MIMO Wireless Communications: An Introduction

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- Single Input Single Output (SISO)

![Fig. 1 SISO system](image)

- What will happen if Tx and Rx employs multiple antennas?
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• Multiple Input Multiple Output (MIMO)?

Fig. 2 $N_T \times N_R$ MIMO system
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• What are advantages of MIMO?
• Capacity (C) for Single Input Single Output (SISO) system
  \[ C = BW \log_2(1 + SNR) \]
• Data rate increase when
  – Bandwidth (BW) and
  – Signal to noise (SNR) power increase

Inherent problems:
• BW is precious, almost and always fixed for different applications
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• Signal power increase
  – battery lifetime decrease
  – creates higher interference
  – needs expensive RF amplifier

• In MIMO, spectral efficiency increase
  – Without increasing BW and
  – Signal power
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• But, how?

• Basically two gains for MIMO systems:
  – (i) MUX/rate gain
  – Capacity behavior \( R \approx r \log_2(SNR) \)

\[
r = \lim_{SNR \to \infty} \frac{R(SNR)}{\log_2(SNR)}
\]
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• For instance
• How much is the achievable rate gain?

\[ R \approx 3 \log_2(SNR) \]

Fig. 3 Achievable rate gain with 3 × 3 MIMO system
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(ii) Diversity gain

• Error probability behavior (minimize it) \( P_e(SNR) \approx SNR^{-d} \)
• Slope of symbol error rate/ bit error rate (SER/BER curve increases)
• Behavior of error probability w.r.t. average transmit power in log-log scale
  • for asymptotically high power

\[
d = - \lim_{SNR \to \infty} \frac{\log_2 \left\{ P_e(SNR) \right\}}{\log_2(SNR)}
\]
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• How much is diversity gain?

\[ P_e(SNR) \approx SNR^{-9} \]

Fig. 4 Diversity gain of $3 \times 3$ MIMO system

• For SISO (Rayleigh fading case), $d=1$, \( P_e(SNR) \approx SNR^{-1} \)
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• How much is rate and diversity gain?

![Diagram of 3x3 MIMO system with two transmit antennas and three receive antennas, showing rate and diversity gain for Case I.]

Fig. 5 Rate and diversity gain of 3 × 3 MIMO system (Case I)

\[ R \approx \log_2(\text{SNR}) \]

\[ P_e(\text{SNR}) \approx \text{SNR}^{-4} \]
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• How much is rate and diversity gain?

Fig. 6 Rate and diversity gain of 3×3 MIMO system (Case II)

\[ R \approx 2 \log_2(SNR) \]

\[ P_e(SNR) \approx SNR^{-1} \]
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- Need for a proper design for MIMO systems
- As $r$ decrease, $d$ increase
- As $r$ increase, $d$ decrease
- Diversity-multiplexing trade-off [1]

$$d_{opt} = (N_T - r)(N_R - r), 0 \leq r \leq \min(N_T, N_R)$$
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• Narrowband MIMO System Model

Fig. 7 Narrowband MIMO system model for $2 \times 2$ MIMO system
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At receiving antenna 1 (receives mixture of signals 1 & 2)
\[ y_1 = h_{11}x_1 + h_{12}x_2 + n_1 \]

At receiving antenna 2 (receives mixture of signals 1 & 2, a major problem in MIMO detection)
\[ y_2 = h_{21}x_1 + h_{22}x_2 + n_2 \]

In matrix form, \( \mathbf{y} = \mathbf{H} \mathbf{x} + \mathbf{n} \)
\[
\begin{bmatrix}
  y_1 \\
  y_2
\end{bmatrix}
= 
\begin{bmatrix}
  h_{11} & h_{12} \\
  h_{21} & h_{22}
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix}
+ 
\begin{bmatrix}
  n_1 \\
  n_2
\end{bmatrix}
\]
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• For $N_T \times N_R$ MIMO system

$$
\begin{bmatrix}
  y_1 \\
  y_2 \\
  \vdots \\
  y_{N_R}
\end{bmatrix}
= 
\begin{bmatrix}
  h_{11} & h_{12} & \cdots & h_{1N_T} \\
  h_{21} & h_{22} & \cdots & h_{2N_T} \\
  \vdots & \vdots & \ddots & \vdots \\
  h_{N_R1} & h_{N_R2} & \cdots & h_{N_RN_T}
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_{N_T}
\end{bmatrix}
+ 
\begin{bmatrix}
  n_1 \\
  n_2 \\
  \vdots \\
  n_{N_R}
\end{bmatrix}
$$

