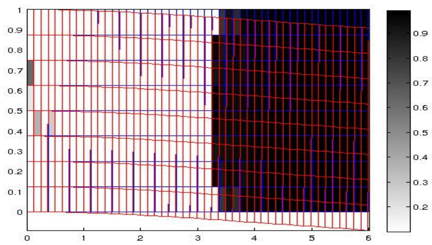
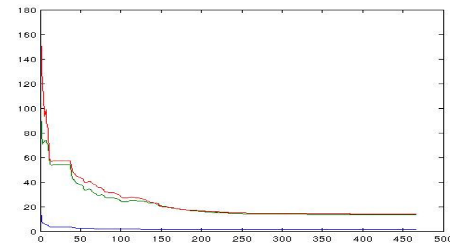


Figure 2. Convergence plot for first natural

<table>
<tr><td>solution</td><td>frequency</td></tr>
<tr><td>
Figure 4. First Eigen mode shape corresponding to optimal topology</td><td>
Figure 5. Iteration histories of the first three natural frequencies</td></tr>
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3.2 Example 2

The beam like structure with both ends are fixed as shown in “Fig. (6)”. The progress of approximate optimal solution with respect to iteration is shown in “Fig. (7)”. In this example, the approximate optimal solution is converged in 110 iterations. The topology of the approximate optimal solution is shown in “Fig. (8)” in which mass is concentrated away from fixed supports. In this example also, these concentrated mass are connected by elements with low density. In other words, the concentrated mass is connected to the support by a flexible structure. First Eigen mode shape corresponding to optimal topology is plotted in “Fig. (9)” and the first three natural frequency variation with iteration is plotted in “Fig. (10)” and it can be observed that all first three natural frequency decreases with iteration. Furthermore, the difference between second and third frequencies becomes very small similar to first example.

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Figure 6. Fixed beam like structure</td><td></td></tr>
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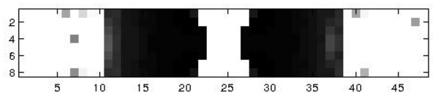


Figure 8. Topology of the approximate optimal solution

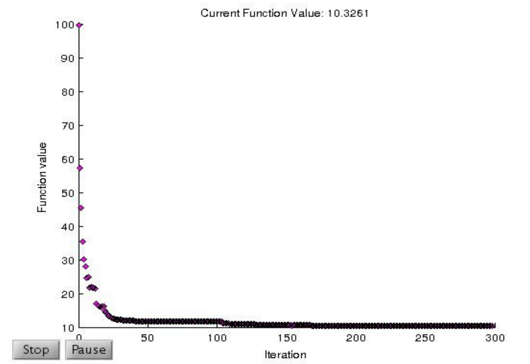


Figure 7. Convergence plot for first natural frequency

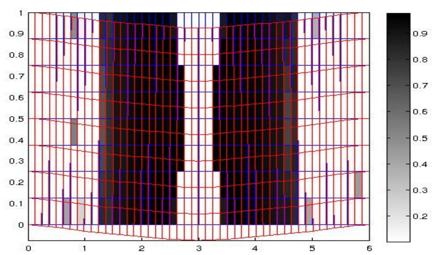


Figure 9. First Eigen mode shape corresponding to optimal topology

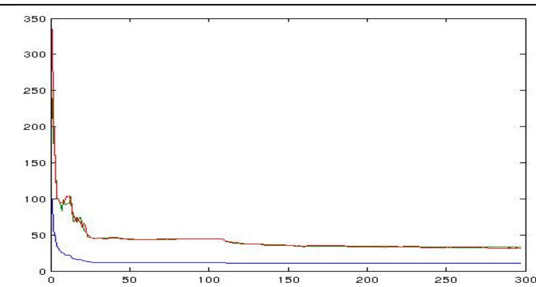


Figure 10. Iteration histories of the first three natural frequencies

In conclusion the low frequency structure is related to the flexible structure with heavy mass far from the support. In other words as the stiffness of the connecting structure between mass and support approaches small value then the frequency also approaches small value.

These results are also verified with genetic algorithm. Although the topology optimization do not provide a clear topologies but it gives the insight into the problem that a flexible structure should be connected to the concentrated mass. The mass should away from the support. We also performed simulation on intuitive design based on the outcome of topology optimization. In next section some intuitive designs are shown for low frequency.

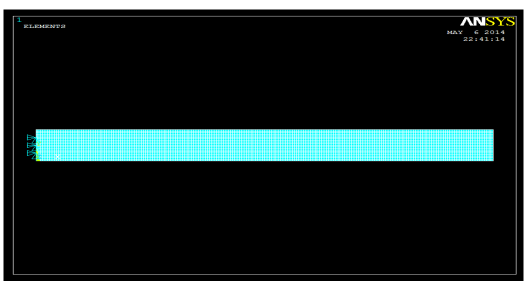
4. INTUITIVE DESIGNS OF THE LOW FREQUENCY STRUCTURES

As concluded in last section, a flexible structure is requires to connect the heavy mass and support. Of course, heavy mass is far from the support. Furthermore, all boundary condition indicates the same result i.e., the connecting structure should be as flexible as possible. Therefore, now our task is to come up with a design based on the observation.

