Line-Prioritized Environmental Selection and Normalization Scheme for Many-Objective Optimization using Reference-Lines-based Framework

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Abstract

The Pareto-dominance-based multi-objective evolutionary algorithms (MOEAs) have been successful in solving many test problems and other engineering optimization problems. However, their performance gets affected when solving more than 3-objective optimization problems due to lack of sufficient selection pressure. Many attempts have been made by the researchers toward improving the environmental selection of those MOEAs. One such attempt is selecting solutions using the reference-lines-based framework. In this paper, an efficient environmental selection and normalization scheme are proposed for this framework. The environmental selection operator is developed to equally prioritize solutions associated with different lines drawn from the origin and the reference points. A normalization scheme is also suggested in which the extreme point is used which gets updated on the designed rules. The framework is referred to as LEAF, and it is tested on 3-, 5-, 10-, and 15-objective DTLZ and WFG test instances. LEAF demonstrates its outperformance on almost all DTLZ instances and shows better performance on most of WFG instances over six MOEAs from the literature.

Keywords: Environmental Selection, Normalization, Many-objective optimization, Reference Lines.

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1. Introduction

A multi-objective optimization problem (MOP) is defined as

$$\begin{array}{ll} \min & \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})), \\ \texttt{s.t.} & \mathbf{x} \in \Omega, \end{array} \tag{1}$$

where M is the number of conflicting objectives, $f_i(\mathbf{x})$ is the i-th objective function, $\mathbf{x}$ is the vector of decision variables, and Ω is the search space. Solving a MOP generates a set of the optimal solutions which are known as the Pareto-optimal (PO) solutions. Evolutionary algorithms (EAs) have been a choice for solving such MOPs because EAs can generate the PO solutions in a single run. Therefore, EAs targeting MOPs, which is referred as MOEAs, are getting attention world-wide from more than two decades. Moreover, the real world problems, such as space trajectory [1], crash worthiness of vehicle [2], water resource management [3], scheduling [4], bulldozer blade in soil cutting [5] etc., have been modeled using multiple objectives that demand efficient MOEAs.

Existing MOEAs can be broadly categorized into three groups, such as Pareto-dominance-based MOEAs, decomposition-based MOEAs, and indicator-based MOEAs [6]. The decomposition-based MOEAs decompose a MOP into a set of single-objective optimization subproblems using aggregate functions. These subproblems are solved simultaneously to evolve a population of solutions. MOEA/D [7] is one such algorithm. Many variants of MOEA/D exist in the literature, such as $\theta-$DEA [8], improved decomposition-based evolutionary algorithm (I-DBEA) [9], use of vector angle distance scaling scheme in multiple single objective Pareto sampling (MSOPS) [10] to name a few. Similar to MOEA/D, the hyperdistance was calculated for each solution and the knee-points were found for each subpopulation, which were classified using the uniform weight vector [11]. The reported limitations of the decomposition-based MOEAs are the generation of uniformly distributed weight vectors for uniformly dividing the objective space and criterion for each sub-problem that can ensure convergence of solutions to the PO front.

The indicator-based MOEAs use performance indicator to assign a composite fitness to solutions ensuring their convergence and diversity on the PO front simultaneously [12]. The hypervolume (HV) indicator is found to be the most successful indicator, but its computation requirement increases exponentially with an increase in the number of objectives. Later, the Monte-Carlo technique was used to quantify HV [13]. Nevertheless, such algorithms need a higher computational effort as compared to other types of MOEAs, which increases with the number of objectives.

Unlike decomposition-based and indicator-based MOEAs, Pareto-dominance-based MOEAs have been successfully used for solving many two- to three-objective optimization problems [14]. However, for more than three objective problems, which are referred to as many-objective optimization problems (MaOPs), such algorithms fail to generate the well converged and diverse solutions over the entire PO front. The main reason is lack of generating enough selection pressure via Pareto-dominance ranking, which drives solutions toward the PO front. The selection pressure gets reduced because almost all solutions become non-dominated for a high-dimensional objective space [15]. Another reason behind reduced selection pressure is the diversity preserving operator, which is either not efficient for MaOPs, like crowding distance operator of NSGA-II [16], or computationally expensive, like $k-$th nearest neighborhood operator of SPEA2 [17]. It is noted that MOEAs for solving MaOPs are now referred to as MaOEAs in this paper.

In the literature, the Pareto-dominance-based MaOEAs have been improved either by modifying or relaxing the Pareto-dominance relation and/or by modifying the diversity preserving operator for a better environmental selection. The aim is to maintain enough selection pressure during the evolution process so that solutions can converge to the PO front. Kukkonen and Lampine [18] proposed a ranking-dominance relation as an alternative to the Pareto-dominance ranking. Many authors have explored a fuzzy dominance [19] in which the fuzzy numbers and arithmetic for $k-$optimality condition have been used [20]. Later, the fuzzy-dominance relation has been developed for comparing a pair of two solutions [21]. The fuzzy fitness is then used to sort solutions in different fronts similar to NSGA-II [16]. Similarly, $L-$dominance relation [22], $\alpha-$dominance relation [23], and angle-dominance criterion [24] have been developed to replace Pareto-dominance relation. From the above literature, it was observed that such modified or relaxed approaches could drive the search toward the PO front. However, the solutions can poorly represent the entire PF for MaOP.

Another approach for maintaining a sufficient environmental selection for the Pareto-dominance-based MOEAs is to enhance diversity. Adra and Fleming [25] proposed two diversity management mechanisms that were coupled with NSGA-II. Li et al. [26] proposed the shift-based density estimation (SDE) to pull the poorly converged solutions into the crowded regions for elimination. Deb and Jain [27] proposed NSGA-III in which a niching technique based on the structured reference points [28] on a unit hyperplane was proposed. Solutions are first normalized and then, associated with the closest reference line, which is drawn from the origin and one of the reference points. Thereafter, a niche count for each reference line is counted, which signifies the number of solutions associated with it. Solutions are then selected according to the ascending order of the niche count of the reference lines. Yang et al. [29] introduced a grid-based EA in which the fitness of each solution is determined by incorporating the grid ranking, the grid crowding distance, and the grid coordinate point distance criteria. Zhang and Li [30] used grids to eliminate dominance resistance solutions. Thereafter, the entropy-based reference distance is used to rank and select non-dominated solutions for the next generation. Chen et al. [31] introduced the dominant solution in the environmental selection in which a hyperplane was created for each solution by its neighboring solutions. After selecting prominent solutions, other non-dominated solutions were selected using the angle-based diversity operator.

Li et al. [32] incorporated decomposition and dominance at the environmental selection. The weight vectors are constructed with the help of structured reference points. A neighborhood for each weight vector is formed, which consists of T closest weight vectors. A restricted mating is then employed for creating an offspring in which a pair of two solutions is chosen from the common neighborhood. A unique subregion is also defined for each weight vector. When an offspring is added to the population, a solution from the most crowded region is removed. Jiang and Yang [33] proposed a composite fitness function which is determined using the local and global fitness functions. The local fitness function consists of a local row fitness and an angle-based density estimation. The reference directions similar to NSGA-III are used and subregions are defined for each reference direction. The local fitness is calculated among the solutions of the same subregion. The global fitness, similar to the raw fitness of SPEA2 [17], is assigned to each solution. Solutions based on the composite fitness are then selected from each subregion to fill the next generation population.

Xiang et al. [34] proposed a vector-angle based MaOEA in which the diversity is maintained through a maximum vector angle between the solutions. After non-dominated sorting the solutions in different fronts, the extreme solutions that have the minimum vector angle with the unit direction of each objective axis are selected to the population P. Thereafter, the first M solutions are copied to P based on the fitness. Rest of the solutions from the last front are copied one by one according to the maximum vector angle between the solution and the solutions in P. Zhang et al. [35] proposed the knee-solutions-based diversity preserving mechanism in which a hyperplane is constructed from the extreme solutions of the non-dominated front. The knee-solutions are identified by determining the maximum distance of these solutions in their neighborhood from the hyperplane. The $k-$ nearest neighbors approach is adopted for calculating the weighted distance of a solution.

Ibrahim et al. [36] proposed to keep an archive of elite solutions which may get eliminated using NSGA-III's environmental selection. The elite solution having the minimum distance from the ideal point for each reference line is selected to update the archive. Bi and Wang [37] improved NSGA-III by diving the objective space into $M-$subspaces. First, the $M-$clusters are created from the $N-$weight vectors using the $k-$means clustering algorithm. The central vector for each subspace is then found. A restricted mating in each subspace is performed to generate offspring. Each offspring solution is then assigned to a subregion by finding the minimum angle between a solution vector and the central weight vector. The niching technique is same as NSGA-III; however, PBI distance of MOEA/D is used, instead of association of NSGA-III. Recently, Chen and Li [38] proposed a diversity ranking method, and a reference vector adaptation method for environmental selection using NSGA-III framework. Liu et al. [39] implemented two strategies at the environmental selection in which the angle-based selection is used to find a pair of solutions with minimum angle between them. The worst solution is deleted using the shift-based density estimation. Liu et al. [40] proposed to generate reference points using the current population. The solutions, which were closest to the reference points, got selected for the next generation. Zhang et al. [41] proposed a dominance-based archiving approach in which a solution got selected from each subspace defined by the weight vectors. Using normalized distance method, redundant solutions from each subspace were deleted. Bai et al. [42] also followed the similar approach of selecting one solution a subspace. The angle-based truncation was also included to remove solutions gradually from the critical layer.

In the above studies, the diversity for selecting solutions has been maintained using the reference lines or vector framework. However, many times solutions are not associated with each reference line that leads to diversity loss. Since the Pareto-dominance ranking is not sufficient when many solutions are non-dominated, the environmental selection needs attention to select at least one solution from each reference line or vector. Some attempts have been made in the literature to focus on the selection of a diverse set of solutions from each reference line. NAEMO [43] is the recent attempt in which sub-archive for each reference line is maintained. Even if a single solution in any archive is dominated by another solution, it is retained to keep diversity. Restricted mating and various mutation strategies are also attempted. Periodic filtering is used to keep the archive size constant. On a similar line, the DoD approach [44] has been proposed in which diversity is given emphasis over dominance. For diversity, clusters of solutions for each reference lines are made using the association operator of NSGA-III and the best solution based on non-dominated sorting and distance to the line gets selected. In case there is no cluster for any reference line, then the solution closest to this line gets selected and makes a cluster. In the previous attempts like $\theta-$DEA [8], MOEA/DD [32] to name a few, the diversity has been given the priority over dominance. However, proper attention has not been given when some lines have no associated solution. Moreover, it has been reported in [8, 44, 45] that normalization is crucial to the reference-lines-based MaOEAs, like NSGA-III in which the population is normalized using the intercepts. However, the degenerate cases are evolved when a unique intercept on each axis cannot be found. Moreover, the negative intercept is also unacceptable. These issues have not been clearly described in NSGA-III. However, many MaOEAs handle this issue by considering the ideal and worst points of the non-dominated set to normalize the population [33, 34, 46], or worst objective value of the population [9, 37, 47, 48].

This paper targets two issues of MaOEAs that are developed using the reference-lines-based framework. The first issue is selecting a diverse set of solutions when some lines have no associated solution, and another issue is normalization of the population. Therefore, the paper has the following contributions. (1) A novel environmental selection is proposed, which equally prioritizes the reference lines and makes clusters of solutions by using association operator of NSGA-III. This selection operator then selects only one solution from each cluster. (2) When some lines have no cluster of solutions, re-association is introduced among the remaining solutions and clusters are made for further selection. The re-association is repeated till all lines have a cluster of at least one solution. (3) Moreover, a normalization scheme is also suggested in which an extreme point is used which gets updated under certain rules. (4) The proposed algorithm is tested on a wide range of DTLZ and WFG test instances up to 15 objectives and the results are compared with the outcome of six MaOEAs from the literature.

The paper is organized into five sections. In Section 2, the framework for MaOEA is described with the proposed environmental selection and normalization scheme. Section 3 presents details of simulation experiments for DTLZ and WFG test problems. The various parameters required to execute a set of MaOEAs are also presented. Section 4 presents a detailed analysis of the proposed algorithm with the existing MaOEAs by solving DTLZ and WFG test problems. Section 5 concludes the paper with some future work.

2. Proposed Algorithm

2.1. General Framework and Overview

The proposed algorithm is developed using the reference-lines-based framework similar to NSGA-III framework, which is shown in Algo. 1. The major inputs required by the framework are M, N, and H. A set of reference points (H) is generated using Das and Dennis approach [28], which is described later in Section 2.2.2. In this framework, a population with N solutions is initialized randomly. The global extreme point ($\mathbf{e}$) is used which is constructed from the maximum objective function values as shown at step 2 of Algo. 1. This extreme point will be utilized later in the proposed normalization scheme. In a typical generation t, a pair of solutions is selected randomly from the parent population (P_t) for performing crossover and mutation. Similarly, other pairs of solutions are selected randomly one by one without repetition for creating a mating pool. An offspring population (Q_t) is then created by using the simulated binary crossover and polynomial mutation operators [49] on the mating pool. In the present form of generational MaOEAs, the parent and offspring populations are combined to create R_t, which is ($P_t \ \cup \ Q_t$) of size $2N$. The combined population R_t is then sorted into the non-dominated fronts $(F_1, F_2, \dots)$ using the non-dominated sorting operator of NSGA-II. The fronts are then copied to a temporary population S_t until the size of S_t is equal to or less than N as shown at step 10. The population P_{t+1} is return, if the size of S_t is equal to N. Otherwise, solutions of F_l is included into S_t as shown at step 16. The proposed line-prioritized environmental selection then selects solutions from S_t to fill the next generation population P_{t+1} at step 17. In the following subsections, the environmental selection, normalization scheme and other operators are described.

2.2. Environmental Selection

Environmental selection is crucial for MaOEAs because it can maintain enough selection pressure when almost all solutions are non-dominated. For the proposed environmental selection, various inputs are required as shown in Algo. 2. The environmental selection involves normalization, association, and line-prioritized selection, which are described in the following sub-sections.

