4. Electrodynamic fields

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4.1 Introduction

Electrodynamics

Faraday’s law

Lenz’s law

Integral form

Differential form

Phasor form

Wave equations

Boundary conditions

Fig. 4.1 Electrodynamics: Faraday’s law and Maxwell’s equations
4.1 Introduction

- We will study Faraday’s law of electromagnetic induction (the 4\textsuperscript{th} Maxwell’s equations)
- Faraday showed that a changing magnetic field creates an electric field
- He was the first person who connected these two forces (magnetic and electric forces)
- Maxwell, in 19\textsuperscript{th} century, combined the works of his predecessors (the four Maxwell’s equations)
4.1 Introduction

- Since everything in nature is symmetric, he postulated that a changing electric field should also produce magnetic field.
- Whenever the dynamic fields interact with a media interface, their behavior is governed by electromagnetic boundary conditions.
- We will discuss how to obtain Helmholtz wave equations from the two Maxwell’s curl equations and
  - generate electromagnetic (EM) waves.
4.2 Faraday’s law of electromagnetic induction

Fig. 4.2 (a) Case I (b) Case II and (c) Case III
4.2 Faraday’s law of electromagnetic induction

- A major advance in electromagnetic theory was made by Michael Faraday in 1831.
- He discovered experimentally that a current was induced in a conducting loop when the magnetic flux linking that loop changes.
- The possible three cases of changing magnetic flux linkage is depicted in Fig. 4.2.
4.2 Faraday’s law of electromagnetic induction

Case I: A stationary circuit in a time varying magnetic field

- It has been observed experimentally that when both the loop and magnet is at rest
- But the magnetic field is time varying (denoted by $B_t$ in Fig. 4.2(a)), a current flows in the loop
- Mathematically:

$$\xi = \int \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \Rightarrow \int \nabla \times \vec{E} \cdot d\vec{s} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$
4.2 Faraday’s law of electromagnetic induction

- It is for any arbitrary surface
- Hence

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

- The emf induced in a stationary closed circuit is equal to the negative rate of increase of magnetic flux linking the circuit
- The negative sign is due to Lenz’s law
4.2 Faraday’s law of electromagnetic induction

Case II: A moving conductor in a static magnetic field

- When a conductor moves with a velocity in a static magnetic field (denoted by $B_s$ in Fig. 4.2(b)), a force

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

- will cause the freely movable electron in the conductor to drift toward one end of the conductor and
  - leave the other end positively charged
4.2 Faraday’s law of electromagnetic induction

- The magnetic force per unit charge
  \[ \frac{\vec{F}_m}{q} = \vec{v} \times \vec{B} \]

- can be interpreted as an induced electric field acting along the conductor and
  - producing a voltage \( V_{21} \)

\[ V_{21} = \int_{1}^{2} \vec{E} \cdot d\vec{l} = \int_{1}^{2} (\vec{v} \times \vec{B}) \cdot d\vec{l} \]
4.2 Faraday’s law of electromagnetic induction

Case III: A moving circuit in a time varying magnetic field

• When a charge moves with a velocity in a region where
  • both an electric field and a magnetic field exists
• The electromagnetic force on \( q \) is given by Lorentz’s force field equation,

\[
\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})
\]
4.2 Faraday’s law of electromagnetic induction

- Hence, when a conducting circuit
  - with contour C and
  - surface S
- moves with a velocity in presence of magnetic and electric field, we obtain,

\[
\oint C \vec{E} \cdot d\vec{l} = -\oint S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} + \oint C (\vec{v} \times \vec{B}) \cdot d\vec{l}
\]

- The line integral in the LHS is the emf induced in the moving frame of reference
4.2 Faraday’s law of electromagnetic induction

- The first term in the right side represents transformer emf
  - due to the variation of magnetic field
- The second term represents the motional emf
  - due the motion of the circuit in magnetic field
- Faraday’s law of electromagnetic induction shows that
  - there is a close connection between electric and magnetic fields
- In other words, a time changing magnetic field produces an electric field
4.2 Faraday’s law of electromagnetic induction

