Single-loop reliability-based design optimization methods for single and multi-objective optimization

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1 Introduction

Real-world optimization problems often involve uncertainties that can be seen in material properties, geometry, operational environment, manufacturing process, etc. Solving such problems using deterministic optimization can generate infeasible or non-reliable solution. Reliability-based Design Optimization (RBDO) (Tu et al., 1999) is one of the efficient tools for dealing with such uncertainties for generating reliable optimal solution. The RBDO model is generally expressed as a minimization of the objective function, which is subjected to probabilistic constraints or performance functions. As a result, the design solution becomes safe and conservative owing to the consideration of the probabilistic constraints and random variables. The aim of this thesis, thus, is to develop RBDO methods that can generate solution with a desired reliability and should also be computationally efficient.

There exists various methods for solving reliability analysis, like simulation based methods, analytical methods, approximate integration methods, etc. The analytical methods are widely used owing to their performance efficiency compared to simulation based methods. These methods can be classified as double-loop methods, decoupled-loop methods, and single-loop methods. The performance of these methods depends on the accurate estimation of most-probable point (MPP). From the literature, these methods consist of two parts: a) optimization and b) reliability analysis. In double-loop method, a nested-loop of optimization is used in which the outer loop is used to estimate the mean values of random variables and the inner loop is used for the MPP estimation of the performance functions. In decoupled-loop method, optimization and reliability analysis are performed sequentially. Both these methods involve complete reliability analysis, making them computationally expensive. Therefore, single-loop methods are developed that approximate reliability analysis. These methods are found effective in generating a reliable solution. However, the convergence of MPP gets diverged for concave and highly non-linear performance functions.

Through an extensive literature review, we identify several key aspects and research gaps with approximate reliability estimation with single-loop methods and its convergence control. In this thesis, we target to fill some of those research gaps by developing efficient RBDO methods. The primary target is to generate the best reliable solution using single-loop method approach that requires less function evaluation (Aoues and Chateauneuf, 2010). This is achieved by incorporating approximate reliability analysis effectively with different strategies and developing proper oscillation criterion to estimate the MPP quickly and accurately. The objectives of this thesis are as follows.

1. Developing a single-loop method using conjugate gradient search and shifting vector approach for improving the accuracy of reliable solution.

2. Enhancing the proposed single-loop method using chaos control theory for estimating MPP efficiently for non-linear performance functions.

3. Developing a hybrid RBDO method incorporating the concept of proposed single-loop method with adaptive differential evolution for global optimization.

4. Developing a multi-objective RBDO formulation using efficient chaos control and solve it using differential evolution with adaptive mutation scheme.

5. Developing a hybrid multi-objective RBDO formulation incorporating modified chaos control theory and shifting vector approach, and solve it using differential evolution.

In the following sections, the proposed strategies are discussed in order to achieve the objectives of the thesis.

2 Single-loop reliability-based design optimization methods

The single-loop methods are found efficient in estimating MPP for convex performance functions. However, these methods sometime face challenges with concave and highly non-linear performance functions. To address this challenge, Karush-Kuhn Tucker (KKT) optimality conditions are used with performance measure approach (PMA) to develop an analytical expression to approximate MPP. This converts the probabilistic constraints to approximate deterministic constraints. In the literature, the steepest descent direction is used to update the design variables. However, the direction seems ineffective while dealing with concave performance functions. Therefore, the conjugate gradient search direction is used for approximating MPP. The shifting vector approach is adopted with the proposed single-loop method that shifts the violated performance function toward the feasible direction. The combination of conjugate gradient search direction and shifting vector approach makes the proposed single-loop method better and computationally efficient.

The proposed single-loop method is tested on four mathematical and four engineering benchmark problems. The results of the proposed method are compared with six different RBDO methods from the literature. Monte-Carlo simulation with a sample size of 1 million is used to quantify the reliability of solutions obtained from the methods. It is found that the proposed method generates the best reliable optimal solution for all eight problems. The computational efficiency is quantified using number of function evaluation. Again, the proposed method is found to be the best among the chosen set of methods from the literature. Therefore, it can be concluded that the proposed method successfully solves both convex and concave performance functions.