Algorithm 1 Framework for proposed algorithm

Input: Parameters, $t = 1$, M : objectives, N : population size, H : set of reference points

Output: A set of non-dominated solutions

Initialize random population (P_t)
Compute extreme point: $\mathbf{e} = (e_1, e_2, \ldots, e_M)^T$ such that $e_j = \max_{\mathbf{x} \in P_t} f_j(\mathbf{x})$
while $t \leq T$ **do**
  $P_t^{'}$ = Random selection (P_t)
  Q_t = Recombination + Mutation $(P_t^{'})$ % Offspring population
  $R_t = P_t \cup Q_t$
  $(F_1, F_2, \ldots)$ = Non-dominated sorting (R_t)
  $S_t = \emptyset$, $i = 1$
  while $|S_t \cup F_i| \leq$ N **do**
    $S_t = S_t \cup F_i$
    $i = i + 1$
  end while
  if $|S_t| = N$ **then**
    $P_{t+1} = S_t$ and return P_{t+1}
  else
    $S_t = S_t \cup F_l$ %F_l is the last front to be included.
    P_{t+1} = Environmental selection $(S_t, H, \mathbf{e})$
  end if
  $t = t + 1$
end while

Algorithm 2 Environmental selection $(S_t, H, \mathbf{e})$

Input: $S_t, H, \mathbf{e}$

Output: P_{t+1} of size N

$\bar{S}_t$ = Normalization $(S_t, \mathbf{e})$
$(\pi,\ d)$ = Associate $(\bar{S}_t, H, \rho)$% Initializing $\rho = \{0, \ldots, 0\}^T$, where $\rho \in R^{|H|}$
P_{t+1} of size N = Line-Prioritized-Selection $(\bar{S}_t, \pi, d)$

2.2.1. Proposed Normalization Scheme

The normalization scheme uses the extreme point ($\mathbf{e}$) which gets updated as presented in Algo. 3. First, the ideal point of population S_t is calculation at step 1. Thereafter, S_t gets translated by subtracting it from the ideal point (step 2). This translation makes the ideal point of translated population at the origin. A set of extreme solutions Z is created by using the augmented scalarizing function using (2) at step 3.

$$\mathbf{z}_j^e = \mathbf{f}^{'}(\mathbf{s}), \mathbf{s} : \min_{\mathbf{s} \in S_t} \left(\max_{i=1}^{M} f_i^{'}(\mathbf{s})/w_i \right), \tag{2}$$

where $\mathbf{z}_j^e$ is the extreme solution corresponding to $j-$th objective, $\mathbf{f}^{'}(\mathbf{s})$ is the translated objective vector of solution $\mathbf{s}$, $f_i^{'}(\mathbf{s})$ is the $i-$th component of $\mathbf{f}^{'}(\mathbf{s})$, and w_i's are the weights for each objective. In order to calculate $j-$ component, that is, $\mathbf{z}_j^e$, $w_j = 1$ and the rest of weights are kept 10^{-6}.

This set constructs a hyperplane for which intercepts are found to normalize the objectives (step 8). It is worth mentioning that the extreme solutions of Z create a system of linear equations which has to be solved for finding intercepts on each objective axis. But due to negative intercept or duplicate solutions in Z, it cannot be utilized for normalization. Therefore, the rules have been made when duplicate solutions in Z are identified. For example, if any duplicate is found, the Nadir point is found from the set of non-dominated solutions of S_t at step 6. Similarly, any intercept is negative as shown at step 11, the Nadir point is found. In the absence of duplicates or negative intercepts, the extreme point $\mathbf{e}$ is updated with the current intercepts as shown at step 14. Otherwise, a rule has been made to check at step 18 in which a component of the extreme point $\mathbf{e}$ gets updated, if the corresponding Nadir point component is smaller (step 19). Lastly, each translated objective function value is normalized by diving it by the extreme point $\mathbf{e}$ at step 22. This normalization scheme keeps the best intercept in each objective axis and gets updated for only two conditions as explained earlier. It is noted that a zero-pivot scenario can arise at step 8, while solving the system of linear equations. In that case, the extreme point $\mathbf{e}$ is calculated the same as mentioned at step 2 of Algo. 1.

2.2.2. Association

The purpose of the association is to assign the closest solution from $\bar{S}_t$ to each of the reference lines. First, a set of reference points (H) is created on a unit hyperplane using Das and Dennis approach [28]. In this approach, the structured points are created which are equally inclined to all objectives. The total number of reference points ($|H|$) that is created by dividing each objective axis into p divisions is given by

$$|H| = \binom{M+p-1}{p} \tag{3}$$

A three-objective case with $p = 5$ divisions is shown in Fig. 1 in which $|H| = \binom{3+5-1}{5}$ or 21 reference points are created. It has been mentioned

Algorithm 3 Normalization $(S_t, \mathbf{e})$

Input: $S_t, \mathbf{e}$
Output: $\bar{S}_t$

Compute ideal point, $\mathbf{z^I} = (z_1^I, z_2^I, \ldots, z_M^I)^T$ such that $z_j^I = \min_{\mathbf{s} \in S_t} f_j(\mathbf{s})$
Translate objectives, $\mathbf{f}^{'}(\mathbf{s}) = (f_1^{'}(\mathbf{s}), f_2^{'}(\mathbf{s}), \ldots, f_M^{'}(\mathbf{s}))^T$ such that $f_j^{'}(\mathbf{s}) = f_j(\mathbf{s}) - z_j^I, \ \forall \mathbf{s} \in S_t$
Compute extreme solutions, $Z = (\mathbf{z_1^e}, \mathbf{z_2^e}, \ldots, \mathbf{z_M^e})$ such that $\mathbf{z}_j^e = \mathbf{f}^{'}(\mathbf{s}), \mathbf{s} : \min_{\mathbf{s} \in S_t} \left(\max_{i=1}^{M} f_i^{'}(\mathbf{s})/w_i \right)$
Compute number of duplicate solutions (d) in Z
if $d > 0$ **then**
    Compute Nadir point, $\mathbf{z^N} = (z_1^N, z_2^N, \ldots, z_M^N)^T$ such that $z_j^N = \max_{\mathbf{s} \in S_t^*} f_j(\mathbf{s})$ and $S_t^* \in S_t$ is the set of the non-dominated solutions.
else
    Compute intercept $\mathbf{a} = (a_1, a_2, \ldots, a_M)^T$ from (Z)
    flag=0
    if $a_i < 0$ **then**
        Compute Nadir point, $\mathbf{z^N} = (z_1^N, z_2^N, \ldots, z_M^N)^T$ such that $z_j^N = \max_{\mathbf{s} \in S_t^*} f_j(\mathbf{s})$ and $S_t^* \in S_t$ is the set of the non-dominated solutions.
        flag $= 1$
    else
        $e_j = a_j, \ \forall j \in \{1, \ldots, M\}$
    end if
end if
if $d > 0$ or flag==1 **then**
    if $z_j^N < e_j$, where $j \in \{1, \ldots, M\}$ **then**
        $e_j = z_j^N$
    end if
end if
$\bar{f}_j(\mathbf{s}) = f_j^{'}(\mathbf{s})/e_j, \forall \mathbf{s} \in S_t, \ \forall j \in \{1, \ldots, M\}$

in [27] that when the number of objectives is higher, Das and Dennis approach [28] will create many reference points, which otherwise are not required. The limited number of reference points are then created using the two-layer approach in which the inner layer is half of the outer layer as shown in Fig. 2. Although the figure shows for the 3-objective case, it has been used for more than 5-objective optimization problems. In the two-layer approach, the reference points are created on those layers only with a relatively small number of divisions. In the figure, the outer layer has $p_1 = 3$ divisions and the inner layer has $p_2 = 1$ division that is creating 13 reference points.

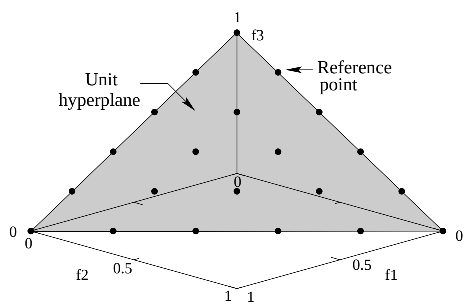

Figure 1: The structured reference points are shown on the unit hyperplane for a three-objective case when the number of divisions is $p = 5$.

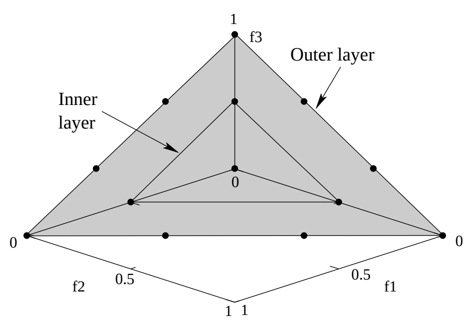

Figure 2: The structured reference points on the outer and inner layers are shown on the unit hyperplane for a three-objective case for understanding and visualization. The outer layer has $p_1 = 3$ divisions and the inner layer has $p_2 = 1$ division. The two-layer approach is used for more than 5-objective optimization problems.

In Algo. 4, a set of reference lines (W) is created such that each reference line passes through the origin and the corresponding reference point at step 2.

Thereafter, the solution from $\bar{S}_t$ are associated with those lines which have zero niche count ($\rho_{\mathbf{r}} = 0$) as shown at step 6 of Algo. 4. The niche count of a line is referred to as the number of solutions associated with it. For each solution $\mathbf{s} \in \bar{S}_t$, the closest reference line is identified using steps 7 and 10. Thereafter, the nearest reference line to solution $\mathbf{s}$ is stored in $\pi(\mathbf{s})$ and its distance is stored in $d(\mathbf{s})$. It can be observed that this association is the same as NSGA-III's association when the niche count of every reference line is zero. However, a line having any associated solution is not considered further for the association. This is referred to as re-association, which is used in the line-prioritized environmental selection in the following subsection.

Algorithm 4 Associate $(\bar{S}_t, H, \rho)$

Input: $\bar{S}_t$, H, $\rho = (\rho_1, \rho_2, \ldots, \rho_H)^T$, initialize $W = \emptyset$.
Output: $\pi,\ d$

for $(i = 0,\ i \leq H,\ i++)$ **do**
  Compute reference line $\mathbf{r}$ and $W = W \cup \mathbf{r}$
end for
for all $\mathbf{s} \in \bar{S}_t$ **do**
  for all $\mathbf{r} \in W$ **do**
    if $\rho_{\mathbf{r}} == 0$ **then**
      Compute $dist(\mathbf{s}, \mathbf{r}) = ||(\mathbf{s} - \mathbf{r}^T\mathbf{sr}/||\mathbf{r}||^2)||$
    end if
  end for
  $\pi(\mathbf{s}) = \mathbf{r} : \operatorname{argmin} dist(\mathbf{s}, \mathbf{r})$ %Associates $\mathbf{s}$ to line $\pi(\mathbf{s})$
  $d(\mathbf{s}) = dist(\mathbf{s}, \pi(\mathbf{s}))$ %Stores minimum distance of $\mathbf{s}$ to $d(\mathbf{s})$
end for

2.2.3. Line-Prioritized Environmental Selection

The line-prioritized environmental selection is proposed to select at least one solution ($\mathbf{s} \in S_t$) representing each reference line to the next generation population P_{t+1}. It is developed in three stages which are shown at steps 1, 12, and 22 of Algo. 5. The purpose of the stage - 1 is to select the best ranked closest solution for every reference line. At the beginning of this stage, P_{t+1} is kept empty and the niche count (ρ) for every reference line is zero. The solutions of $\bar{S}_t$ are now associated with the reference lines as shown at step 3. The front-wise selection of solutions is initiated at step 4 in which the solutions of F_1 are considered first. Since these solutions can be associated with some reference lines, the solutions which are closest to those reference lines $\mathbf{r} \in H$ are selected. The niche count of those reference lines is increased by one. A condition is included at step 6 which ensures that only one solution for every reference line is selected into P_{t+1}. It is noted that solutions of F_1 may not be associated with some of the reference lines. In order to select one solution for those lines, the solutions from F_2 followed by other fronts $(F_3, \ldots, F_l)$ are considered. The solutions closest to the reference lines having zero niche count are selected into P_{t+1} and the niche count is updated. It can be observed that the selected best ranked closest solution to the reference lines may or may not belong to the front 1. However, diversity is preserved among the selected solutions of P_{t+1}.

The stage - 2 is then initiated at step 12, which has a purpose of selecting remaining solutions for those reference lines which has no associated solution in the stage - 1. Therefore, the remaining solutions from $\bar{S}_t$ are re-associated at step 14. It is important to note that these remaining solutions will be associated with those lines, which have zero niche count after the stage - 1 as shown at step 6 of Algo. 4. After re-association, the solutions closest to the reference lines having zero niche count are selected to P_{t+1} and the niche count is updated by one. In this stage also, only one solution is selected for the reference lines. It is important to note that the closest solution to the reference line is selected irrespective of its rank.

At last, the stage - 3 is initiated at step 22 which only has the purpose to fill the P_{t+1} up to its maximum size when $|H| < N$. As per the last re-association of the stage - 2, the closest solutions to any reference line is selected to P_{t+1}.

Algorithm 5 Line-Prioritized-Selection ($\bar{S}_t, \pi, d$)

Input: $\bar{S}_t, \pi, d$
Output: P_{t+1} of size N

1: % Stage 1: Selecting the best ranked closest solution to each line
2: $P_{t+1} = \emptyset$, $\rho = \{0, \ldots, 0\}^T$, where $|\rho| = |H|$
3: $(\pi,\ d) =$ Associate $(\bar{S}_t, H, \rho)$
4: **for** $i = 1 \to l$ **do** {% l refers to the index of the last front F_l}
5: **for all** $\mathbf{r} \in W$ **do**
6: **if** $\rho_{\mathbf{r}} == 0$ **then**
7: $P_{t+1} = P_{t+1} \cup \mathbf{s}\ :\ \operatorname{argmin} dist(\mathbf{s}, \mathbf{r}),\ \mathbf{s} \in F_i$
8: $\rho_{\mathbf{r}} = \rho_{\mathbf{r}} + 1,\ \bar{S}_t = \bar{S}_t / \mathbf{s}$
9: **end if**
10: **end for**
11: **end for**
12: % Stage 2: Re-associating and selecting solution to the remaining lines
13: **while** $|P_{t+1}| < N$ **do**
14: $(\pi,\ d) =$ Associate $(\bar{S}_t, H, \rho)$ % Re-associate the solutions which are not selected yet to the references lines which have zero niche count
15: **for all** $\mathbf{r} \in W$ **do**
16: **if** $\rho_{\mathbf{r}} == 0$ **then**
17: $P_{t+1} = P_{t+1} \cup \mathbf{s}\ :\ \operatorname{argmin} dist(\mathbf{s}, \mathbf{r})$
18: $\rho_{\mathbf{r}} = \rho_{\mathbf{r}} + 1,\ \bar{S}_t = \bar{S}_t / \mathbf{s}$
19: **end if**
20: **end for**
21: **end while**
22: % Stage 3: Filling P_{t+1} when $|H| < N$
23: **while** $|P_{t+1}| < N$ **do**
24: **for all** $\mathbf{r} \in W$ **do**
25: $P_{t+1} = P_{t+1} \cup \mathbf{s}\ :\ \operatorname{argmin} dist(\mathbf{s}, \mathbf{r})$
26: $\rho_{\mathbf{r}} = \rho_{\mathbf{r}} + 1,\ \bar{S}_t = \bar{S}_t / \mathbf{s}$
27: **end for**
28: **end while**

The graphical illustration of environmental selection of NSGA-III and LEAF for different cases is shown in Fig. 3. A combined population of $|P \cup Q| = 14$ solutions are shown in the figure from which $N = 7$ solutions will be selected using seven reference lines $(L_1, L_2, \ldots, L_7)$. In case - 1, the solutions are sorted in different fronts, such as $F_1 = 3, F_2 = 3, F_3 = 7, F_4 = 1$. Here, NSGA-III selects $\{1, 2, 3, 4, 5, 6\}$ from F_1 and F_2, and one random solution ($\{12\}$) from F_3. LEAF in this case selects $\{1, 3\}$ from lines L_1 and L_7. Now, the solutions from other lines are selected, such as $\{5\}$ from L_2 and $\{11, 12\}$ from lines L_4 and L_5 respectively. The stage-1 of Algo. 5 is over now. As of now, there is no solution selected from lines L_3 and L_6. LEAF then re-associates the remaining solutions with these lines and selects $\{9, 2\}$ as discussed in the stage-2 of Algo. 5.