Lenz’s law

Fig. 4.3 Lenz’s law
4.2 Faraday’s law of electromagnetic induction

- The induced current creates a magnetic flux
  - which prevents the variation of the magnetic flux generating the induced emf
- For example,
  - the loop L is drawn to the coil c,
  - the magnetic flux through the loop increases
- The current induced in the loop in this case is in the counterclockwise direction
4.3 Maxwell’s Equations

- The theory of EM field was mathematically completed by Maxwell.
- It was James Clerk Maxwell who has combined the previous works of
  - Carl Federick Gauss,
  - Andre Marie Ampere and
  - Michael Faraday
- into four laws
4.3 Maxwell’s Equations

- which completely explains the entire electromagnetic phenomenon in nature
  - except quantum mechanics

4.3.1 Electrodynamics before Maxwell

Table 4.1 Electrodynamics before Maxwell (next page)
<table>
<thead>
<tr>
<th>Equation No.</th>
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<th>Laws</th>
</tr>
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<tbody>
<tr>
<td>1.</td>
<td>$\nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon_0}$</td>
<td>Gauss law for electric field</td>
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<td>2.</td>
<td>$\nabla \cdot \vec{B} = 0$</td>
<td>Gauss law for magnetic field</td>
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<tr>
<td>3.</td>
<td>$\nabla \times \vec{B} = \mu_0 \vec{J}$</td>
<td>Ampere’s law</td>
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<td>4.</td>
<td>$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$</td>
<td>Faraday’s law</td>
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</tbody>
</table>
4.3 Maxwell’s Equations

- There is fatal inconsistency in one of these formulae
- It has to do with the rule that the divergence of curl is always zero
- Apply this rule to 4, we get,

\[ \nabla \cdot (\nabla \times \vec{E}) = -\frac{\partial (\nabla \cdot \vec{B})}{\partial t} = 0 \]

- This is consistent
- LHS is zero
  - since it is the divergence of curl of electric field vector
4.3 Maxwell’s Equations

- RHS is also zero from Gauss’s law for magnetic field which states that divergence of a magnetic flux density vector is zero.
- Let us apply this rule to 3, we get,

\[ \nabla \cdot (\nabla \times \vec{B}) = \nabla \cdot (\mu_0 \vec{J}) = \mu_0 (\nabla \cdot \vec{J}) \]
4.3 Maxwell’s Equations

- The RHS is zero for
  - steady state currents only
- whereas the LHS is zero for all cases
  - since it is the divergence of curl of magnetic flux density vector
- One fundamental question is that
  - what is happening in between the parallel plates of the capacitor while
    - charging or discharging
4.3 Maxwell’s Equations

• From our circuit analysis,
  • no current flows between the two plates since they are disconnected

• In accordance with Gauss theorem,
  • for time varying electric flux density and
  • fixed or static closed surface

\[ \oint \mathbf{D} \cdot d\mathbf{s} = q \Rightarrow \oint \frac{\partial}{\partial t} (\mathbf{D} \cdot d\mathbf{s}) = \frac{\partial q}{\partial t} \]
4.3 Maxwell’s Equations

- So the term on the LHS of the above equation is
  - rate of change of electric flux or
  - it can be also termed as displacement current
- we can add this extra term in the Ampere’s law and we get,

\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \]

- The 2\textsuperscript{nd} term on the RHS can also be termed as displacement current density
- Both terms on the RHS together is known as the total current.
4.3 Maxwell’s Equations

4.3.2 Maxwell’s equations in integral form

- Maxwell equations are the elegant mathematical expression of decades of experimental observations
  - of the electric and
  - magnetic effects
    - of charges and
    - currents
- by many scientists viz.,
  - Gauss,
  - Ampere and
  - Faraday
4.3 Maxwell’s Equations

- Maxwell’s own contribution indeed is
  - the last term of the third equation (refer to Table 4.2)
- But this term had profound impact on the electromagnetic theory
- It made evident for the first time that
  - varying electric and magnetic fields could produce or generate each other
- and these fields could propagate indefinitely through free space,
  - far from the varying charges and currents
    - where they are originated
4.3 Maxwell’s Equations

- Previously the fields had been envisioned
  - bound to the charges and currents giving rise to them
- Maxwell’s new term displacement current freed them
  - to move through space in a self-sustaining fashion, and
- even predicted their velocity of motion was
  - the speed of light
- Electrodynamics after Maxwell can be represented by the
  four Maxwell’s equations in integral form (listed in Table 4.2)