Received signal vector  Channel matrix  Transmitted signal vector  Noise signal vector
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• In order to have performance analysis of MIMO channel
  – we need Analytical MIMO channel models

• **Analytical MIMO channel model**
  – i.i.d. MIMO channel model
  – Separately correlated MIMO channel model
  – Uncorrelated keyhole MIMO channel model
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- i.i.d. MIMO channel model (each element of the channel matrix $\mathbf{H}$ is complex random variable)

$$h_{ij} = h_{ij}^{real} + jh_{ij}^{imag} ; i = 1,2, \cdots N_R; j = 1,2, \cdots N_T$$

$$h_{ij}^{real/imag} \sim N\left(0, \frac{1}{2}\right)$$

$$\Rightarrow p(h_{ij}^{real/imag}) = \frac{1}{\sqrt{2\pi \times \frac{1}{2}}} \exp \left(- \frac{(h_{ij}^{real/imag})^2}{2 \times \frac{1}{2}}\right)$$
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Complex Gaussian distribution
→ Joint distribution of real and imaginary part
→ They are assumed independent
→ PDF can be multiplied

\[ h_{ij} \sim N_C(0,1) \]

\[ \Rightarrow p(h_{ij}) = \frac{1}{\sqrt{\pi}} \exp\left(-\left(h_{ij}^{\text{real}}\right)^2\right) \frac{1}{\sqrt{\pi}} \exp\left(-\left(h_{ij}^{\text{imag}}\right)^2\right) \]

\[ \Rightarrow p(h_{ij}) = \frac{1}{\pi} \exp\left(-\left(h_{ij}^{\text{real}}\right)^2 + \left(h_{ij}^{\text{imag}}\right)^2\right) \]

\[ \Rightarrow p(h_{ij}) = \frac{1}{\pi} \exp\left(-|h_{ij}|^2\right) \]
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For i.i.d. case,

\[ p_H(H) = \prod_{i,j=1}^{N_R,N_T} \frac{1}{\pi} \exp\left\{-|h_{i,j}|^2\right\} = \frac{1}{\pi^{N_RN_T}} \exp\left\{-\sum_{i,j=1}^{N_R,N_T} |h_{i,j}|^2\right\} \]

\[ p_H(H) = \frac{1}{\pi^{N_RN_T}} \exp\left(-\text{Trace}\left(HH^H\right)\right) \]

“etr” is the abbreviation for “exponential trace”.

\[ p_H(H) = \pi^{-N_RN_T} \text{etr}\left\{-HH^H\right\} \]
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\[ \text{trace}(HH^H) \]

\[ = \text{trace} \left( \begin{bmatrix}
  h_{11} & h_{12} & \cdots & h_{1N_T} \\
  h_{21} & h_{22} & \cdots & h_{2N_T} \\
  \vdots & \ddots & \ddots & \vdots \\
  h_{N_R1} & h_{N_R2} & \cdots & h_{N_RN_T}
\end{bmatrix} \begin{bmatrix}
  h_{11} & h_{21} & \cdots & h_{N_R1} \\
  h_{12} & h_{22} & \cdots & h_{N_R2} \\
  \vdots & \ddots & \ddots & \vdots \\
  h_{N_T1} & h_{N_T2} & \cdots & h_{N_RN_T}
\end{bmatrix}^* \right) \]

\[ = \text{trace} \left( \begin{bmatrix}
  |h_{11}|^2 + |h_{12}|^2 + \cdots + |h_{1N_T}|^2 \\
  \vdots & \vdots & \vdots & \vdots \\
  |h_{21}|^2 + |h_{22}|^2 + \cdots + |h_{2N_T}|^2 \\
  \vdots & \vdots & \vdots & \vdots \\
  |h_{N_{R1}}|^2 + |h_{N_{R2}}|^2 + \cdots + |h_{N_{RN_{T}}}|^2
\end{bmatrix} \right) \]

\[ = \sum_{i,j=1}^{N_R \cdot N_T} |h_{i,j}|^2 \]
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- MIMO channel parallel decomposition
- To see how much is the capacity increase in MIMO systems