In case - 2, the solutions are sorted in $F_1 = 10, F_2 = 4$ fronts. NSGA-III selects solutions $\{1, 3, 6, 8, 10\}$ and randomly selects solutions $\{4, 5\}$. For this case, LEAF selects solutions $\{1, 3, 6, 8, 13, 10\}$ from lines $L_1, L_2, L_4, L_5, L_6, L_7$ using the stage-1 of Algo. 5. Using re-association at stage-2 of the same algorithm, it selects $\{11\}$ for the remaining line L_3.

In case - 3, all solutions are non-dominated, that is, $F_1 = 14$. NSGA-III selects solutions $\{1, 4, 7, 10, 14\}$ and can randomly select $\{3, 9\}$. LEAF in this case selects solutions $\{1, 4, 7, 10, 14\}$ from lines L_1, L_3, L_4, L_5, L_7. For selecting solutions for the remaining lines L_2 and L_6, LEAF re-associates the remaining solutions with these two lines and selects $\{2, 12\}$. In all of the above cases, LEAF gives priority to select a diverse set of solutions from each reference line by associating and re-associating them.

2.3. Computational Complexity

The worst computational complexity is determined by considering population of 2N or $S_t = F_1 = 2$N. The environmental selection of LEAF involves normalization, association, and line-based selection. The worst complexity of normalization is $O(MN)$. The worst complexity of association is $O(MNH)$. The worst complexity of line-based selection is $O(N^2M)$. Therefore, the worst computational complexity of LEAF for one generation is either $O(N^2 log^{M-2} N)$ (non-dominated sorting) or $O(N^2M)$ (association), whichever is larger.

3. Details for Simulation Experiment

In this section, the details of various experiments performed for analyzing the performance of LEAF are presented. First, the test problems used for performance evaluation are discussed. Thereafter, the performance metrics are described. We also present a brief discussion on MaOEAs which have been taken for comparison. The settings for experiments are provided thereafter.

3.1. Test Problems

For comparison, two well-known test suits, DTLZ [50] and WFG [51], are used. Since both the test suits are scalable, the number of objectives considering in this paper is $M \in \{3, 5, 8, 10, 15\}$ for DTLZ problems and $M \in \{3, 5, 8, 10\}$ for

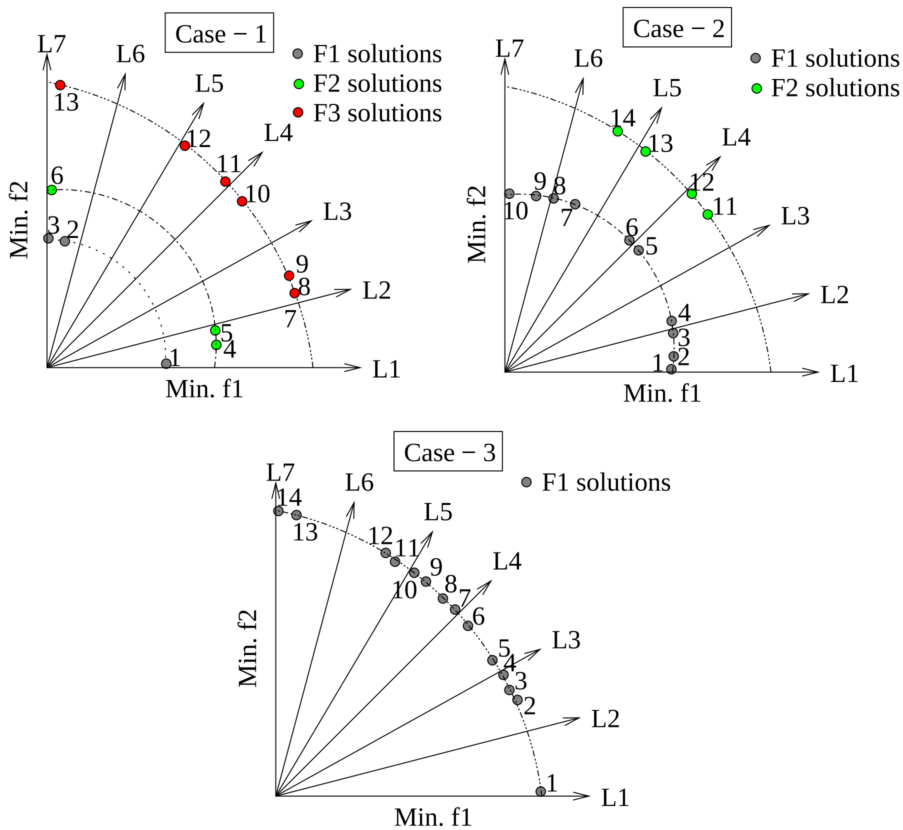

Figure 3: Different cases for comparing NSGA-III and LEAF. (1) Case - 1: $F_1 = 3, F_2 = 3, F_3 = 7, F_4 = 1$, (2) Case - 2: $F_1 = 10, F_2 = 4$, (3) Case - 3: $F_1 = 14$.

WFG problems. For DTLZ problems, the number of decision variables is given as $n = M + k - 1$, where $k = 5$ for DTLZ1, and $k = 10$ for DTLZ2-4 problems. For WFG1-9 problems, the number of decision variables is set to $n = k + l$ in which the position-related variable is $k = 2 \times (M - 1)$, and the distance-related variable is $l = 20$. The above test problems pose various challenges for MaOEAs in generating a well-converge and well-diverse set of solutions on the Pareto-optimal front. The characteristics of these problems are listed in Table 1.

Table 1: Characteristics of test problems

Test problems	Characteristics
DTLZ1	linear, multi-modal
DTLZ2	concave
DTLZ3	concave, multi-modal
DTLZ4	concave, biased
WFG1	mixed, biased
WFG2	convex, disconnected, multi-modal, non-separable
WFG3	linear, degenerate, non-separable
WFG4	concave, multi-modal
WFG5	concave, deceptive
WFG6	concave, non-separable
WFG7	concave, biased
WFG8	concave, biased, non-separable
WFG9	concave, biased, multi-modal, deceptive, non-separable

3.2. Performance Indicators

Two performance indicators are used to examine the performance of the proposed algorithm with the existing MaOEAs. The inverse generalized distance (IGD) indicator [7, 52] and hypervolume (HV) indicator [53] are used which can measure convergence, diversity, and spread of the obtained non-dominated solutions.

IGD indicator is calculated as,

$$IGD(\mathbf{Q}, \mathbf{P}^*) = \frac{\sum_{i=1}^{|P^*|} \min_{j=1}^{|Q|} d(p_i, q_j)}{|P^*|}, \tag{4}$$

where P^* is the set of PO solutions, Q is the set of obtained non-dominated solutions, $|P^*|$ is the cardinality of P^*, $|Q|$ is the cardinality of Q, and $d(p_i, q_j) = ||p_i - q_j||^2$. IGD indicator can measure convergence and diversity of the obtained non-dominated solutions with respect to the PO solutions. It has been a common choice for many recent studies presented in Section 1.

Another indicator is HV which measures the size of the objective space dominated by the solutions in Q and bounded by $\mathbf{z}^r$. It is determined as

$$HV(Q) = VOL\left(\bigcup_{\mathbf{x}\in\Omega}[f_1((x), z_i^r] \times \ldots [f_M((x), z_M^r]\right), \tag{5}$$

where $VOL(.)$ indicates the Lebesgue measure, and $\mathbf{z}^r = (z_1^r, \ldots, z_M^r)^T$ is the reference point in the objective space that is dominated by all Pareto-optimal solutions. The large is the HV value, the better is the quality of Q for approximating the PO front. For DTLZ1, $\mathbf{z}^r = (1, \ldots, 1)^T$ is chosen. For other problems DTLZ and WFG problems, $\mathbf{z}^r = (2, \ldots, 2)^T$ is considered. The HV values presented in this paper are normalized to $[0, 1]$ by dividing $z = \prod_{i=1}^{M} z_i^r$. Both the indicators are determined by normalizing Q, except for DTLZ1.

For IGD indicator, a set of the PO solutions P^* is required, which is calculated with the help of the reference lines created at step 2 of Algo. 4. The reference lines pass through the origin and their respective reference points. The points of intersection of these reference lines with the PO front is then evaluated which constitutes a set of the PO solutions, P^*. For DTLZ1 problem, the Pareto-optimal front is the hyperplane having intercepts at 0.5 in each objective in the first quadrant, that is, $\sum_{i=1}^{M} f_i = 0.5, \forall f_i \geq 0$. DTLZ2-4 problems have the hypersphere of radius one as the PO front in the first quadrant, that is, $\sum_{i=1}^{M} f_i^2 = 1, \forall f_i \geq 0$. WFG4-9 problems have the same PO front as of DTLZ2.

3.3. Significance Test

A difference for statistical significance is tested using the Wilcoxon signed-rank test [54] at 5% significance level for the assessment of obtained results from two competing MaOEAs.

3.4. Algorithms for Comparison

Six algorithms from the literature have been chosen for the comparison with LEAF, that are, NSGA-III [27], MOEA/D [7], SPEA2+SDE [26], SPEA/R [33], VaEA [34], and GrEA [29].

Since LEAF is using a similar framework of NSGA-III, the non-dominated sorting and association are the same. However, the environmental selection and normalization are different. The environmental selection of NSGA-III involves niche preservation in which the niche count for every reference line is determined for S_t population. Thereafter, the solutions from F_l are selected based on the least niche count of the reference lines. In LEAF, the environmental selection is performed by equally prioritizing the reference lines and selecting one solution for each line. Association and re-association are performed in three stages so that a diverse set of solutions gets selected. In normalization, determining intercept on each objective axis is same in both MaOEAs. However, for degenerate cases LEAF proposes a set of rules for updating the extreme point.

MOEA/D is a decomposition-based MaOEA in which MaOP is decomposed into many single-objective optimization subproblems using the aggregate function (penalty-based boundary interaction, PBI). All subproblems are solved simultaneously for predefined weight vectors. For performing crossover and local environmental selection, a neighborhood is defined for subproblem. MOEA/D and NSGA-III are the common choices for comparing the new algorithm in the literature [55].

SPEA2+SDE employs the shift-based density estimation as described in Section 1 for diversity and uses SPEA2 fitness for the environmental selection.

SPEA/R uses the $k-$layer reference direction generation approach, instead of [28]'s approach. It is developed on the diversity-first and convergence-second strategy. Therefore, solutions are associated with the reference direction first and then, the fitness, as described in Section 1, is used in the environmental selection. SPEA/R uses normalization using the ideal and Nadir points from the non-dominated front. It also allows restricted mating for crossover.

VaEA uses a similar framework of NSGA-III in which the worst solutions from the combined population of parent and offspring populations are used for normalization. The association is performed using the vector angle. Therefore, the environmental selection is done based on the maximum-vector-angle and worse-elimination principles.

GrEA uses the grid-based criteria for the fitness assignment in which the grid ranking, the grid crowding distance, and the grid coordinate point distance are used. In the environmental selection, the Pareto-ranking is used and the solutions of F_l is chosen based on the grid-based fitness.

3.5. Experiment Settings

3.5.1. Population Size

The population sizes, divisions and reference points for all MaOEAs are given in Table 2.

Table 2: Number of reference points and corresponding population sizes for MaOEAs

No. of obj. (M)	divisions p or (p_1, p_2)	No. of ref. points (H)	Population (N)
3	12	91	92
5	6	210	210
8	$(3, 2)$	156	156
10	$(3, 2)$	275	276
15	$(2, 1)$	135	136

3.5.2. Runs and Termination Criterion

All MaOEAs are run for 20 times with different initial population. The outcomes of 20 runs are stored for evaluating the performance indicators. The termination criterion is set for the maximum number of generation, which is the same as mentioned in [27]. Table 3 summarizes termination conditions for all problems.

Table 3: Maximum number of generations for terminating MaOEAs.

No. of objectives	DTLZ1	DTLZ2	DTLZ3	DTLZ4	WFG (all)
3	400	250	1000	600	1000
5	600	350	1000	1000	1250
8	750	500	1000	1250	1500
10	1000	750	1500	2000	2000
15	1500	1000	2000	3000	3000

3.5.3. Other Parameters

In all MaOEAs, the SBX and polynomial mutation operators [49] are used for generating offspring. The probability of crossover is set to 1.0, and the probability of mutation is $1/n$, where n is the number of variables. The distribution index for SBX operator is $\eta_c = 30$, and the distribution index for polynomial mutation operator is $\eta_m = 20$.

The number of division is set to 10 for GrEA. The archive size is set same as the population size for SPEA/R, and the number of $k-$layers for 3, 5, 8, 10 and 15 objectives for all problems is k =7, 8, 5, 6 and 3 respectively. The population size is determined as $N = 4 \times ceil((((M \times k \times (k+3)/2)+1)/4)$. The PBI approach is used in MOEA/D for which T is set to 20 and θ =5.

4. Results and Discussion

In this section, LEAF is compared with the existing MaOEAs and its performance is evaluated using IGD and HV indicators on DTLZ and WFG problems. First, the proposed normalization is implemented with NSGA-III code developed by the authors and the results are compared with the results of [27]. Later, the LEAF is compared with the set of MaOEAs.