Table 4.2 Electrodynamics after Maxwell (next page)
## 4.3 Maxwell’s Equations

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<td>$\oint \vec{B} \cdot d\vec{s} = 0$</td>
<td>Gauss law for magnetic field</td>
</tr>
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<td>3.</td>
<td>$\oint \vec{H} \cdot d\vec{l} = \int \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{s}$</td>
<td>Ampere’s law</td>
</tr>
<tr>
<td>4.</td>
<td>$\oint \vec{E} \cdot d\vec{l} = - \int \left( \frac{\partial \vec{B}}{\partial t} \right) \cdot d\vec{s}$</td>
<td>Faraday’s law</td>
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4.3 Maxwell’s Equations

- The relations for linear, isotropic and non-dispersive materials can be written as:

\[ \vec{D} = \varepsilon \vec{E}, \vec{B} = \mu \vec{H} \]

- where \( \varepsilon \) is the permittivity and

- \( \mu \) is the permeability of the material

- Suppose, we restrict ourselves to time-independent situations
  - That means nothing is varying with time and
  - all fields are stationary
4.3 Maxwell’s Equations

- If the fields are stationary (electric and magnetic field are constant with time),
- Maxwell’s equations reduces to four groups of independent equations

\[
\begin{align*}
1. \quad \oint_D \cdot d\vec{s} &= q \\
2. \quad \oint_B \cdot d\vec{s} &= 0 \\
3. \quad \oint_H \cdot d\vec{l} &= I \\
4. \quad \oint_E \cdot d\vec{l} &= 0
\end{align*}
\]
4.3 Maxwell’s Equations

- In this case, the electric and magnetic fields are independent of one another
- These are laws for electrostatics and magnetostatics

4.3.3 Differential form of Maxwell’s equations

- We can obtain the 4 laws of Maxwell’s equations in differential form listed below from the integral form of Maxwell’s equations by applying
  - Divergence and
  - Stokes theorems
4.3 Maxwell’s Equations

1. \( \nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon} \)

2. \( \nabla \cdot \vec{B} = 0 \)

3. \( \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \)

4. \( \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \)
4.3 Maxwell’s Equations

- Equation 1 means that static or dynamic charges in a given volume are responsible for a diverging electric field.
- Equation 2 means that there is no physical medium which makes a magnetic field diverge.
- Equation 3 means either a current flow through a medium or a time-varying electric field produces a spatially curling magnetic field.
- Equations 4 mean that a spatially varying (curling) electric field will cause or produce a time-varying magnetic field.
4.3 Maxwell’s Equations

- Maxwell equation include the continuity equation

\[ \oint \left( \vec{j} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{s} = 0 \]

4.3.4 Time harmonic fields and Maxwell’s equations in phasor form

- For time harmonic fields

\[ \vec{F}(x, y, z, t) = \vec{F}(x, y, z)e^{j\omega t} \]

- where \( \omega \) is the angular frequency of the time varying field
4.3 Maxwell’s Equations

\[ \frac{\partial}{\partial t} \vec{F}(x, y, z, t) = j\omega \vec{F}(x, y, z, t); \quad \int \vec{F}(x, y, z, t) dt = \frac{\vec{F}(x, y, z, t)}{j\omega} + c_1 \]

- where \( c_1 \) is a constant of integration
- Since we are interested only in time varying quantities, we can take, \( c_1 = 0 \)

\[ \frac{\partial}{\partial t} \equiv j\omega, \quad \frac{\partial^2}{\partial t^2} = j\omega \times j\omega = -\omega^2 \]
4.3 Maxwell’s Equations

Maxwell’s equations in phasor form

- If we take the time variation explicitly out, then we can write

1. \( \nabla \times \vec{E} = -j\omega \vec{B} \)

2. \( \nabla \times \vec{H} = j\omega \vec{D}\) + \(\vec{J}\)

3. \( \nabla \cdot \vec{D} = \rho_v \)