Fig. 8 Transmit precoding and receiver shaping (needs CSIR and CSIT)
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• From SVD of the channel matrix $\mathbf{H}$, we have,

\[
\mathbf{H} = \mathbf{U} \Sigma \mathbf{V}^H
\]

\[
\tilde{\mathbf{y}} = \mathbf{U}^H \mathbf{y} = \mathbf{U}^H (\mathbf{H} \mathbf{x} + \mathbf{n}) = \mathbf{U}^H \left( \mathbf{U} \Sigma \mathbf{V}^H \mathbf{x} + \mathbf{n} \right)
\]

\[
\Rightarrow \tilde{\mathbf{y}} = \mathbf{U}^H \left( \mathbf{U} \Sigma \mathbf{V}^H \mathbf{V} \tilde{\mathbf{x}} + \mathbf{n} \right)
\]

\[
\Rightarrow \tilde{\mathbf{y}} = \Sigma \tilde{\mathbf{x}} + \tilde{\mathbf{n}}
\]
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- Component-wise

\[
\begin{bmatrix}
\tilde{y}_1 \\
\tilde{y}_2 \\
\vdots \\
\tilde{y}_{R_H} \\
\vdots \\
\tilde{y}_{N_R}
\end{bmatrix} =
\begin{bmatrix}
\sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \ddots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_{R_H} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_{N_T} & 0
\end{bmatrix}
\begin{bmatrix}
\tilde{x}_1 \\
\tilde{x}_2 \\
\vdots \\
\tilde{x}_{R_H} \\
\vdots \\
\tilde{x}_{N_T}
\end{bmatrix} +
\begin{bmatrix}
\tilde{n}_1 \\
\tilde{n}_2 \\
\vdots \\
\tilde{n}_{R_H} \\
\vdots \\
\tilde{n}_{N_R}
\end{bmatrix}
\]
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• Parallel $R_H$ Gaussian channels

\[
\begin{align*}
\tilde{y}_1 &= \sigma_1 \tilde{x}_1 + \tilde{n}_1 \\
\tilde{y}_2 &= \sigma_2 \tilde{x}_2 + \tilde{n}_2 \\
&\vdots \\
\tilde{y}_{R_H} &= \sigma_{R_H} \tilde{x}_{R_H} + \tilde{n}_{R_H} \\
&\vdots \\
\tilde{y}_{N_R} &= \tilde{n}_{N_R}
\end{align*}
\]

• Capacity?

• increases $R_H$ fold
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• Rate Gain $\sqrt{\text{ }}$
• Diversity gain $\times$

**Space-time codes**

Implemented at the transmitter side, needs CSIR and block fading

• Why space-time codes?

\[ \overline{P_e} \approx \frac{c}{(G_c S)^{G_d}} \]
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- where $S$ is the SNR
- $c$ is a scaling constant specific to the
  - modulation employed and
  - the nature of the channel
- $G_c \geq 1$ denotes the coding gain and
- $G_d$ is the diversity order of the system
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- Diversity gain/order determines the:
  - negative slope of an error rate curve plotted vs SNR on a log-log scale

\[
\log_2 \left( \bar{P}_e \right) \approx \log_2 c - G_d \log_2 G_c - G_d \log_2 S
\]