4.1. Comparison with NSGA-III

We develop NSGA-III code[1] using the c-programming framework of NSGA-II[2]. The operators as defined in [27] are implemented, except normalization because it is not defined clearly for the degenerate cases. Therefore, the normalization proposed in this paper is used and referred to as NNSGA-III in which an extra 'N' stands for normalization. The same set of test problem instances is solved, which is reported in [27] with the same set of parameters for

[1]Source codes of LEAF and NNSGA-III at https://www.iitg.ac.in/dsharma/pub.html.
[2]http://www.egr.msu.edu/~kdeb/codes.shtml, Version: NSGA-II in C with gnuplot (Real + Binary + Constraint Handling), Revision 1.1

algorithm and test problems. Table 4 presents IGD values obtained from [27] study and from NNSGA-III. It can be seen that NNSGA-III is able to generate equivalent results reported in [27]. NNSGA-III shows better IGD values for 88 instances, and NSGA-III shows better IGD values for 47 instances. During the implementation of NNSGA-III, it was observed that the performance of the algorithm is sensitive toward the normalization technique. Since the source code of NSGA-III is not made available by its authors, many versions of NSGA-III codes are available made by other researchers and the results are compared in [8, 33, 34] and those results are not similar to the original NSGA-III results. In that scenario, NNSGA-III source can be used for a relative comparison.

4.2. Performance on DTLZ problems

Table 5 presents the statistical IGD values obtained from MaOEAs. It can be seen that LEAF shows better IGD values in 54 instances of DTLZ problems. In the remaining six instances, NSGA-III is the winner by showing better IGD values. Other MaOEAs fail to generate better IGD value for any instance. The table also shows a comparison of results based on the outcome of the Wilcoxon signed-rank test in which '+' indicates significantly better performance of LEAF over the corresponding MaOEA. Similarly, '−' and '=' signs indicate significantly bad performance and equivalent performance of LEAF over the corresponding MaOEA, respectively. A relative performance of LEAF is also found based on the Wilcoxon signed-rank test in which if LEAF is significantly better than MaOEA (i) ('+' sign in Table 5), then the score of MaOEA (i) is incremented by one. If MaOEA (i) is significantly better than LEAF ('−' sign in the same table), then the score of LEAF is incremented by one. In the case of equivalent performance ('=' sign in the table), the score is unchanged. This relative performance is similar to the performance score used in [8]. Table 9 presents a relative performance of MaOEAs based on IGD values in which the ratio indicates win/loss of LEAF over the corresponding MaOEA. It can be clearly seen that LEAF outperforms all MaOEAs in each objective dimension of all DTLZ problems.

Table 6 presents the statistical values of HV obtained from MaOEAs on DTLZ problems. SPEA2+SDE shows better HV values for 23 instances, mainly in 8- and 10-objective DTLZ2 and all objectives of DTLZ3 problems. LEAF shows better HV values for 21 instances, mainly in all objectives of DTLZ1, 3- and 5- objective of DTLZ2, and 5-objective DTLZ4 problems. NSGA-III and MOEA/D show better HV value for only one instance. The Wilcoxon signed-rank test is again used to compare results of MaOEAs, which is shown in the same table. A relative performance of MaOEAs using the same test is shown in Table 10 in which LEAF outperforms MOEA/D, VaEA and GrEA in each objective dimension of all DTLZ problems. Except for one instance, LEAF outperforms SPEA/R also. SPEA2+SDE shows better performance for 8- and 10-objective DTLZ2 and DTLZ3 problems over LEAF.

The parallel coordinates of 10-objective DTLZ1 obtained from MaOEAs are shown in Fig. 4. The non-dominated solutions shown in this figure is associated with the run of median IGD value. It can be seen that LEAF and VaEA are converged to the true PO front, whereas the rest of MaOEAs fail. For 10-objective DTLZ3 problem, the parallel coordinates are shown in Fig. 5 in which LEAF and SPEA2+SDE are converged to the true PO front. For 10-objective DTLZ4 problem, the parallel coordinates are shown in Fig. 6 in which all MaOEAs are converged to the true PO front.

4.3. Performance on WFG problems

Table 7 presents the statistical values of IGD obtained from MaOEAs for WFG problems. LEAF shows better IGD values for 58 instances, mainly in 5-objective WFG3, 3-, 5- and 8-objective WFG4, and complete WFG5, WFG6 and WFG7 problem instances. SPEA/R then shows better IGD values for 45 instances, mainly in 3-, 5-, 8- and 10-objective WFG2, 10- and 15-objective WFG4, 3-, 5-, 8- and 10-objective WFG8, and complete WFG9 problem. VaEA shows better IGD values for 20 instances, mainly in 3-, 8-, 10-, and 15-objective WFG1, 15-objective WFG2, and 8- and 15-objective WFG3 problems. SPEA2+SDE shows better IGD values for 12 instances, mainly in 5-objective WFG1, 3-objective WFG3, and 15-objective WFG8 problems.

The table also shows results from the Wilcoxon signed-rank test. A relative performance of MaOEAs based on the same test is presented in Table 11. LEAF performs significantly better than MOEA/D and GrEA in all objective dimensions. In comparison to SPEA2+SDE and VaEA, LEAF shows more wins in all objective dimensions. SPEA/R performs better than LEAF in 10-objective dimension and equivalent in 8-objective. LEAF performs significantly better than SPEA/R in 3-, 5- and 15-objective dimensions. In the problem-wise comparison, LEAF outperforms MOEA/D and GrEA in all WFG problems. SPEA2+SDE is better than LEAF in only WFG1 and WFG3 problems, otherwise it is outperformed by LEAF in rest of WFG problems. LEAF is better than SPEA/R in WFG1, WFG3, WFG5, WFG6 and WFG7 problems and shows equivalent performance in WFG4 problem. For WFG2, WFG8 and WFG9 problems, SPEA/R performs better than LEAF. VaEA performs better than LEAF in WFG1 problem and equivalent in WFG2 and WFG3 problems. For the rest of WFG problems, LEAF performs better than VaEA.

Table 8 presents the statistical HV values obtained from MaOEAs. LEAF shows better HV value for 52 instances, mainly in 10-objective WFG1, 5-objective WFG2, 3- and 5-objective WFG3 and WFG4, 3-, 5- and 10-objective WFG5, 3- and 5-objective WFG7, 10-objective WFG9 problems. SPEA/R then shows better HV values for 34 instances, mainly in 3- and 8-objective WFG2, 8- and 10-objective WFG4, 3- and 10-objective WFG6, 8- and 10-objective WFG7, and complete WFG8 problems. VaEA shows better HV values for 17 instances, mainly in 3- and 8-objective WFG1, 8- and 10-objective WFG3, and 8-objective WFG5. SPEA2+SDE show better HV values for 14 instances, in 5-objective WFG1 and WFG6, and 5- and 8-objective WFG9 problems. GrEA shows better HV value for 1 instance in WFG1.

The same table also shows the Wilcoxon signed-rank test results. A relative performance of MaOEAs based on the same test is presented in Table 12. For 3- and 5-objective dimensions, LEAF performs better than other MaOEAs.

For 8-objective, LEAF is unable to perform better than SPEA/R and VaEA. However, LEAF performs significantly better than MOEA/D, SPEA2+SDE and GrEA. For 10-objective dimension, LEAF performs better than MOEA/D, SPEA2+SDE, VaEA and GrEA, whereas SPEA/R performs equivalently. A relative comparison based on WFG problems is also shown in the table. LEAF performs significantly better than MOEA/D and GrEA in all WFG problems. In comparison with SPEA2+SDE, LEAF is better in WFG4 to WFG8 problems, and it is equivalent in WFG2 problem. LEAF is outperformed by SPEA2+SDE in WFG1, WFG3 and WFG9 problems. LEAF performs better than SPEA/R in WFG1, WFG3, WFG5, WFG6 and WFG9 problems. It is outperformed by SPEA/R in WFG2, WFG4 and WFG8 problems. Both of them are equivalent in WFG7 problem. LEAF is better than VaEA in WFG4 to WFG9 problems. LEAF is not better in WFG1 problem and equivalent in WFG2 and WFG3 problems against VaEA.

5. Conclusion

The line-prioritized environmental selection and normalization have been proposed and coupled with NSGA-III framework. The environmental selection operator selected a diverse set of solutions representing each reference line by associating and re-associating solutions. The external point was introduced, which got updated with the rules proposed in normalization. Based on IGD values, it can be concluded that LEAF outperformed all MaOEAs on all DTLZ problems' instances. For WFG problems, LEAF showed better performance in almost all instances. Based on HV values, LEAF again showed better performance over MaOEAs for most of the DTLZ and WFG problems' instances. Overall, it can be concluded that LEAF is emerged as one of the competitive algorithms and can be an alternative for preserving a diverse set of solutions for many-objective optimization. In the future, the concept of SPEA/R can be used with LEAF for better performance in which the diversity is preserved first over the dominance. Moreover, LEAF can be extended for solving constraint multi-objective optimization problems. Also, a parametric study can be done with LEAF.

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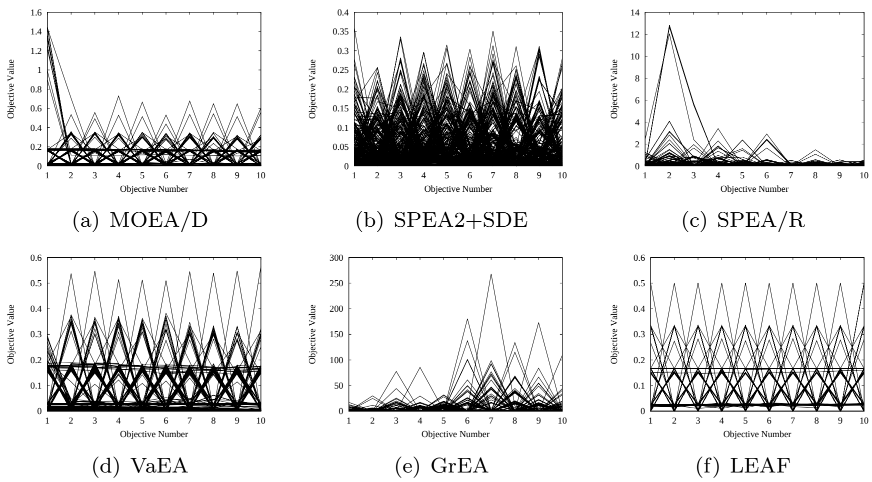
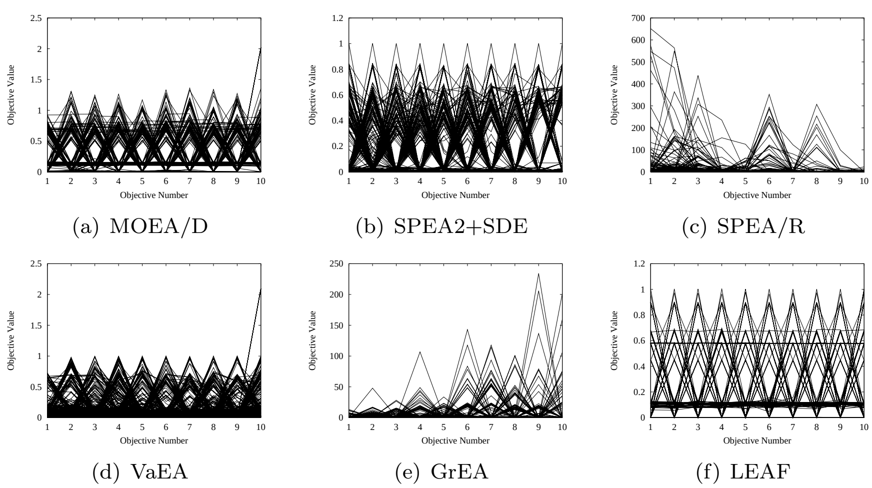


(a) MOEA/D (b) SPEA2+SDE (c) SPEA/R

(d) VaEA (e) GrEA (f) LEAF

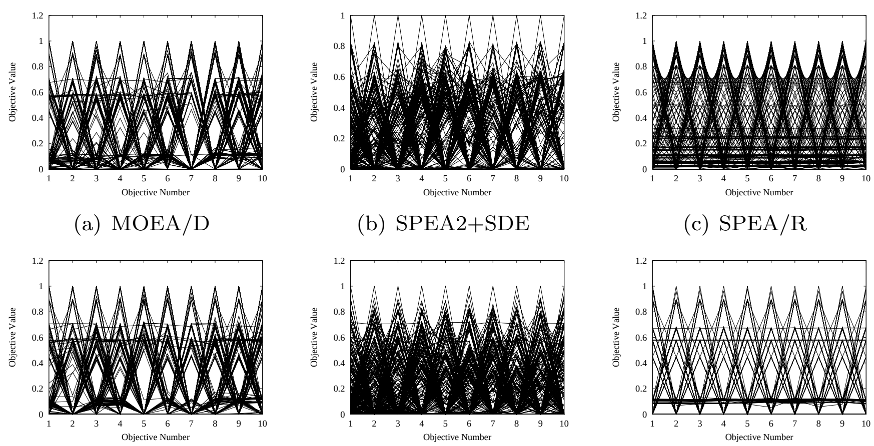
Figure 4: Parallel coordinates of non-dominated front obtained from MaOEAs for 10-objective DTLZ1 problem.



(a) MOEA/D (b) SPEA2+SDE (c) SPEA/R

(d) VaEA (e) GrEA (f) LEAF

Figure 5: Parallel coordinates of non-dominated front obtained from MaOEAs for 10-objective DTLZ3 problem.



(a) MOEA/D (b) SPEA2+SDE (c) SPEA/R

(d) VaEA (e) GrEA (f) LEAF

Figure 6: Parallel coordinates of non-dominated front obtained from MaOEAs for 10-objective DTLZ4 problem.

Table 4: Best, median and worst IGD values obtained for each objective from original NSGA-III and NNSGA-III on DTLZ1-4, WFG6-7, SDTLZ1-2 and CDTLZ1 instances with different number of objectives are presented. The best performances are highlighted in bold face with gray background.