When the proposed local-search methods solved various RBDO example, it was found that the convergence for estimating MPP gets oscillated for highly non-linear performance function. This lead to the second objective of this thesis in which these oscillations need to be traced and appropriate theory must be applied to overcome the challenge. Therefore, an approximate single-loop chaos control method is proposed to handle periodic oscillation during the convergence of MPP. In every iteration, MPP is estimated using conjugate gradient search direction. When oscillation among MPPs generated in the current iteration and previous iterations is observed, the current MPP is updated using chaos control theory. An oscillation criterion is also proposed to trace oscillation among MPPs in which the vector difference between the three most recent MPPs is used to check the criterion.

The proposed method is also tested on eight benchmark problems from the literature. The results are compared with eight RBDO methods including chaos control based methods from the literature. The results reveal that the chaos among the MPPs is controlled and the convergence is achieved effectively. The proposed method is also able to generate the best reliable solution among the chosen set of RBDO methods. The computational efficiency is also found better than the other methods. Results also demonstrate that the method is insensitive to the initial value of chaos control factor.

The above proposed single-loop methods have significantly improved the performance in generating reliable solutions with less function evaluation. However, they face several challenges while dealing with non-linear objective and performance functions that can result in multiple optimal solutions. Moreover, the conjugate search directions is also susceptible to converge to the local optimal solution. This leads to the use of global optimization technique called differential evolution (DE), which is a population based meta-heuristic optimization technique. RBDO methods with differential evolution are discussed in the following sections for achieving global optimization.

3 Hybrid single-loop RBDO using differential evolution

From the literature of RBDO using evolutionary optimization techniques, it can be observed that several studies have been done to improve effectiveness of these techniques for obtaining reliable global optimal solution. Most of the works is done on coupling evolutionary optimization techniques such as genetic algorithm, differential evolution, etc. with the double-loop method. Since both evolutionary optimization techniques and reliability analysis are computationally expensive, this leads to the third objective of the thesis. In this objective, a single-loop RBDO formulation is developed using differential evolution. A single-loop RBDO formulation using shifting vector approach is proposed that can give direction to violated performance functions toward the feasible direction for generating reliable solution. The proposed formulation is different than the formulation of the first objective of the thesis since DE is used instead of conjugate gradient direction for approximating MPP. Further, the formulation also incorporates target and trial vectors of DE for guiding the algorithm. A heuristic parameter is also developed for proper selection of mutation operator for exploration and exploitation of search space. In our case, two mutation operators from the following DE variants are used, i.e., DE/rand/1/bin and DE/best/1/bin. The heuristic parameter is tuned such that a random vector is selected for generating the mutant vector in the early generations of DE. Later, the best vector is used.

The proposed RBDO method is tested on three mathematical and two engineering benchmark examples. It is also compared with four RBDO methods from the literature and a double-loop based DE method. The PMA approach is used the double-loop based DE method for estimating the reliability of MPP. The reliability of obtained solutions from these methods is verified using Monte-Carlo simulation with a sample size of 1 million. The results demonstrate that the proposed method successfully converges to the reliable global optimal solution for the chosen set of examples. The proposed method is also found computationally efficient than the double-loop based DE method.

It has been observed that real-world problems also consist of multiple objectives apart from uncertainty. In the following sections, the multi-objective reliability-based design optimization is discussed.

4 Single-loop multi-objective RBDO using differential evolution

The fourth objective of the thesis is to develop a single-loop multi-objective reliability-based deign optimization method. The challenge is to obtain a reliable Pareto optimal front. To address this challenge, a single-loop formulation is proposed in which the reliability analysis is approximated using KKT optimality conditions similar to the first objective. However, in this case a pseudo MPP is estimated with respect to all probabilistic constraints that is determined through the steepest descent direction. This pseudo MPP replaces the vector of response function in chaos control formulation so that the chance of oscillation among MPPs gets reduced. Therefore, the concept of a modified chaos control theory is adapted with the proposed single-loop multi-objective RBDO formulation. The proposed formulation is solved using multi-objective DE that is designed using non-dominated sorting and crowding distance. The algorithm is made adaptive for better convergence by introducing a heuristic parameter for generating mutant vectors from different variants of DE as mentioned earlier. The heuristic parameter is estimated in every generation using hypervolume performance indicator. Since the best vector is chosen for adaptive mutation scheme in the later generations of DE, it is found by selecting the closest non-dominated solution with respect to the target vector in the objective space.