- Space-time coded scheme with diversity order \( G_d \) has
  - an error probability at high SNR behaving as

\[
\bar{P}_e \approx (S)^{-G_d}
\]
If there is some coding gain, then

– average probability of error will be of the form

$$
\bar{P}_e \approx \frac{1}{(G_c S)^{G_d}}
$$

• If there were no array or power gain then

– the probability of error expression will be of the form

$$
\bar{P}_e \approx \frac{1}{G_c (S)^{G_d}}
$$
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• The coding gain determines the
  – horizontal shift of uncoded system error rate curve to the
    • space time coded error rate curve
    • plotted on a log-log scale obtained for the same diversity order
Fig. 9 Illustration of diversity and coding gains

- BER curves are usually waterfall type but
  - we have shown straight lines for illustration purpose only
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• Alamouti Space-Time Codes

\[
\langle s^1, s^2 \rangle = s^1 \left( s^2 \right)^H = 0
\]

\[
s^1 = [s_1 - (s_2)^*]^T \quad s^2 = [s_2 (s_1)^*]^T
\]

Fig. 10 A block diagram of Alamouti space-time encoder
Fig. 11 Alamouti’s space-time decoding.
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- Received signal vector
  \[ \mathbf{r} = \mathbf{Hs} + \mathbf{n} \]

- Received signals in two time intervals
  \[
  \begin{bmatrix}
  r_1 \\
  r_2
  \end{bmatrix} =
  \begin{bmatrix}
  s_1 & s_2 \\
  -s_2^* & s_1^*
  \end{bmatrix}
  \begin{bmatrix}
  h_1 \\
  h_2
  \end{bmatrix}
  +
  \begin{bmatrix}
  n_1 \\
  n_2
  \end{bmatrix}
  \]

- The output of the combiner
  \[ \tilde{\mathbf{r}} = \mathbf{H}^H \mathbf{Hs} + \tilde{\mathbf{n}} \]

  \[
  \begin{bmatrix}
  \tilde{r}_1 \\
  \tilde{r}_2
  \end{bmatrix} =
  \begin{bmatrix}
  h_1^* & h_2 \\
  h_2^* & -h_1
  \end{bmatrix}
  \begin{bmatrix}
  r_1 \\
  r_2
  \end{bmatrix}
  =
  \begin{bmatrix}
  h_1^* r_1 + h_2 r_2^* \\
  h_2^* r_1 - h_1 r_2^*
  \end{bmatrix}
  \]
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\[ \tilde{r}_1 = (|h_1|^2 + |h_2|^2)s_1 + \tilde{n}_1, \quad \tilde{r}_2 = (|h_1|^2 + |h_2|^2)s_2 + \tilde{n}_2 \]

• For 2×1 MIMO system, two signals are picked up by
  – the receiving antenna at the receiver
• Two signals are completely decoupled after the combining operation
  – for Alamouti Space Time Codes
• Simplifies greatly the detection strategy in comparison
  – to conventional MIMO detection
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• Applying MLD
• For \(0 \leq t \leq T\), we have

\[
\hat{s}_1 = \arg\min_{m} \left\| \tilde{r}_1 - \left( |h_1|^2 + |h_2|^2 \right) s_m \right\|, \quad s_m \in \{s_k\}_{k=1}^{M}
\]

• For \(T \leq t \leq 2T\), we have,

\[
\hat{s}_2 = \arg\min_{m} \left\| \tilde{r}_2 - \left( |h_1|^2 + |h_2|^2 \right) s_m \right\|
\]
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MIMO detection

• Detect signals jointly
  – since many signals are transmitted from the transmitter to the receiver

• Maximum likelihood (ML) detection outputs the vector which
  minimizes the Euclidean distance between
  – the received vector and
  – all possible combinations of the transmitted symbol vectors

\[
\hat{s} = \arg \min_x \|r - Hs\|^2
\]
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• Consider 2 \times 2 MIMO system

\[\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}; \mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}; \mathbf{s} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}; \mathbf{n} = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}\]

\[\mathbf{r} = \mathbf{Hs} + \mathbf{n}\]

\[\Rightarrow \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}\]
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• At the detector,
  – we want to detect $s_1$ and $s_2$ at time $t$,
  – but there exist interference between these two signals