	M	NSGA-III	NNSGA-III		M	NSGA-III	NNSGA-III
DTLZ1	3	4.880E-04	**3.510E-04**	DTLZ2	3	1.262E-03	**1.045E-03**
		1.308E-03	1.536E-03			1.357E-03	**1.270E-03**
		4.880E-03	5.787E-03			**2.114E-03**	2.870E-03
	5	5.116E-04	**4.962E-04**		5	4.254E-03	**3.058E-03**
		9.799E-04	**7.431E-04**			4.982E-03	**4.481E-03**
		1.979E-03	**1.246E-03**			**5.862E-03**	1.128E-02
	8	**2.044E-03**	2.175E-03		8	1.371E-02	**1.152E-02**
		3.979E-03	**3.582E-03**			1.571E-02	**1.293E-02**
		8.721E-03	6.645E-02			1.811E-02	**1.691E-02**
	10	**2.215E-03**	2.279E-03		10	1.350E-02	**1.142E-02**
		3.462E-03	**2.583E-03**			1.528E-02	**1.279E-02**
		6.869E-03	9.297E-02			1.697E-02	**1.486E-02**
	15	2.649E-03	**1.922E-03**		15	1.360E-02	**1.052E-02**
		5.063E-03	**2.853E-03**			1.726E-02	**1.428E-02**
		1.123E-02	**4.324E-03**			2.114E-02	**1.758E-02**

	M	NSGA-III	NNSGA-III		M	NSGA-III	NNSGA-III
DTLZ3	3	9.751E-04	**8.723E-04**	DTLZ4	3	**2.915E-04**	3.113E-04
		4.007E-03	**3.991E-03**			5.970E-04	**3.918E-04**
		6.665E-03	9.847E-03			**4.286E-01**	5.314E-01
	5	3.086E-03	**2.174E-03**		5	9.849E-04	**3.641E-04**
		5.960E-03	**3.675E-03**			1.255E-03	**4.334E-04**
		1.196E-02	**1.014E-02**			1.721E-03	**5.072E-04**
	8	**1.244E-02**	1.256E-02		8	5.079E-03	**2.541E-03**
		2.375E-02	2.444E-02			7.054E-03	**3.442E-03**
		9.649E-02	**5.287E-02**			6.051E-01	**5.319E-03**
	10	8.849E-03	**8.236E-03**		10	5.694E-03	**3.578E-03**
		1.188E-02	**1.069E-02**			6.337E-03	**4.228E-03**
		2.083E-02	**1.929E-02**			1.076E-01	**5.174E-03**
	15	1.401E-02	**1.121E-02**		15	7.110E-03	**5.257E-03**
		2.145E-02	**1.766E-02**			3.431E-01	**7.298E-03**
		4.195E-02	**3.671E-02**			1.073E+00	**9.578E-03**

	M	NSGA-III	NNSGA-III		M	NSGA-III	NNSGA-III
SDTLZ1	3	**3.853E-04**	5.146E-04	SDTLZ2	3	1.347E-03	**1.102E-03**
		1.214E-03	1.293E-03			2.069E-03	**1.790E-03**
		1.103E-02	**4.572E-03**			5.284E-03	**2.395E-03**
	5	1.099E-03	**7.883E-04**		5	**1.005E-02**	1.447E-02
		2.500E-03	**1.753E-03**			**2.564E-02**	5.579E-02
		3.921E-02	**2.572E-02**			**8.430E-02**	1.134E-01
	8	4.659E-03	**2.472E-03**		8	**1.582E-02**	1.733E-02
		1.051E-02	**6.086E-03**			**1.788E-02**	4.704E-02
		1.167E-01	**2.792E-02**			**2.089E-02**	2.256E-01
	10	3.403E-03	**2.562E-03**		10	**2.113E-02**	2.382E-02
		5.577E-03	**4.341E-03**			**3.334E-02**	4.699E-02
		3.617E-02	**3.136E-02**			2.095E-01	**1.756E-01**
	15	3.450E-03	**2.860E-03**		15	**2.165E-02**	2.520E-02
		6.183E-03	**3.786E-03**			**2.531E-02**	8.814E-02
		1.367E-02	**5.115E-03**			**4.450E-02**	3.780E-01

	M	NSGA-III	NNSGA-III
WFG6	3	**4.828E-03**	2.447E-02
		1.224E-02	2.831E-02
		5.486E-02	**3.511E-02**
	5	**5.065E-03**	2.903E-02
		1.965E-02	3.479E-02
		4.475E-02	**4.261E-02**
	8	**1.009E-02**	3.350E-02
		2.922E-02	3.837E-02
		7.098E-02	**4.294E-02**
	10	**1.060E-02**	2.793E-02
		2.491E-02	3.728E-02
		6.129E-02	**4.548E-02**
	15	**1.368E-02**	2.911E-02
		2.877E-02	3.599E-02
		6.970E-02	**4.709E-02**

	M	NSGA-III	NNSGA-III
WFG7	3	2.789E-03	**2.136E-03**
		3.692E-03	**2.572E-03**
		4.787E-03	**3.344E-03**
	5	8.249E-03	**6.704E-03**
		9.111E-03	**8.688E-03**
		1.050E-02	1.727E-02
	8	2.452E-02	**1.720E-02**
		2.911E-02	**2.039E-02**
		6.198E-02	**2.553E-02**
	10	3.228E-02	**2.167E-02**
		4.292E-02	**2.292E-02**
		9.071E-02	**2.441E-02**
	15	**3.457E-02**	7.278E-02
		5.450E-02	1.312E-01
		8.826E-02	4.640E-01

	M	NSGA-III	NNSGA-III
CDTLZ2	3	**2.603E-03**	5.045E-03
		4.404E-03	5.512E-03
		8.055E-03	1.998E-02
	5	**7.950E-03**	7.964E-03
		1.341E-02	**9.757E-03**
		1.917E-02	**1.613E-02**
	8	2.225E-02	**1.789E-02**
		2.986E-02	**2.557E-02**
		4.234E-02	**3.336E-02**
	10	7.389E-02	**1.552E-02**
		9.126E-02	**1.962E-02**
		1.051E-01	**2.711E-02**
	15	**2.169E-03**	3.063E-02
		2.769E-02	3.880E-02
		4.985E-02	2.369E-01

Table 5: Best, median and worst IGD values obtained for each objective by LEAF and other algorithms on DTLZ instances with different number of objectives. Best performances are highlighted in bold face with gray background.

	M	NSGA-III	MOEA-D	SPEA2+SDE	SPEA/R	VaEA	GrEA	LEAF
DTLZ1	3	**4.880E-04**	3.013E-02	2.133E-02	4.447E-03	1.280E-02	3.025E-02	5.286E-04
		1.308E-03	3.068E-02 +	2.251E-02 +	2.138E-02 +	4.899E-02 +	5.267E-02 +	**1.249E-03**
		4.880E-03	3.503E-02	2.431E-02	9.910E-02	4.039E-01	3.284E-01	**2.867E-03**
	5	5.116E-04	2.073E-01	4.919E-02	1.517E-02	1.898E-02	2.554E-01	**4.148E-04**
		9.799E-04	2.240E-01 +	5.154E-02 +	4.038E-02 +	3.401E-02 +	4.083E-01 +	**6.975E-04**
		1.979E-03	2.304E-01	5.266E-02	1.323E-01	6.372E-02	6.990E-01	**1.170E-03**
	8	2.044E-03	1.881E-01	8.934E-02	6.328E-02	1.933E-02	1.051E-01	**2.030E-03**
		3.979E-03	2.087E-01 +	9.485E-02 +	1.490E-01 +	2.722E-02 +	2.390E-01 +	**2.638E-03**
		8.721E-03	2.207E-01	9.838E-02	6.193E-01	4.981E-02	5.160E-01	**4.431E-03**
	10	2.215E-03	2.168E-01	9.308E-02	4.146E-02	2.214E-02	3.440E-01	**1.784E-03**
		3.462E-03	2.573E-01 +	9.772E-02 +	1.018E-01 +	2.939E-02 +	4.116E-01 +	**2.280E-03**
		6.869E-03	2.591E-01	1.058E-01	2.897E-01	3.972E-02	5.173E-01	**4.756E-03**
	15	2.649E-03	2.858E-01	1.487E-01	2.409E-01	4.884E-02	1.062E+02	**2.015E-03**
		5.063E-03	2.885E-01 +	1.571E-01 +	4.356E-01 +	5.362E-02 +	1.550E+02+	**3.019E-03**
		1.123E-02	2.887E-01	1.633E-01	2.969E+00	5.938E-02	3.746E+02	**4.356E-03**
DTLZ2	3	1.262E-03	7.068E-02	6.924E-02	3.125E-03	8.292E-03	6.904E-02	**1.029E-03**
		1.357E-03	7.199E-02 +	7.506E-02 +	5.074E-03 +	1.485E-02 +	7.355E-02 +	**1.308E-03**
		2.114E-03	7.366E-02	8.007E-02	1.128E-02	2.572E-02	7.626E-02	**1.912E-03**
	5	4.254E-03	7.321E-01	1.682E-01	9.941E-03	1.334E-02	1.362E-01	**3.265E-03**
		4.982E-03	7.323E-01 +	1.740E-01 +	1.308E-02 +	1.606E-02 +	1.439E-01 +	**4.359E-03**
		5.862E-03	7.324E-01	1.925E-01	2.120E-02	2.064E-02	1.520E-01	6.989E-03
	8	1.371E-02	6.267E-01	2.597E-01	2.307E-02	2.830E-02	2.931E-01	**1.065E-02**
		1.571E-02	6.754E-01 +	2.706E-01 +	2.849E-02 +	3.524E-02 +	3.015E-01 +	**1.326E-02**
		1.811E-02	7.166E-01	3.158E-01	3.342E-02	4.904E-02	3.116E-01	**1.474E-02**
	10	1.350E-02	9.178E-01	2.612E-01	2.455E-02	2.270E-02	3.379E-01	**1.030E-02**
		1.528E-02	9.183E-01 +	2.808E-01 +	2.893E-02 +	3.838E-02 +	3.459E-01 +	**1.195E-02**
		1.697E-02	9.184E-01	2.922E-01	3.689E-02	4.143E-02	3.511E-01	2.006E-02
	15	1.360E-02	1.026E+00	3.094E-01	4.607E-02	3.692E-02	4.701E-01	**1.055E-02**
		1.726E-02	1.029E+00+	3.340E-01 +	5.477E-02 +	6.588E-02 +	4.777E-01 +	**1.207E-02**
		2.114E-02	1.076E+00	3.529E-01	7.102E-02	1.485E-01	4.883E-01	**1.731E-02**
DTLZ3	3	**9.751E-04**	7.145E-02	6.824E-02	6.758E-03	1.955E-01	6.646E-02	1.600E-03
		4.007E-03	7.247E-02 +	7.413E-02 +	3.334E-02 +	1.052E+00+	7.371E-02 +	**3.049E-03**
		6.665E-03	7.983E-02	8.052E-02	2.413E-01	4.125E+00	4.205E-01	9.425E-03
	5	3.086E-03	6.892E-01	1.661E-01	7.925E-02	2.226E-02	5.402E-01	**1.447E-03**
		5.960E-03	7.323E-01 +	1.734E-01 +	2.060E-01 +	1.936E-01 +	7.693E-01 +	**3.637E-03**
		1.196E-02	2.435E+00	1.890E-01	3.801E-01	5.687E-01	1.123E+00	**7.537E-03**
	8	**1.244E-02**	6.348E-01	2.610E-01	3.811E-01	8.289E-02	3.100E-01	1.405E-02
		2.375E-02	6.792E-01 +	2.805E-01 +	2.296E+00+	8.487E-01 +	1.022E+00+	**2.236E-02**
		9.649E-02	8.422E-01	3.035E-01	4.493E+00	1.145E+00	1.229E+00	**5.302E-02**
	10	8.849E-03	8.950E-01	2.663E-01	3.882E-01	5.758E-02	9.290E-01	**7.087E-03**
		1.188E-02	9.174E-01 +	2.791E-01 +	6.790E-01 +	3.287E-01 +	1.219E+00+	**9.583E-03**
		2.083E-02	9.192E-01	3.065E-01	4.395E+00	1.169E+00	1.267E+00	**1.601E-02**
	15	1.401E-02	1.019E+00	3.295E-01	5.443E+00	6.600E-02	9.488E+01	**9.958E-03**
		2.145E-02	1.033E+00+	3.536E-01 +	1.207E+01+	1.280E+00+	2.027E+02+	**1.387E-02**
		4.195E-02	1.037E+00	3.755E-01	3.338E+01	1.301E+00	3.000E+02	**2.184E-02**
DTLZ4	3	2.915E-04	7.075E-02	7.268E-02	4.001E-04	7.698E-03	6.875E-02	**2.668E-04**
		5.970E-04	7.360E-02 +	7.525E-02 +	1.837E-03 +	2.267E-01 +	7.399E-02 +	**3.907E-04**
		4.286E-01	2.584E-01	5.399E-01	**4.983E-03**	9.503E-01	9.503E-01	5.306E-01
	5	9.849E-04	6.900E-01	1.596E-01	2.182E-03	1.641E-02	1.385E-01	**3.641E-04**
		1.255E-03	7.325E-01 +	1.747E-01 +	4.001E-03 +	1.939E-01 +	1.441E-01 +	**4.391E-04**
		1.721E-03	8.265E-01	3.967E-01	9.661E-03	3.947E-01	1.496E-01	**6.257E-04**
	8	5.079E-03	7.037E-01	2.553E-01	7.315E-03	3.326E-02	1.385E-01	**2.963E-03**
		7.054E-03	7.360E-01 +	2.741E-01 +	9.148E-03 +	2.380E-01 +	1.441E-01 +	**3.434E-03**
		6.051E-01	7.971E-01	3.815E-01	1.220E-02	6.228E-01	1.496E-01	**4.451E-03**
	10	5.694E-03	8.954E-01	2.641E-01	6.907E-03	4.088E-02	3.406E-01	**3.333E-03**
		6.337E-03	9.369E-01 +	2.780E-01 +	8.963E-03 +	1.843E-01 +	3.460E-01 +	**3.951E-03**
		1.076E-01	1.015E+00	2.944E-01	1.191E-02	3.770E-01	3.519E-01	**4.637E-03**
	15	7.110E-03	1.036E+00	2.974E-01	9.225E-03	1.226E-01	4.513E-01	**5.613E-03**
		3.431E-01	1.077E+00+	3.171E-01 +	1.115E-02 +	2.898E-01 +	4.613E-01 +	**8.156E-03**
		1.073E+00	1.142E+00	3.647E-01	1.475E-02	9.067E-01	4.741E-01	**1.319E-02**

Table 6: Best, median and worst HV values obtained for each objective by LEAF and other algorithms on DTLZ instances with different number of objectives. Best performances are highlighted in bold face with gray background.