4. \( \nabla \cdot \vec{B} = 0 \)
4.3 Maxwell’s Equations

- Some points to be noted on perfect conductors and magnetic fields:
  - A perfect conductor or metal can’t have time-varying magnetic fields inside it
  - At the surface of a perfect metal there can be no component of a time-varying magnetic field that is normal to the surface
  - Time-varying currents can only flow at the surface of a perfect metal but not inside it

4.3.5 Electromagnetic boundary conditions
<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Scalar form</th>
<th>Vector form</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$E_{t1} = E_{t2}$</td>
<td>$\hat{n} \times (\tilde{E}_1 - \tilde{E}_2) = 0$</td>
</tr>
<tr>
<td>2.</td>
<td>$H_{t1} - H_{t2} = J_s$</td>
<td>$\hat{n} \times (\tilde{H}_1 - \tilde{H}_2) = \tilde{J}_s$</td>
</tr>
<tr>
<td>3.</td>
<td>$B_{n1} = B_{n2}$</td>
<td>$\hat{n} \cdot (\tilde{B}_1 - \tilde{B}_2) = 0$</td>
</tr>
<tr>
<td>4.</td>
<td>$\hat{n} \cdot (\tilde{D}_1 - \tilde{D}_2) = \rho_s$</td>
<td>$D_{n1} - D_{n2} = \rho_s$</td>
</tr>
<tr>
<td>5.</td>
<td>$J_{n1} = J_{n2}$</td>
<td>$\hat{n} \cdot (\tilde{J}_1 - \tilde{J}_2) = 0$</td>
</tr>
<tr>
<td>6.</td>
<td>$\frac{J_{t1}}{J_{t2}} = \frac{\sigma_1}{\sigma_2}$</td>
<td>$\hat{n} \times \left( \frac{\tilde{J}_1}{\tilde{J}_2} \right) = \frac{\sigma_1}{\sigma_2}$</td>
</tr>
</tbody>
</table>
4.3 Maxwell’s Equations

- The tangential component of the electric field is continuous across the boundary between two dielectrics.
- The tangential component of the magnetic field is discontinuous across the boundary between two magnetic materials by the surface current density flowing along the boundary.
- The normal component of the magnetic flux density is continuous across the boundary between two magnetic materials.
4.3 Maxwell’s Equations

- The normal component of the electric flux density is discontinuous across the boundary between two dielectrics by the surface charge density at the boundary.
- It states that the normal component of electric current density is continuous across the boundary.
- The ratio of the tangential components of the current densities at the interface is equal to the ratio of the conductivities.
4.3 Maxwell’s Equations

4.4 Wave equations from Maxwell’s equations

4.4.1 Helmholtz wave equations

- In linear isotropic medium, the two Maxwell curl equations in phasor form are

\[ \nabla \times \vec{E} = -j \omega \mu \vec{H} \]

\[ \nabla \times \vec{H} = j \omega \varepsilon \vec{E} + \vec{J} = j \omega \varepsilon \vec{E} + \sigma \vec{E} = \left( j \omega \varepsilon + \sigma \right) \vec{E} \]
4.3 Maxwell’s Equations

- To solve for electric and magnetic fields,
- we can take curl on the first equation and
- eliminate the curl of magnetic field vector in the RHS of the first equation by using the second equation given above

\[ \nabla \times \nabla \times \vec{E} = \nabla \times \nabla \times \vec{E} = \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} \]
4.3 Maxwell’s Equations

- For a charge free region,

\[ \nabla \cdot \vec{E} = 0 \]

\[ \therefore \nabla \times \nabla \times \vec{E} = -\nabla^2 \vec{E} = -j\omega\mu(\nabla \times \vec{H}) = -j\omega\mu(j\omega\varepsilon + \sigma) \vec{E} = -\gamma^2 \vec{E} \]

- Hence,

\[ \nabla^2 \vec{E} - \gamma^2 \vec{E} = 0 \]

- Similarly,

\[ \nabla^2 \vec{H} - \gamma^2 \vec{H} = 0 \]
4.3 Maxwell’s Equations

- These two equations are known as Helmholtz equations or wave equations.
- $\gamma = \alpha + j\beta$ is a complex number and it is known as propagation constant.
- The real part $\alpha$ is attenuation constant, it will show how fast the wave will attenuate.
- Whereas the imaginary part $\beta$ is the phase constant.