$$r_1 = h_{11}s_1 + h_{12}s_2 + n_1; r_2 = h_{21}s_1 + h_{22}s_2 + n_2$$

• Assume that $s_k$ are modulated in M-ary constellation i.e., $s_k \in \{s_1, s_2, ..., s_M\}$
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• We need to find the minimum metric of the Euclidean distance

\[
\min_{i,j\in\{1,2,\ldots,M\}} \left[ \|r_1 - \left( h_{11}s_i + h_{12}s_j \right) \|^2 + \|r_2 - \left( h_{21}s_i + h_{22}s_j \right) \|^2 \right]
\]

\[
\min_{i,j\in\{1,2,\ldots,M\}} \left[ \left\| (h_{11}s_i + h_{12}s_2 + n_1) - \left( h_{11}s_i + h_{12}s_j \right) \right\|^2 + \left\| (h_{21}s_1 + h_{22}s_2 + n_2) - \left( h_{21}s_i + h_{22}s_j \right) \right\|^2 \right]
\]

• For instance, 16-QAM, \((s_1,s_2)\) are (1 of 16 symbols, 1 of 16 symbols) implies 16×16 pairs
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• Metric calculations of 256 are required
• For 3×3 MIMO system,
  – $N_T=N_R=3$, metric calculations of $16^3=4096$ are required
• For 5×5 MIMO system,
  – $N_T=N_R=5$, metric calculations of $16^5=10,48,576$ are required
• which is obviously impractical
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- ML detectors are optimal but impractical
- Low complexity suboptimal detectors like
  - Zero forcing (ZF) and
  - Minimum mean square error (MMSE)
- are preferable
- In linear detector,
- a linear preprocessor \( \mathbf{W} \) is first applied
  - to the received signal vector

\[
\hat{s} = \mathbf{W}^H \mathbf{r}
\]
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• For instance in ZF

\[ \hat{s} = W_{ZF}^H r = H^+ r = s + H^+ n \]

• \( H^+ \) is the Moore Penrose pseudo-inverse of \( H \)

\[ W_{ZF}^H = H^+ = (H^H H)^{-1} H^H \]

• Performance is not so good

• Consider a 2×2 MIMO system with \( s \in S = \{-3, -1, 1, 3\} \)
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- The channel matrix is given by $H = [2 \ 0.5; 1 \ 2]$.
- Suppose the received signal vector is $r = [1 \ 0.9]^T$.
- The ZF detector’s output is given by
  \[
  \hat{s} = H^+ r = [0.5 \ 0.2]^T
  \]
- Thus the hard decision of for $s$ becomes $[1 \ 1]^T$
  – which is different from the ML decision of $[1 \ -1]^T$
- ZF less complex than MLD, but poor performance.
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Antenna selection [2]

• Reduce hardware complexity

• A subset of total available antennas selected
  – Based on capacity maximization
  – Based on maximum received SNR
  – Done at transmitter, at receiver or both
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- Transmit Antenna Selection/Maximal Ratio Combining (TAS/MRC) [3]

**Fig. 12 3 × 3 TAS/MRC MIMO system**
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• How to select antenna?
• By maximizing the received SNR \( I = \arg \max \{ C_i = \sum_{j=1}^{N_r} |h_{i,j}|^2 \} \)
• For Rayleigh fading channel,
  \(- C_i \) are i.i.d. Chi square distributed with the probability density function (PDF) and cumulative distribution function (CDF) as
  \[
p(x) = \frac{1}{(N_r - 1)!} x^{N_r-1} e^{-x}, \quad x \geq 0
  \]
  \[
P(x) = 1 - e^{-x} \sum_{i=0}^{N_r-1} \frac{x^i}{i!}, \quad x \geq 0
  \]
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- Order statistics
- PDF of maximum received SNR $C_{(N_t)}$ such that $C_{(1)} \leq C_{(2)} \leq \ldots \leq C_{(N_t)}$ can be given as [4]

$$p_{(N_t)}(x) = N_t[P(x)]^{N_t-1} p(x)$$

$$= \frac{N_t}{(N_t - 1)!} \left(1 - e^{-x} \sum_{i=0}^{N_t-1} \frac{x^i}{i!}\right)^{N_t-1} \cdot x^{N_t-1} e^{-x}$$
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• Assume binary phase shift keying (BPSK) for TAS/MRC MIMO system