	M	NSGA-III	MOEA-D	SPEA2+SDE	SPEA/R	VaEA	GrEA	LEAF
DTLZ1	3	9.73519E-01	9.67358E-01	9.67749E-01	9.73376E-01	9.70545E-01	9.64834E-01	**9.73583E-01**
		9.73217E-01	9.66735E-01^{+}	9.63163E-01 $^{+}$	9.70047E-01 $^{+}$	9.57151E-01^{+}	9.42057E-01^{+}	**9.73434E-01**
		9.71931E-01	9.63765E-01	9.56540E-01	9.44695E-01	4.78116E-01	6.78237E-01	**9.72982E-01**
	5	9.98971E-01	6.96282E-01	9.95495E-01	9.97918E-01	9.98560E-01	9.59152E-01	**9.98982E-01**
		9.98963E-01	6.12610E-01^{+}	9.93187E-01 $^{+}$	9.97159E-01 $^{+}$	9.98216E-01^{+}	6.00213E-01^{+}	**9.98975E-01**
		9.98673E-01	5.81437E-01	9.90103E-01	9.91088E-01	9.97263E-01	2.59548E-01	**9.98963E-01**
	8	9.99975E-01	9.96381E-01	9.95808E-01	9.99786E-01	9.99824E-01	9.98724E-01	**9.99977E-01**
		9.93549E-01	9.95112E-01^{+}	9.94518E-01 $^{+}$	9.96402E-01 $^{+}$	9.99570E-01^{+}	9.40772E-01^{+}	**9.99972E-01**
		9.66432E-01	9.92048E-01	9.90652E-01	4.83954E-01	9.98418E-01	5.12831E-01	**9.99966E-01**
	10	9.99991E-01	8.50548E-01	9.98305E-01	9.99974E-01	9.99932E-01	9.51489E-01	**9.99998E-01**
		9.99985E-01	6.23685E-01^{+}	9.96976E-01 $^{+}$	9.99923E-01 $^{+}$	9.99864E-01^{+}	8.47343E-01^{+}	**9.99997E-01**
		9.99969E-01	6.01849E-01	9.95322E-01	9.74683E-01	9.99656E-01	5.19166E-01	**9.99996E-01**
DTLZ2	3	9.26626E-01	9.21545E-01	**9.26943E-01**	9.26671E-01	9.25552E-01	9.24230E-01	9.26662E-01
		9.26536E-01	9.21144E-01^{+}	9.26518E-01 $^{+}$	9.26565E-01 $^{+}$	9.24474E-01^{+}	9.24030E-01^{+}	**9.26636E-01**
		9.26395E-01	9.20489E-01	9.26104E-01	9.26319E-01	9.22430E-01	9.23636E-01	**9.26577E-01**
	5	9.90459E-01	2.82518E-01	**9.90577E-01**	9.86881E-01	9.90383E-01	9.90335E-01	9.90492E-01
		9.90400E-01	2.82326E-01^{+}	9.90376E-01 $^{+}$	9.86822E-01 $^{+}$	9.90274E-01^{+}	9.90202E-01^{+}	**9.90462E-01**
		9.90328E-01	2.82203E-01	9.90158E-01	9.86721E-01	9.90117E-01	9.89796E-01	**9.90443E-01**
	8	9.99320E-01	9.44133E-01	**9.99410E-01**	9.98713E-01	9.99325E-01	9.99297E-01	9.99339E-01
		9.78936E-01	9.23437E-01^{+}	**9.99383E-01**$^{-}$	9.98658E-01 $^{+}$	9.99312E-01^{+}	9.99237E-01^{+}	9.99329E-01
		9.19680E-01	9.05961E-01	**9.99339E-01**	9.98475E-01	9.99286E-01	9.99092E-01	9.99316E-01
	10	9.99918E-01	1.80323E-01	**9.99927E-01**	9.99764E-01	9.99919E-01	9.99581E-01	9.99920E-01
		9.99916E-01	1.79345E-01^{+}	**9.99925E-01**$^{-}$	9.99744E-01 $^{+}$	9.99876E-01^{+}	9.99473E-01^{+}	9.99919E-01
		9.99915E-01	1.79030E-01	**9.99918E-01**	9.99721E-01	9.99872E-01	9.99195E-01	9.99915E-01
DTLZ3	3	9.26480E-01	9.21508E-01	**9.26988E-01**	9.26316E-01	3.29451E-03	9.23958E-01	9.26466E-01
		9.25805E-01	9.20488E-01^{+}	**9.26390E-01**$^{-}$	9.24816E-01 $^{+}$	6.83835E-03^{+}	9.23162E-01^{+}	9.26084E-01
		9.24234E-01	9.17104E-01	**9.25501E-01**	8.99569E-01	1.11064E-01	6.23475E-01	9.24700E-01
	5	9.90453E-01	3.37452E-01	**9.90585E-01**	9.86510E-01	9.90161E-01	9.58108E-01	9.90536E-01
		9.90344E-01	2.82245E-01^{+}	**9.90477E-01**$^{=}$	9.82919E-01 $^{+}$	9.80306E-01^{+}	8.33530E-01^{+}	9.90425E-01
		9.89510E-01	3.31567E-03	**9.90278E-01**	9.72970E-01	8.19713E-01	4.99916E-01	9.90252E-01
	8	9.99300E-01	9.43275E-01	**9.99409E-01**	9.86510E-01	9.99072E-01	9.95604E-01	9.99323E-01
		9.24059E-01	9.27554E-01^{+}	**9.99351E-01**$^{-}$	9.82919E-01 $^{+}$	6.58526E-01^{+}	7.82551E-01^{+}	9.99281E-01
		9.04182E-01	8.68136E-01	**9.99263E-01**	9.72970E-01	5.03707E-01	4.99999E-01	9.99162E-01
	10	9.99921E-01	2.91770E-01	**9.99929E-01**	9.86510E-01	9.99856E-01	9.40403E-01	9.99922E-01
		9.99918E-01	1.82485E-01^{+}	**9.99925E-01**$^{-}$	9.82919E-01 $^{+}$	9.96896E-01^{+}	5.62597E-01^{+}	9.99919E-01
		9.99910E-01	1.78772E-01	9.99911E-01	9.72970E-01	5.07743E-01	4.97573E-01	**9.99914E-01**
DTLZ4	3	9.26659E-01	9.22481E-01	**9.26963E-01**	9.26883E-01	9.26598E-01	9.24353E-01	9.26730E-01
		9.26705E-01	9.21754E-01^{+}	9.26495E-01 $^{+}$	**9.26823E-01**$^{-}$	9.14404E-01^{+}	9.23974E-01^{+}	9.26729E-01
		7.99572E-01	9.12762E-01	8.02347E-01	**9.26724E-01**	5.00000E-01	5.00000E-01	8.01492E-01
	5	**9.91102E-01**	4.58744E-01	9.90628E-01	9.87093E-01	9.90628E-01	9.90614E-01	9.90571E-01
		9.90413E-01	2.85835E-01^{+}	9.90513E-01 $^{+}$	9.87067E-01 $^{+}$	9.88983E-01^{+}	9.90547E-01^{+}	**9.90570E-01**
		9.90156E-01	2.82104E-01	9.74519E-01	9.87043E-01	9.71855E-01	9.90429E-01	**9.90568E-01**
	8	9.99363E-01	9.52367E-01	**9.99405E-01**	9.98833E-01	9.99380E-01	9.90614E-01	9.99365E-01
		9.99361E-01	9.38329E-01^{+}	**9.99388E-01**$^{-}$	9.98828E-01 $^{+}$	9.98877E-01^{+}	9.90547E-01^{+}	9.99364E-01
		9.94784E-01	9.26924E-01	9.98479E-01	9.98811E-01	9.87270E-01	9.90429E-01	**9.99363E-01**
	10	9.99915E-01	3.20842E-01	**9.99926E-01**	9.99793E-01	9.99925E-01	9.99908E-01	9.99923E-01
		9.99910E-01	2.37897E-01^{+}	**9.99923E-01**$^{=}$	9.99792E-01 $^{+}$	9.99918E-01^{+}	9.99903E-01^{+}	9.99923E-01
		9.99827E-01	2.11147E-01	9.99919E-01	9.99790E-01	9.99454E-01	9.99896E-01	**9.99923E-01**

Table 7: Best, median and worst IGD values obtained for each objective by LEAF and other algorithms on WFG instances with different number of objectives. Best performances are highlighted in bold face with gray background.

	M	MOEA-D	SPEA2+SDE	SPEA/R	VaEA	GrEA	LEAF
WFG1	3	5.062E-01 5.362E-01 + 5.457E-01	2.316E-01 2.895E-01 − 3.409E-01	4.010E-01 4.230E-01 + 4.317E-01	**1.544E-01** **1.830E-01** − **2.345E-01**	2.769E-01 3.001E-01 − 3.201E-01	3.385E-01 3.674E-01 3.753E-01
	5	2.746E-01 3.181E-01 − 3.606E-01	**2.123E-01** **2.385E-01** − **2.866E-01**	4.304E-01 4.582E-01 + 4.662E-01	3.126E-01 3.803E-01 − 4.301E-01	5.550E-01 5.680E-01 + 5.926E-01	3.729E-01 3.992E-01 4.053E-01
	8	2.535E-01 3.339E-01 + 4.096E-01	3.032E-01 3.382E-01 + 3.587E-01	3.464E-01 4.010E-01 + 6.962E-01	3.214E-01 3.332E-01 + **3.507E-01**	6.221E-01 6.550E-01 + 6.770E-01	**1.940E-01** **2.203E-01** 3.710E-01
	10	3.312E-01 3.513E-01 + 3.894E-01	3.046E-01 3.385E-01 + 3.615E-01	2.934E-01 3.192E-01 + 5.884E-01	2.806E-01 2.924E-01 + **3.066E-01**	6.064E-01 6.123E-01 + 6.194E-01	**1.647E-01** **1.747E-01** 8.374E-01
	15	4.575E-01 4.639E-01 + 4.788E-01	4.809E-01 4.988E-01 + 5.359E-01	3.326E-01 6.362E-01 + 6.421E-01	4.091E-01 4.141E-01 + 4.187E-01	6.737E-01 6.789E-01 + 6.874E-01	**3.211E-01** **3.390E-01** **3.601E-01**
WFG2	3	7.832E-02 8.624E-02 + **9.304E-02**	4.895E-02 5.619E-02 = 1.108E-01	1.745E-02 2.050E-02 = 9.922E-02	4.304E-02 4.989E-02 + 1.111E-01	6.131E-01 6.141E-01 + 7.146E-01	**1.602E-02** **2.036E-02** 9.824E-02
	5	3.231E-01 3.295E-01 + 3.409E-01	7.183E-02 7.670E-02 + 1.898E-01	**4.746E-02** **4.972E-02** − **5.139E-02**	7.106E-02 7.732E-02 + 1.736E-01	7.265E-01 7.276E-01 + 8.208E-01	5.673E-02 5.928E-02 1.599E-01
	8	2.224E-01 2.357E-01 + 2.520E-01	1.707E-01 1.895E-01 = 2.983E-01	**6.683E-02** **7.197E-02** − **2.049E-01**	1.110E-01 1.203E-01 − 2.143E-01	7.636E-01 7.643E-01 + 8.534E-01	8.843E-02 1.976E-01 2.397E-01
	10	4.440E-01 4.495E-01 + 4.530E-01	2.480E-01 2.844E-01 = 3.028E-01	**6.505E-02** **7.483E-02** − 2.403E-01	1.896E-01 2.042E-01 − **2.211E-01**	8.384E-01 8.387E-01 + 8.392E-01	1.731E-01 2.619E-01 3.165E-01
	15	9.878E-01 9.922E-01 + 9.925E-01	8.561E-01 9.125E-01 + 9.189E-01	**3.381E-01** 1.094E+00 + 1.128E+00	4.713E-01 **5.550E-01** − **7.147E-01**	1.142E+00 1.142E+00 + 1.207E+00	5.929E-01 6.426E-01 8.046E-01
WFG3	3	2.847E-02 4.245E-02 + 6.349E-02	**1.084E-02** **1.440E-02** − **1.737E-02**	3.485E-02 4.155E-02 + 6.476E-02	3.387E-02 4.451E-02 + 5.633E-02	4.516E-01 4.652E-01 + 4.803E-01	1.567E-02 1.901E-02 2.736E-02
	5	7.538E-01 7.825E-01 + 8.263E-01	5.858E-02 7.556E-02 + 9.310E-02	9.751E-02 1.149E-01 + 1.386E-01	6.074E-02 8.556E-02 + 1.585E-01	5.050E-01 5.233E-01 + 5.422E-01	**3.503E-02** **4.667E-02** **6.037E-02**
	8	1.255E-01 2.154E-01 − 2.552E-01	6.986E-02 1.446E-01 − 4.712E-01	2.677E-01 4.197E-01 + 6.170E-01	7.622E-02 **1.108E-01** − **1.882E-01**	5.497E-01 5.746E-01 + 6.068E-01	**5.955E-02** 3.004E-01 5.617E-01
	10	9.405E-01 9.591E-01 + 9.979E-01	7.161E-02 1.566E-01 + 4.316E-01	1.056E-01 2.262E-01 + 5.069E-01	7.716E-02 1.723E-01 + 2.733E-01	5.687E-01 5.833E-01 + 6.076E-01	**4.436E-02** **8.091E-02** **2.346E-01**
	15	9.674E-01 1.019E+00 + 1.053E+00	6.070E-02 **1.076E-01** − 7.521E-01	3.815E-01 4.350E-01 + 5.666E-01	**5.268E-02** 2.071E-01 − **2.794E-01**	5.790E-01 6.072E-01 + 6.313E-01	8.268E-02 3.211E-01 5.716E-01
WFG4	3	1.099E-01 1.137E-01 + 1.195E-01	6.959E-02 7.450E-02 + 8.100E-02	7.735E-03 9.383E-03 + 1.134E-02	5.244E-02 5.576E-02 + 6.114E-02	4.155E-01 4.171E-01 + 4.197E-01	**4.662E-03** **6.102E-03** **7.890E-03**
	5	8.438E-01 8.515E-01 + 8.591E-01	1.644E-01 1.723E-01 + 1.863E-01	1.994E-02 2.158E-02 + 2.502E-02	1.601E-01 1.681E-01 + 1.774E-01	6.264E-01 6.304E-01 + 6.360E-01	**1.697E-02** **1.981E-02** **2.327E-02**
	8	4.350E-01 5.023E-01 + 5.575E-01	2.763E-01 2.894E-01 + 3.112E-01	3.166E-02 3.783E-02 + 8.833E-02	2.460E-01 2.759E-01 + 2.961E-01	8.910E-01 9.071E-01 + 9.299E-01	**2.925E-02** **3.602E-02** **4.398E-02**
	10	1.053E+00 1.059E+00 + 1.063E+00	2.856E-01 2.988E-01 + 3.746E-01	**3.083E-02** **3.722E-02** = **4.268E-02**	3.191E-01 3.355E-01 + 3.522E-01	9.752E-01 9.854E-01 + 9.938E-01	3.345E-02 3.833E-02 4.921E-02
	15	1.160E+00 1.163E+00 + 1.167E+00	4.078E-01 5.141E-01 − 6.745E-01	**3.081E-02** **3.180E-01** − 6.991E-01	4.893E-01 5.151E-01 − **5.278E-01**	1.232E+00 1.246E+00 + 1.280E+00	5.195E-01 5.701E-01 6.860E-01
WFG5	3	9.738E-02 9.985E-02 + 1.012E-01	7.614E-02 7.969E-02 + 8.488E-02	3.330E-02 3.457E-02 + 3.851E-02	5.927E-02 6.271E-02 + 6.667E-02	2.673E-01 2.718E-01 + 2.740E-01	**2.956E-02** **3.158E-02** **3.563E-02**
	5	8.612E-01 8.679E-01 + 8.824E-01	1.691E-01 1.744E-01 + 1.952E-01	4.140E-02 4.335E-02 + 4.474E-02	1.592E-01 1.638E-01 + 1.707E-01	7.221E-01 7.298E-01 + 7.336E-01	**3.501E-02** **3.990E-02** **4.426E-02**