It has been observed from the previous sections that the concepts of shifting vector and chaos control theory were effective with single-loop formulation. Therefore, in the last objective of the thesis these concepts are coupled to develop a new single-loop multi-objective RBDO formulation. By incorporating these changes, the trial vector and target vector become the part of RBDO formulation. The same adaptive multi-objective DE is used for solving the proposed formulation.

The developed methods are tested on two bi-objective mathematical and one bi-objective engineering examples and the results of differential evolution with adaptive mutation scheme are compared with a double-loop multi-objective differential evolution. The double-loop multi-objective DE is developed using PMA for reliability analysis. Results demonstrate that irrespective of the non-linearity in the performance functions and objectives, the methods generated a set of reliable Pareto-optimal solutions. With respect to the double-loop multi-objective DE, the proposed methods generated the obtained Pareto-optimal solutions in less function evaluation.

5 Organization of the thesis

The thesis is organized in the following manner.

- **Chapter 1** begins with the introduction of the problem statement and mathematical formulation. This is followed by the challenges and a brief description of the previous efforts in the literature. The research gaps are then discussed, which lead to the motivation and the objectives of the thesis.

- **Chapter 2** An exhaustive literature review of RBDO methods and different types of reliability analysis are presented in this chapter.

- **Chapter 3** The formulations of the single-loop reliability-based design optimization using conjugate gradient direction, shifting vector, and chaos control theory are described in detail. The oscillation criterion for MPP is also discussed. The results and discussion are presented on various RBDO examples.

- **Chapter 4** The formulation of hybrid single-loop reliability-based design optimization is discussed. The adaptive differential evolution and its implementation with different mutation schemes are discussed in detail. The results are presented and compared with the existing RBDO methods.

- **Chapter 5** presents the detailed discussion of the single-loop multi-objective reliability-based design optimization formulations. Efficient reliability analysis with modified chaos control theory and shifting vector is discussed in detail. The implementation of DE is discussed by incorporating hypervolume performance indicator. The obtained Pareto-optimal solutions are generated and compared with the double-loop multi-objective DE with PMA for reliability analysis.

- **Chapter 6** concludes the thesis with a note on future work.

Journal

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- Raktim Biswas and Deepak Sharma (2021). A single-loop shifting vector method with conjugate gradient search for reliability-based design optimization. *Engineering Optimization* 53(6), 1044-1063, DOI: 10.1080/0305215X.2020.1770745.

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- Raktim Biswas and Deepak Sharma (2021). An approximate single-loop chaos control method for reliability-based design optimization using conjugate gradient search directions. *Engineering Optimization* (*Accepted*),
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Under review

- Raktim Biswas and Deepak Sharma. A single-loop reliability-based design optimization using adaptive differential evolution. *Applied Soft Computing* (*Under Review*)

- Raktim Biswas and Deepak Sharma. Chaos control assisted single-loop multi-objective reliability-based design optimization using differential evolution *Swarm and Evolutionary Computation* (*Under Review*)

In preparation

- Raktim Biswas and Deepak Sharma. An efficient multi-objective reliability-based design optimization method using shifting vector approach (*In Preparation*)

Conference

- Raktim Biswas and Deepak Sharma. A Single-Loop Reliability-based Design Optimization Method using Iteratively updating Hessian. *EUROGEN, 19*

- Raktim Biswas and Deepak Sharma. Reliability-based design optimization using adaptive sequential optimization and reliability assessment. (*In Preparation*)

References

Aoues, Y. and A. Chateauneuf (2010, Mar). Benchmark study of numerical methods for reliability-based design optimization. *Structural and Multidisciplinary Optimization 41*(2), 277–294.

Tu, J., K. K. Choi, and Y. H. Park (1999, 12). A New Study on Reliability-Based Design Optimization. *Journal of Mechanical Design 121*(4), 557–564.