4.4.2 Propagation in rectangular coordinates
4.3 Maxwell’s Equations

- The Helmholtz vector wave equation can be solved in any orthogonal coordinate system by substituting the appropriate Laplacian operator.

- Let us first solve it in Cartesian coordinate systems.

- Let us assume that the electric field is polarized along the x-axis and it is propagating along the z-direction.

\[ \vec{E}(x, y, z, t) = E_x(z, t) \hat{x} \]
4.3 Maxwell’s Equations

- Therefore, the wave equation now reduces to

\[ \nabla^2 \vec{E} - \gamma^2 \vec{E} = 0 \Rightarrow \frac{\partial^2 \vec{E}}{\partial z^2} - \gamma^2 \vec{E} = 0 \]

- Hence, the solution of the wave equation is of the form

\[ \vec{E}(x, y, z) = E_x(z) \hat{x} = \left( E_0^+ e^{-\gamma z} + E_0^- e^{+\gamma z} \right) \hat{x} \]

- where superscript + and – for \( E_0 \) the arbitrary constants denote for wave propagating along + and – z-axis respectively
4.3 Maxwell’s Equations

- Assuming that the electric field propagates only in the positive z-direction and of finite value at infinity
- then the arbitrary constant for wave traveling along $-z$ axis must be equal to zero, which means

$$\vec{E}(x, y, z) = E_x(z) \hat{x} = E_0^+ e^{-\gamma z} \hat{x}$$

- Dropping the superscript $+$ in the constant $E_0$ of the above expression, we can write

$$\vec{E}(x, y, z) = E_0 e^{-\gamma z} \hat{x}$$
4.3 Maxwell’s Equations

- Let us put the time dependence now by multiplying the above expression by $e^{j\omega t}$, which gives

$$\vec{E}(x, y, z, t) = E_x(z, t)\hat{x} = (E_0 e^{-\gamma z})e^{j\omega t}\hat{x} = (E_0 e^{-\alpha z})e^{-j\beta z}e^{j\omega t}\hat{x}$$

- The real part of this electric field becomes

$$\text{Re}\{\vec{E}(x, y, z, t)\} = \text{Re}\{E_x(z, t)\hat{x}\} = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\hat{x}$$
4.3 Maxwell’s Equations

- Some points to be noted:
  - First point is that the solution of the wave equation is a vector.
  - Secondly, Fig. 4.5 shows a typical plot of the normalized propagating electric field at a fixed point in time.
  - Thirdly, its phase $\phi = \omega t - \beta z$ is a function of both time, frequency and propagation distance.
4.3 Maxwell’s Equations

- So, if we fix the phase and let the wave travel a distance of $\Delta z$ over a period of time $\Delta t$
- Mathematically, this can be expressed as
  - $\Delta \varphi = \omega \Delta t - \beta \Delta z = 0$,
- therefore,
  - $v_p = \Delta z / \Delta t = \omega / \beta$,
- which is the phase velocity of the wave
- Hence,
  - $v_p = \Delta z / \Delta t = \omega / \left\{ \omega \sqrt{\mu \varepsilon} \right\} = 1 / \sqrt{\mu \varepsilon}$
4.3 Maxwell’s Equations

- For free space this turns out to be $v_p = 1/\sqrt{\frac{\mu_0 \varepsilon_0}{}} \approx 3 \times 10^8$ m/s
- Note that the phase velocity can be greater than the speed of light
- For instance, hollow metallic pipe in the form of rectangular waveguide has phase velocity

$$v_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$
4.3 Maxwell’s Equations

- For guided-wave propagation inside rectangular waveguide $f_c < f$ and
- correspondingly such guided waves propagate with phase velocity greater than the speed of light
- this doesn’t contradict the Eiensten’s theory of relativity because
  - there is no energy or information transfer associated with this velocity
4.3 Maxwell’s Equations

- Note that beside phase velocity, we have another term called group velocity which is associated with the energy or information transfer.

- Inside rectangular waveguide, in fact, the group velocity is much lower than the speed of light.