	M	MOEA-D	SPEA2+SDE	SPEA/R	VaEA	GrEA	LEAF
	8	4.189E-01	2.899E-01	4.904E-02	2.549E-01	9.708E-01	**4.612E-02**
		4.742E-01 +	3.109E-01 +	5.219E-02 +	2.823E-01+	9.757E-01 +	**5.127E-02**
		5.241E-01	3.515E-01	5.425E-02	2.953E-01	9.829E-01	**5.356E-02**
	10	1.067E+00	2.922E-01	4.961E-02	3.197E-01	1.052E+00	**4.386E-02**
		1.073E+00+	3.092E-01 +	5.238E-02 +	3.323E-01+	1.055E+00+	**4.948E-02**
		1.077E+00	3.182E-01	5.565E-02	3.480E-01	1.057E+00	**5.475E-02**
	15	1.173E+00	7.259E-01	6.776E-02	5.023E-01	1.188E+00	**3.685E-02**
		1.178E+00+	8.256E-01 +	2.125E-01 +	5.228E-01+	1.192E+00+	**3.872E-02**
		1.181E+00	9.058E-01	9.339E-01	5.380E-01	1.195E+00	**4.269E-02**
WFG6	3	1.094E-01	7.364E-02	**1.904E-02**	6.184E-02	6.211E-01	2.090E-02
		1.167E-01 +	7.822E-02 +	2.820E-02 =	6.474E-02+	6.255E-01 +	**2.810E-02**
		1.210E-01	8.457E-02	3.417E-02	6.728E-02	6.268E-01	**3.307E-02**
	5	8.806E-01	1.594E-01	2.855E-02	1.518E-01	8.877E-01	**2.719E-02**
		9.041E-01 +	1.759E-01 +	3.485E-02 =	1.598E-01+	9.111E-01 +	**3.419E-02**
		9.173E-01	1.831E-01	**3.874E-02**	1.665E-01	9.217E-01	4.051E-02
	8	4.448E-01	2.870E-01	3.854E-02	2.217E-01	1.082E+00	**3.474E-02**
		5.792E-01 +	3.149E-01 +	4.378E-02 +	2.472E-01+	1.085E+00+	**3.936E-02**
		6.495E-01	3.668E-01	5.208E-02	2.730E-01	1.100E+00	**4.964E-02**
	10	1.073E+00	2.995E-01	3.624E-02	2.957E-01	1.143E+00	**3.043E-02**
		1.088E+00+	3.178E-01 +	4.392E-02 +	3.189E-01+	1.146E+00+	**3.719E-02**
		1.108E+00	3.374E-01	5.207E-02	3.290E-01	1.159E+00	**4.184E-02**
	15	1.175E+00	7.050E-01	5.071E-02	5.190E-01	1.248E+00	**3.043E-02**
		1.188E+00+	8.204E-01 +	4.863E-01 +	5.275E-01+	1.252E+00+	**3.719E-02**
		1.203E+00	8.431E-01	1.109E+00	5.425E-01	1.259E+00	**4.184E-02**
WFG7	3	9.769E-02	7.056E-02	3.927E-03	5.002E-02	4.623E-01	**1.679E-03**
		1.031E-01 +	7.465E-02 +	4.800E-03 +	5.353E-02+	4.682E-01 +	**2.606E-03**
		1.098E-01	8.522E-02	6.481E-03	5.731E-02	4.711E-01	**3.288E-03**
	5	8.446E-01	1.692E-01	1.002E-02	1.407E-01	5.933E-01	**6.518E-03**
		8.528E-01 +	1.773E-01 +	1.162E-02 +	1.487E-01+	5.979E-01 +	**7.325E-03**
		8.622E-01	1.869E-01	**1.381E-02**	1.540E-01	6.036E-01	1.424E-02
	8	4.595E-01	2.882E-01	3.053E-02	2.347E-01	8.331E-01	**1.721E-02**
		5.342E-01 +	3.128E-01 +	3.868E-02 +	2.623E-01+	9.230E-01 +	**1.953E-02**
		5.921E-01	3.472E-01	7.388E-02	2.899E-01	1.003E+00	**2.211E-02**
	10	1.059E+00	2.995E-01	3.458E-02	3.089E-01	8.742E-01	**1.944E-02**
		1.064E+00+	3.103E-01 +	4.152E-02 +	3.158E-01+	9.027E-01 +	**2.065E-02**
		1.072E+00	3.221E-01	4.932E-02	3.350E-01	9.511E-01	**2.310E-02**
	15	1.592E+00	4.102E-01	3.375E-01	4.973E-01	1.154E+00	**3.739E-02**
		1.600E+00+	4.643E-01 +	6.146E-01 +	5.163E-01+	1.187E+00+	**7.754E-02**
		1.607E+00	5.111E-01	1.104E+00	5.233E-01	1.252E+00	**4.482E-01**
WFG8	3	1.338E-01	9.087E-02	**4.154E-02**	9.408E-02	6.739E-01	6.510E-02
		1.431E-01 +	9.245E-02 +	**4.488E-02**−	9.869E-02+	6.787E-01 +	7.235E-02
		1.638E-01	9.598E-02	**5.137E-02**	1.030E-01	6.874E-01	7.515E-02
	5	9.020E-01	1.862E-01	**5.621E-02**	2.044E-01	9.599E-01	1.203E-01
		9.927E-01 +	1.913E-01 +	**7.042E-02**−	2.195E-01+	9.667E-01 +	1.302E-01
		1.054E+00	2.056E-01	**7.473E-02**	2.291E-01	9.717E-01	1.370E-01
	8	5.232E-01	3.197E-01	**1.205E-01**	3.931E-01	1.121E+00	2.049E-01
		5.761E-01 +	3.326E-01 +	**1.327E-01**−	4.070E-01+	1.128E+00+	2.211E-01
		6.256E-01	3.402E-01	**1.409E-01**	4.259E-01	1.134E+00	2.413E-01
	10	1.095E+00	3.544E-01	1.361E-01	3.785E-01	1.161E+00	**1.174E-01**
		1.201E+00+	3.837E-01 +	**1.549E-01**−	4.452E-01+	1.180E+00+	2.025E-01
		1.311E+00	3.957E-01	**1.618E-01**	4.767E-01	1.184E+00	2.660E-01
	15	1.105E+00	**5.249E-01**	6.018E-01	5.788E-01	1.262E+00	5.429E-01
		1.134E+00+	**5.742E-01**=	9.638E-01 +	5.949E-01=	1.271E+00+	5.917E-01
		1.359E+00	6.253E-01	1.113E+00	**6.115E-01**	1.276E+00	6.231E-01
WFG9	3	1.029E-01	6.979E-02	2.943E-02	6.125E-02	1.204E-01	**2.867E-02**
		1.113E-01 +	8.395E-02 +	**5.358E-02**−	7.244E-02+	1.292E-01 +	6.484E-02
		1.123E-01	8.912E-02	**5.840E-02**	8.152E-02	1.401E-01	6.542E-02
	5	8.319E-01	1.554E-01	**5.311E-02**	1.683E-01	2.325E-01	5.670E-02
		8.532E-01 +	1.648E-01 +	6.368E-02 =	1.867E-01+	2.400E-01 +	**6.101E-02**
		8.979E-01	1.689E-01	**7.271E-02**	1.953E-01	2.478E-01	9.265E-02
	8	4.179E-01	2.557E-01	**8.827E-02**	2.596E-01	5.830E-01	9.049E-02
		4.677E-01 +	2.812E-01 +	1.188E-01 +	2.944E-01+	6.581E-01 +	**1.001E-01**
		5.147E-01	3.156E-01	1.980E-01	3.239E-01	6.974E-01	**1.372E-01**
	10	1.053E+00	2.672E-01	**9.076E-02**	3.271E-01	6.357E-01	9.885E-02
		1.079E+00+	2.921E-01 +	1.233E-01 +	3.489E-01+	6.515E-01 +	**1.071E-01**
		1.168E+00	3.661E-01	1.675E-01	3.639E-01	6.717E-01	**1.358E-01**
	15	1.074E+00	4.503E-01	**1.036E-01**	4.975E-01	9.371E-01	1.272E-01
		1.175E+00+	7.921E-01 +	1.688E-01 +	5.239E-01+	9.739E-01 +	**1.385E-01**
		1.416E+00	8.517E-01	**2.898E-01**	5.425E-01	9.822E-01	3.000E-01

Table 8: Best, median and worst HV values obtained for each objective by LEAF and other algorithms on WFG instances with different number of objectives. Best performances are highlighted in bold face with gray background.

	M	MOEA-D	SPEA2+SDE	SPEA/R	VaEA	GrEA	LEAF
WFG1	3	6.27270E-01 6.15260E-01 + 6.06940E-01	8.00190E-01 7.48320E-01 − 7.01480E-01	6.82790E-01 6.70940E-01 + 6.65730E-01	**8.74760E-01** **8.32610E-01** − 7.94030E-01	8.29960E-01 8.17710E-01 − **8.05710E-01**	7.30150E-01 7.05510E-01 6.97660E-01
	5	7.88910E-01 7.09790E-01 − 6.58270E-01	**8.99140E-01** **8.53140E-01** − **8.00680E-01**	6.40660E-01 6.23950E-01 + 6.19950E-01	7.67260E-01 6.95360E-01 − 6.51580E-01	5.23600E-01 5.18940E-01 + 5.07860E-01	6.69450E-01 6.49440E-01 6.45800E-01
	8	**9.18730E-01** 8.36930E-01 = 7.90200E-01	8.49570E-01 8.14900E-01 = 7.84130E-01	6.74480E-01 6.34840E-01 + 4.66590E-01	8.75640E-01 **8.59300E-01** − **8.27990E-01**	4.65280E-01 4.52820E-01 + 4.42950E-01	9.05230E-01 8.29650E-01 6.55680E-01
	10	8.69470E-01 8.12030E-01 + 7.39700E-01	9.39170E-01 8.85890E-01 + 8.26950E-01	6.93930E-01 6.78220E-01 + 4.51850E-01	9.32610E-01 9.20530E-01 + **9.08160E-01**	4.49830E-01 4.47750E-01 + 4.45970E-01	**9.84430E-01** **9.59740E-01** 5.78980E-01
WFG2	3	9.71550E-01 9.66700E-01 = **9.53690E-01**	9.83120E-01 9.77650E-01 = 8.82680E-01	9.86730E-01 9.84940E-01 = 8.92270E-01	9.81860E-01 9.78080E-01 + 8.88500E-01	4.77680E-01 4.77410E-01 + 4.34210E-01	**9.87550E-01** **9.85110E-01** 8.92750E-01
	5	8.21540E-01 8.10370E-01 + 8.05090E-01	9.93970E-01 9.91120E-01 + 8.94440E-01	9.96080E-01 9.95120E-01 + **9.93820E-01**	9.94770E-01 9.90850E-01 + 8.94370E-01	4.45760E-01 4.45650E-01 + 4.07370E-01	**9.97470E-01** **9.96330E-01** 8.97050E-01
	8	9.95910E-01 9.92990E-01 − **9.81730E-01**	9.92700E-01 9.89940E-01 − 8.95060E-01	**9.97090E-01** **9.95430E-01** − 8.97440E-01	9.95660E-01 9.91880E-01 = 8.92850E-01	4.14660E-01 4.14470E-01 + 3.79710E-01	9.96200E-01 8.97450E-01 8.92770E-01
	10	9.05990E-01 8.96940E-01 + 8.91430E-01	9.96550E-01 9.94460E-01 − **9.93090E-01**	**9.98120E-01** **9.97130E-01** − 8.97380E-01	9.97080E-01 9.95500E-01 − 9.92410E-01	3.99920E-01 3.99820E-01 + 3.99720E-01	9.96150E-01 9.44820E-01 8.93820E-01
WFG3	3	8.63110E-01 8.50160E-01 + 8.29680E-01	8.73990E-01 8.69330E-01 + 8.40140E-01	8.74280E-01 8.67970E-01 + 8.53310E-01	8.71150E-01 8.61310E-01 + 8.52490E-01	4.87770E-01 4.80640E-01 + 4.73490E-01	**8.80340E-01** **8.76760E-01** **8.71800E-01**
	5	3.61990E-01 3.52270E-01 + 3.39140E-01	8.37540E-01 7.88680E-01 + 7.76590E-01	8.60600E-01 8.38480E-01 + 8.20580E-01	8.65350E-01 8.43920E-01 + 8.28910E-01	4.53680E-01 4.46950E-01 + 4.40490E-01	**8.88090E-01** **8.79870E-01** **8.71050E-01**
	8	**8.82450E-01** **8.72070E-01** − **8.67080E-01**	8.33210E-01 7.53380E-01 = 5.59410E-01	7.13170E-01 6.62100E-01 + 5.89310E-01	8.64960E-01 8.48120E-01 − 8.29240E-01	4.23950E-01 4.15440E-01 + 4.05210E-01	8.56560E-01 7.12880E-01 6.13980E-01
	10	2.83190E-01 2.78890E-01 + 2.69330E-01	8.12910E-01 7.68010E-01 + 5.71020E-01	7.70670E-01 7.33000E-01 + 6.32980E-01	**8.68470E-01** **8.38090E-01** − **8.27910E-01**	4.06920E-01 4.02100E-01 + 3.94690E-01	8.67630E-01 8.11510E-01 7.43330E-01
WFG4	3	8.94690E-01 8.86890E-01 + 8.75160E-01	9.23370E-01 9.22290E-01 = 9.19470E-01	9.22330E-01 9.20690E-01 + 9.17950E-01	9.20650E-01 9.18680E-01 + 9.14610E-01	5.95130E-01 5.94020E-01 + 5.92100E-01	**9.23760E-01** **9.22510E-01** **9.21010E-01**
	5	4.05230E-01 3.55980E-01 + 3.41010E-01	9.80820E-01 9.77770E-01 + 9.74470E-01	9.80220E-01 9.78940E-01 + 9.76700E-01	9.73930E-01 9.69580E-01 + 9.64310E-01	5.63010E-01 5.59250E-01 + 5.49340E-01	**9.81810E-01** **9.80020E-01** **9.78700E-01**
	8	9.50970E-01 9.35540E-01 + 9.25210E-01	9.85960E-01 9.76560E-01 + 9.70260E-01	**9.92110E-01** **9.89240E-01** − **9.85780E-01**	9.88010E-01 9.80910E-01 = 9.74290E-01	4.94280E-01 4.86850E-01 + 4.78290E-01	9.86880E-01 9.82270E-01 9.77270E-01
	10	3.89470E-01 3.36420E-01 + 2.93030E-01	9.87590E-01 9.84020E-01 + 9.73940E-01	**9.95840E-01** **9.94740E-01** − **9.93070E-01**	9.85140E-01 9.81830E-01 + 9.78790E-01	4.70830E-01 4.62710E-01 + 4.52510E-01	9.89800E-01 9.86470E-01 9.81940E-01
WFG5	3	8.82950E-01 8.80890E-01 + 8.79540E-01	9.03320E-01 9.01220E-01 = 8.94090E-01	8.98040E-01 8.93180E-01 + 8.89130E-01	9.02990E-01 8.98100E-01 + 8.93790E-01	7.32620E-01 7.30130E-01 + 7.24970E-01	**9.03510E-01** **9.02710E-01** **8.97040E-01**
	5	3.49760E-01 3.21680E-01 + 3.17350E-01	9.57580E-01 9.51690E-01 + 9.46870E-01	9.50670E-01 9.47760E-01 + 9.46240E-01	9.56040E-01 9.53650E-01 + 9.49460E-01	7.24480E-01 7.22990E-01 + 7.14660E-01	**9.60070E-01** **9.58880E-01** **9.57390E-01**
	8	9.31020E-01 9.23240E-01 + 9.15960E-01	9.55790E-01 9.50110E-01 + 9.43710E-01	9.58040E-01 9.55180E-01 + 9.51790E-01	**9.62240E-01** 9.60370E-01 = **9.58730E-01**	7.20810E-01 7.19180E-01 + 7.06000E-01	9.61710E-01 **9.60780E-01** 9.58600E-01
	10	2.95180E-01 2.55770E-01 + 2.51260E-01	9.57650E-01 9.54870E-01 + 9.46620E-01	9.58640E-01 9.56310E-01 + 9.52040E-01	9.60640E-01 9.58770E-01 + 9.53100E-01	7.21330E-01 7.18190E-01 + 7.16580E-01	**9.61450E-01** **9.60010E-01** **9.57330E-01**