We know that the cantilever beam is flexible among the cases that we dealt previously. Therefore, we present few designs based on the observation on optimal topology of cantilever beam. In all the designs we consider the solid mass occupies 50% of the domain towards the free end and flexible structure occupies 50% of the domain towards the fixed end. We also present the performance of cantilever beam in order to compare the result of our designs.

4.1 Example 1 Cantilever Beam

Here, we consider domain of size 200 mm and 20 mm along length and width respectively as shown in "Fig. (11a)". The out of plane thickness is taken as one mm. The Young's modulus, Poisson's ration and density of material are taken as 150 kPa, 0.17 and 2330 kg/mm^3. This problem has been solved using ANSYS finite element analysis (FEA) package using 4 node quadrilateral element assuming plane stress condition. We observed the first natural frequency as 20.354 Hz and the mode shape is shown in "Fig. (11b)". Later design will be compared with this example.



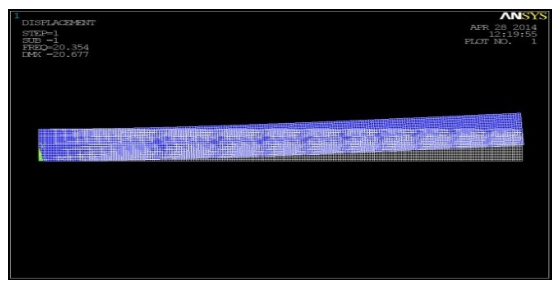


Figure 11. (a) Structure with cantilever beam boundary condition (b) First mode shape corresponding to natural frequency 20.354 Hz

In all the following examples the domain is considered as 200 mm and 20 mm along length and width, similar to the cantilever beam. The only difference is the material distribution or design of structure that connects mass and support. As mentioned, the 50% of domain occupies the solid mass towards the free end.

4.2 Example 2

Here, we consider thin beam like structures connected to solid material on the top and bottom as shown in "Fig. (12a)". The each beam consists the width 2 mm. We found the frequency corresponding to first natural mode is 3.364 Hz. The mode shape is shown in "Fig. (12b)".

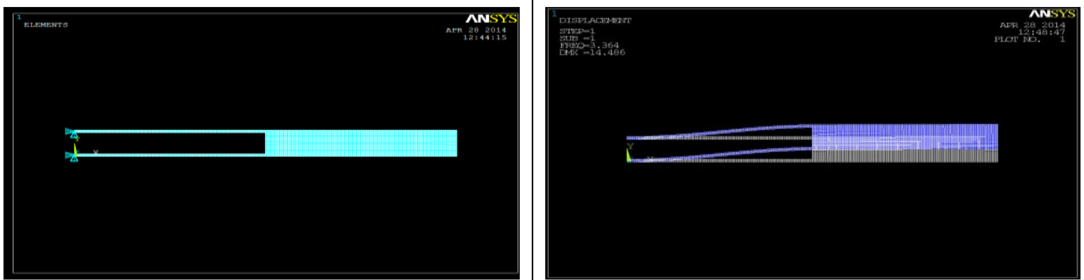


Figure 12. (a) Flexible structure with two beam in parallel (b) First mode shape corresponding to natural frequency 3.364 Hz

4.3 Example 3

As one can observe two beams are parallel in previous example. We now improved the flexibility of connecting structure by introducing folded beam like structure as shown in "Fig. (13a)". The folded beam can be thought of three beam are in series. Thus, improves the flexibility. The first frequency of the structure is observed to be 0.416 Hz whereas the mode shape is presented in "Fig. (13b)". In this example also beam thickness in folded beam is 2 mm. We also provided 7 mm gap between beams and 10 mm gap between heavy mass and right extreme of beam.

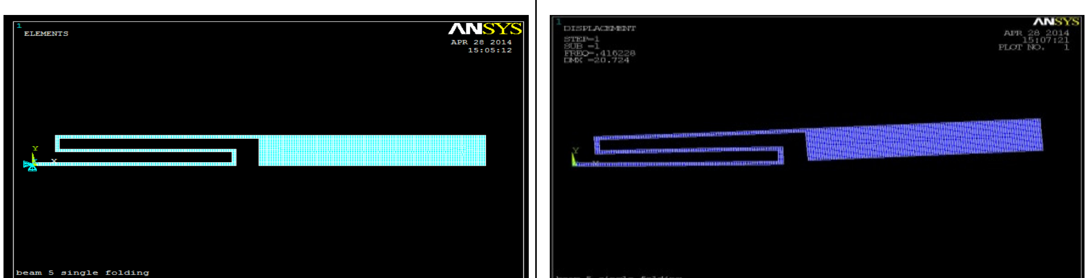


Figure 13. (a) Folded beam with three beams in series (b) First mode shape corresponding to natural frequency 0.416 Hz