- This can be seen from the another relation between the phase and group velocity inside rectangular waveguide $v_p v_g = c^2$. 
4.3 Maxwell’s Equations

- phase velocity $v_p = \omega / \beta$
- group velocity $v_g = d\omega / d\beta$
- Fourthly, phase constant is also generally known as wave number
- $\beta = \omega / v_p = 2 \pi f / v_p = 2 \pi / \lambda$
- Fifth point is that ratio of the amplitude of the electric field to the magnetic field is known as intrinsic or wave impedance of the medium
4.3 Maxwell’s Equations

- $\eta = \frac{|E|}{|H|} = \frac{\omega \mu}{\beta} = \frac{\omega \mu}{\{\omega \sqrt{\mu\varepsilon} = \mu}{\{\sqrt{\mu\varepsilon}} = \sqrt{\mu/\varepsilon}$

- For free space $\eta = \sqrt{\mu_0/\varepsilon_0} \approx 377 \Omega$

4.4.3 Propagation in spherical coordinates

- We can also calculate the propagation of electric fields from an isotropic point source in spherical coordinates
- We will assume that the source is isotropic and the electric field solution is independent of $(\theta, \phi)$
- Assuming that the electric field is polarized along $\theta$-direction and it is only a function of $r$
4.3 Maxwell’s Equations

\[ \vec{E}(r, \theta, \varphi, t) = E_\theta (r, t) \hat{\vartheta} \]

- we can write the vector wave equation as follows:

\[ \nabla^2 \vec{E} - \gamma^2 \vec{E} = 0 \Rightarrow \frac{d}{dr} \left( r^2 \frac{d \vec{E}}{d r} \right) - \gamma^2 r^2 \vec{E} = 0 \]

- For finite fields, the solution is of the form

\[ \vec{E}(r, \theta, \varphi) = E_\theta (r) \hat{\theta} = \frac{E_0}{r} e^{-\gamma r} \]
4.3 Maxwell’s Equations

- Like before, we can add the time dependence to the phasor and get the real part of the electric field as follows

\[
\text{Re}\{\tilde{E}(r, \theta, \varphi, t)\} = \text{Re}\{E_\theta (r, t) \hat{\theta}\} = \frac{E_0}{r} e^{-\alpha r} \cos(\omega t - \beta r) \hat{\theta}
\]

- Note that there is an additional \(1/r\) dependence term in the expression for electric field here

- this factor is termed as spherical spreading for the isotropic point source
4.5 Summary

Electrodynamics

- Faraday's law
  \[ \oint E \cdot dl = -\int \frac{\partial B}{\partial t} \cdot ds + \oint (\mathbf{\nabla} \times \mathbf{B}) \cdot d\mathbf{l} \]

- Lenz's law
  \[ \oint D \cdot d\mathbf{s} = \int \rho_s dv \]
  \[ \oint B \cdot d\mathbf{s} = 0 \]
  \[ \oint H \cdot d\mathbf{l} = \int \left( \mathbf{j} + \frac{\partial D}{\partial t} \right) \cdot d\mathbf{s} \]
  \[ \oint \mathbf{E} \cdot d\mathbf{l} = -\int \left( \frac{\partial B}{\partial t} \right) \cdot d\mathbf{s} \]

- Integral form

- Differential form

- Phasor form

Maxwell's equations

\[ \nabla \times E = \frac{\rho_s}{\varepsilon} \]
\[ \nabla \times B = 0 \]
\[ \nabla \times H = \mathbf{j} + j\omega B \]
\[ \nabla \times \mathbf{E} = -j\omega \mathbf{B} \]

Wave equations

\[ \nabla^2 \mathbf{E} - \gamma^2 \mathbf{E} = 0 \]
\[ \nabla^2 \mathbf{H} - \gamma^2 \mathbf{H} = 0 \]

Boundary conditions

- \( \mathbf{n} \times (\mathbf{E}_1 - \mathbf{E}_2) = 0 \)
- \( \mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{j}_s \)
- \( \mathbf{n} \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0 \)
- \( D_{n1} - D_{n2} = \rho_s \)
- \( \mathbf{n} \cdot (\mathbf{j}_1 - \mathbf{j}_2) = 0 \)
- \( \mathbf{n} \times \left( \frac{\mathbf{j}_1}{\mathbf{j}_2} \right) = \frac{\sigma_1}{\sigma_2} \)

Fig. 4.6 Electrodynamics in a nutshell