	M	MOEA-D	SPEA2+SDE	SPEA/R	VaEA	GrEA	LEAF
WFG6	3	8.90020E-01 8.74750E-01+ 8.73820E-01	9.11430E-01 **9.06340E-01**= 8.99830E-01	**9.12380E-01** 9.04200E-01 = 9.00880E-01	9.06840E-01 9.01570E-01 + 8.97230E-01	5.81050E-01 5.80840E-01+ 5.80370E-01	9.10700E-01 9.04950E-01 **9.01000E-01**
	5	3.25520E-01 3.20350E-01+ 3.08050E-01	9.68310E-01 **9.63000E-01**= **9.59340E-01**	**9.68320E-01** 9.58200E-01 + 9.54280E-01	9.62430E-01 9.56510E-01 + 9.51670E-01	5.76270E-01 5.75740E-01+ 5.75090E-01	9.67870E-01 9.61550E-01 9.56620E-01
	8	9.54790E-01 8.94460E-01+ 8.63360E-01	9.66710E-01 9.59900E-01 + 9.55500E-01	**9.76720E-01** 9.65940E-01 = 9.58560E-01	9.74290E-01 **9.65950E-01**= **9.59030E-01**	5.70510E-01 5.70120E-01+ 5.69430E-01	9.71610E-01 9.64650E-01 9.56080E-01
	10	2.58050E-01 2.45840E-01+ 2.36260E-01	9.66500E-01 9.61140E-01 + 9.55910E-01	**9.80160E-01** **9.67310E-01**= 9.56200E-01	9.73060E-01 9.64340E-01 = 9.50880E-01	5.67900E-01 5.67370E-01+ 5.67120E-01	9.75080E-01 9.65710E-01 **9.60820E-01**
WFG7	3	9.10830E-01 9.06110E-01+ 8.97420E-01	9.25220E-01 9.24750E-01 + 9.23580E-01	9.24960E-01 9.24160E-01 + 9.23570E-01	9.22930E-01 9.22650E-01 + 9.21630E-01	5.19440E-01 5.13800E-01+ 5.08770E-01	**9.25790E-01** **9.25170E-01** **9.24760E-01**
	5	3.46370E-01 3.32900E-01+ 3.29180E-01	9.85880E-01 9.84990E-01 + 9.83410E-01	9.84000E-01 9.83520E-01 + 9.81920E-01	9.83410E-01 9.80370E-01 + 9.77780E-01	4.64280E-01 4.60730E-01+ 4.55460E-01	**9.87850E-01** **9.87120E-01** **9.86030E-01**
	8	9.71600E-01 9.54080E-01+ 9.33240E-01	9.90760E-01 9.86010E-01 + 9.82070E-01	**9.95600E-01** **9.94890E-01**= **9.93800E-01**	9.95010E-01 9.94400E-01 = 9.93570E-01	3.85120E-01 3.52080E-01+ 3.14980E-01	9.95600E-01 9.94250E-01 9.93550E-01
	10	3.01270E-01 2.78160E-01+ 2.62150E-01	9.94830E-01 9.93150E-01 + 9.89310E-01	**9.97810E-01** **9.97570E-01**− **9.96790E-01**	9.96620E-01 9.95440E-01 + 9.93230E-01	3.50130E-01 3.36260E-01+ 3.16590E-01	9.97630E-01 9.96650E-01 9.96060E-01
WFG8	3	8.73560E-01 8.58990E-01+ 8.44030E-01	8.99640E-01 8.96730E-01 + 8.93910E-01	**9.12060E-01** **9.09470E-01**− **9.05760E-01**	8.93340E-01 8.90140E-01 + 8.86740E-01	5.51060E-01 5.49260E-01+ 5.46000E-01	9.08500E-01 9.06290E-01 9.04610E-01
	5	2.77240E-01 2.20730E-01+ 1.90960E-01	9.64760E-01 9.62690E-01 + 9.60450E-01	**9.73780E-01** **9.70780E-01**− **9.66910E-01**	9.53750E-01 9.45730E-01 + 9.35660E-01	5.64610E-01 5.63130E-01+ 5.58190E-01	9.70340E-01 9.66480E-01 9.63020E-01
	8	9.40600E-01 9.20960E-01+ 8.90010E-01	9.79330E-01 9.77170E-01 − 9.74240E-01	**9.89670E-01** **9.88220E-01**− **9.85050E-01**	9.62950E-01 9.52950E-01 + 9.36220E-01	5.66570E-01 5.65540E-01+ 5.62900E-01	9.65800E-01 9.59860E-01 9.48000E-01
	10	2.73470E-01 1.73300E-01+ 1.29770E-01	9.84750E-01 9.81510E-01 − 9.77900E-01	**9.94820E-01** **9.94030E-01**− **9.92960E-01**	9.67200E-01 9.56930E-01 + 9.39640E-01	5.67200E-01 5.66450E-01+ 5.64710E-01	9.79960E-01 9.66620E-01 9.56930E-01
WFG9	3	8.61620E-01 8.53330E-01+ 8.52070E-01	**8.96670E-01** 8.60220E-01 = 8.57900E-01	8.89040E-01 8.62340E-01 = 8.59100E-01	8.93400E-01 **8.73110E-01**= 8.53690E-01	7.98790E-01 7.83540E-01+ 7.56110E-01	8.96170E-01 8.61500E-01 **8.60710E-01**
	5	3.28210E-01 3.09430E-01+ 2.73970E-01	9.46620E-01 9.10110E-01 = **9.06070E-01**	9.21170E-01 9.01670E-01 + 8.96610E-01	9.35730E-01 9.01900E-01 + 8.97920E-01	8.28430E-01 8.09330E-01+ 7.98530E-01	**9.48550E-01** **9.43320E-01** 9.05420E-01
	8	8.89640E-01 8.80480E-01+ 8.70290E-01	**9.46820E-01** 9.01370E-01 = 8.89310E-01	9.03290E-01 8.91790E-01 + 8.80880E-01	9.39060E-01 8.97570E-01 + 8.90390E-01	8.15890E-01 7.94770E-01+ 7.84480E-01	9.42990E-01 **9.33660E-01** **9.03180E-01**
	10	2.84860E-01 2.43830E-01+ 2.10400E-01	**9.47320E-01** 9.40070E-01 = 8.90080E-01	9.22390E-01 8.94760E-01 + 8.87560E-01	9.44250E-01 8.98490E-01 + 8.91280E-01	8.16590E-01 8.05180E-01+ 7.80360E-01	9.47210E-01 **9.40410E-01** **9.00730E-01**

Table 9: A relative performance of MaOEAs over all objective dimensions for DTLZ problems, namely DTLZ (Dx) based on IGD is presented. The ratio (win/loss) suggests win/loss of LEA over the corresponding MaOEA based on the outcome of the Wilcoxon signed-rank test.

	M	3	5	8	10	15
IGD (win/loss)	MOEA/D	4/0	4/0	4/0	4/0	4/0
	SPEA2+SDE	4/0	4/0	4/0	4/0	4/0
	SPEA/R	4/0	4/0	4/0	4/0	4/0
	VaEA	4/0	4/0	4/0	4/0	4/0
	GrEA	4/0	4/0	4/0	4/0	4/0
Overall		20/0	20/0	20/0	20/0	20/0

	Problems	D1	D2	D3	D4
IGD (win/loss)	MOEA/D	5/0	5/0	5/0	5/0
	SPEA2+SDE	5/0	5/0	5/0	5/0
	SPEA/R	5/0	5/0	5/0	5/0
	VaEA	5/0	5/0	5/0	5/0
	GrEA	5/0	5/0	5/0	5/0
Overall		25/0	25/0	25/0	25/0

Table 10: A relative performance of MaOEAs over all objective dimensions for DTLZ problems, namely DTLZ (Dx) based on HV is presented. The ratio (win/loss) suggests win/loss of LEA over the corresponding MaOEA based on the outcome of the Wilcoxon signed-rank test.

	M	3	5	8	10
HV (win/loss)	MOEA/D	4/0	4/0	4/0	4/0
	SPEA2+SDE	3/1	3/0	1/3	1/2
	SPEA/R	3/1	4/0	4/0	4/0
	VaEA	4/0	4/0	4/0	4/0
	GrEA	4/0	4/0	4/0	4/0
Overall		17/2	19/0	17/3	17/2

	Problems	D1	D2	D3	D4
HV (win/loss)	MOEA/D	4/0	4/0	4/0	4/0
	SPEA2+SDE	4/0	2/2	0/3	2/1
	SPEA/R	4/0	4/0	4/0	3/1
	VaEA	4/0	4/0	4/0	4/0
	GrEA	4/0	4/0	4/0	4/0
Overall		20/0	18/2	16/3	17/2

Table 11: A relative performance of MaOEAs over all objective dimensions for WFG problems, namely WFG (Wx) based on IGD is presented. The ratio (win/loss) suggests win/loss of LEA over the corresponding MaOEA based on the outcome of the Wilcoxon signed-rank test.

	M	3	5	8	10	15
IGD (win/loss)	MOEA/D	9/0	8/1	8/1	9/0	9/0
	SPEA2+SDE	6/2	8/1	7/1	8/0	6/2
	SPEA/R	5/2	5/2	7/2	6/2	8/1
	VaEA	8/1	8/1	7/2	8/1	5/3
	GrEA	8/1	9/0	9/0	9/0	8/0
Overall		36/6	38/5	38/6	40/3	37/6

	Problems	W1	W2	W3	W4	W5	W6	W7	W8	W9
IGD (win/loss)	MOEA/D	4/1	5/0	4/1	5/0	5/0	5/0	5/0	5/0	5/0
	SPEA2+SDE	3/2	2/0	2/3	4/1	5/0	5/0	5/0	4/0	5/0
	SPEA/R	5/0	1/3	5/0	3/1	5/0	3/0	5/0	1/4	3/1
	VaEA	3/2	2/3	3/2	4/1	5/0	5/0	5/0	4/0	5/0
	GrEA	4/1	5/0	5/0	5/0	5/0	5/0	5/0	5/0	5/0
Overall		19/6	15/6	19/6	21/3	25/0	23/0	25/0	19/4	23/1

Table 12: A relative performance of MaOEAs over all objective dimensions for WFG problems, namely WFG (Wx) based on HV is presented. The ratio (win/loss) suggests win/loss of LEA over the corresponding MaOEA based on the outcome of the Wilcoxon signed-rank test.

	M	3	5	8	10
HV (win/loss)	MOEA/D	8/0	8/1	6/2	9/0
	SPEA2+SDE	3/1	6/1	4/2	6/2
	SPEA/R	5/1	8/1	4/3	4/4
	VaEA	7/1	8/1	2/2	6/2
	GrEA	8/1	9/0	9/0	9/0
Overall		31/4	39/4	25/9	34/8

	Problems	W1	W2	W3	W4	W5	W6	W7	W8	W9
HV (win/loss)	MOEA/D	2/1	2/1	3/1	4/0	4/0	4/0	4/0	4/0	4/0
	SPEA2+SDE	1/2	1/2	3/0	3/0	3/0	2/0	4/0	2/2	0/0
	SPEA/R	4/0	1/2	4/0	2/2	4/0	1/0	2/1	0/4	3/0
	VaEA	1/3	2/1	2/2	3/0	3/0	2/0	3/0	4/0	3/0
	GrEA	3/1	4/0	4/0	4/0	4/0	4/0	4/0	4/0	4/0
Overall		11/7	10/6	16/1	16/2	18/0	13/0	17/1	14/6	14/0