4.4 Example 4

It clears that the no of folds in the folded beam increases then the frequency goes down. This fact is verified in this example by taking five beams in series as shown in "Fig. (14a)".Here also we consider 2mm thick beam in supporting structure. As the domain size is constant we decrease the gap between beams to 2.5 mm and other parameter remains same. The frequency is observed to be 0.3333 Hz whereas mode shape is shown in "Fig. (14b)". Now the problem comes with the manufacturing constraints and contact. The problem of contact is not taken care in linear analysis. While increasing the beam in supporting structure one should take care of this constraint.

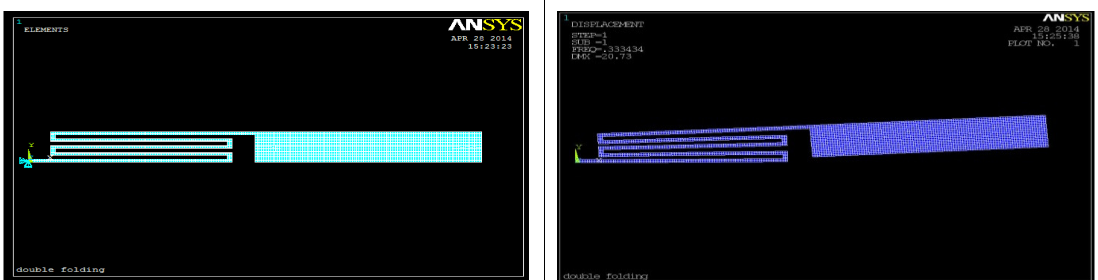


Figure 14. (a) Folded beam with five beams in series (b) First mode shape corresponding to natural frequency 0.333 Hz

4.5 Example 5

Previous two examples were not symmetric structures. Here we provided a symmetric structure that is flexible than the first example. Three folded beams structure is attached in parallel in order to get symmetric and as well as flexible structure as shown in "Fig. (15a)" Here, we decrease the gap between beams to 1.6 mm and other parameter remains same. The frequency is observed to be 1.234 Hz whereas mode shape is shown in "Fig. (15b)"

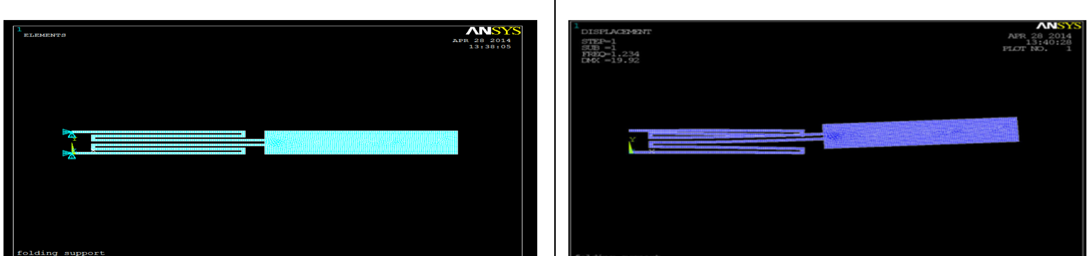


Figure 15. (a) Structure with three folded beams in parallel (b) First mode shape corresponding to natural frequency 1.234 Hz

From above examples we can attain the stiffness as low as possible by decreasing the beam thickness in supporting structure and increasing the number of beams in series. However manufacturing constraints will not allow vary low thickness beam. Furthermore, contact also becomes a problem while designing these flexible structures. Therefore, designer should take care of these factors while designing the low frequency structures.

5. CONCLUSION

In conclusion, beam like structure with two sets of boundary condition i.e. one end fixed and both ends fixed are solved for minimizing the first natural frequency. In both cases 50% volume fraction of design domain is considered where high minimum frequency was observed in case of volume fraction other than 50% of design domain. With 50% volume fraction of design domain the low frequency structure is related to the flexible structure with heavy mass far from the support means stiffness of the connecting structure between mass and support approaches small value then the frequency also approaches small value.

Based on the outcome of topology optimization, few intuitive designs were simulated where 50% of volume fraction is connected with boundaries with flexible structure. Foldable beam like structure is used as flexible structure to connect 50% of volume fraction to boundaries and was found that as we increase the number of folds in beam leads to decrease in frequency. Minimum first natural frequency of 0.33Hz is obtained by connecting 50% volume fraction with flexible structure includes 5 beams folded together with 2 mm beam thickness and gap of 2.5 mm. The problem of contact is not taken care in linear analysis. While increasing the beam in supporting structure one should take care of this constraint.

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