• Instantaneous SNR at the MRC receiver

\[ \gamma_b = \gamma \sum_{j=1}^{N_r} |h_{N_t,j}|^2 = \gamma C_{(N_t)} \]

• The average BER

\[ P_2 = \int_{0}^{\infty} Q(\sqrt{2 \gamma_b}) p_{\gamma_b}(\gamma_b) d\gamma_b \]

Conditional error probability (CEP) for BPSK
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• Thus, closed form expression for BER Rayleigh fading is

\[
P_2 = \frac{N_t}{(N_r-1)!} \sum_{k=0}^{N_t-1} \left[ \frac{(-1)^k \binom{N_t-1}{k}}{[2(k+1)]^{N_r}} \right] \times \sum_{t=0}^{k(N_r-1)} \left[ a_t(N_r,k)(N_r+t-1) \times \left( 1 - \sqrt{\frac{\gamma}{\gamma+k+1}} \right)^{N_r+t} \right]
\]

\[
\times \sum_{j=0}^{N_r+t-1} 2^{-j} \binom{N_r+t-1+j}{j} \times \left( 1 + \sqrt{\frac{\gamma}{\gamma+k+1}} \right)^j \right] \}
\]

• where, \( a_t(N_r,k) \) is the coefficient of \( z^{2t} \) in the expansion of

\[
\left\{ \sum_{i=0}^{N_r-1} \frac{z^i}{2(k+1)\ i!} \right\}^k
\]
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• $\eta - \mu$ distribution proposed by M. Yacoub in 2000 (can analyze non LOS propagation)
• Can model many distributions
• Some of the special cases of $\eta - \mu$ distribution are
  – Rayleigh distribution for $\eta \to 1, \mu = 0.5$,
  – one sided Gaussian distribution for $\eta \to 1, \mu = 0.25$,
  – Nakagami-m distribution for $\eta \to 1, \mu = m/2$ and
  – Hoyt distribution for $\eta \to q^2, \mu = 0.5$
• BER analysis for TAS/MRC over $\eta - \mu$ fading channel can be carried out in the similar way
Fig. 13: BER performance of (2, 1; 1) TAS/MRC system over $\eta - \mu$ fading channels for $\eta = 1$ with BPSK modulation
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Spatial modulation

- Two information bearing units
  - Transmit antenna index: estimated at receiver
  - A symbol from signal constellation: transmitted from antenna corresponding to transmit antenna index

- Advantages
  - Higher capacity
  - Reduced hardware complexity
  - Avoidance of transmit antenna synchronization

- How?
# MIMO Wireless Communications

## Table 1: SM Mapping Table for two cases

<table>
<thead>
<tr>
<th>Input Bits</th>
<th>$N_t = 2, M = 4$ (QAM)</th>
<th>$N_t = 4, M = 2$ (BPSK)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Antenna Number</td>
<td>Transmit Symbol</td>
</tr>
<tr>
<td>000</td>
<td>1</td>
<td>+1+j</td>
</tr>
<tr>
<td>001</td>
<td>1</td>
<td>+1-j</td>
</tr>
<tr>
<td>010</td>
<td>1</td>
<td>-1-j</td>
</tr>
<tr>
<td>011</td>
<td>1</td>
<td>-1+j</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>+1+j</td>
</tr>
<tr>
<td>101</td>
<td>2</td>
<td>+1-j</td>
</tr>
<tr>
<td>110</td>
<td>2</td>
<td>-1-j</td>
</tr>
<tr>
<td>111</td>
<td>2</td>
<td>-1+j</td>
</tr>
</tbody>
</table>
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Fig. 14 SM system model [5]
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• $k$ bit information blocks
• Sub blocks of $m$ bits and $n$ bits
  • $n$ bits spatially modulated
  • $m$ bits modulated using digital modulation schemes
• $m$ bits are transmitted physically, effectively similar to transmitting $k=m+n$ bits
• Restriction on the number of transmit antennas
  • Integer exponent of 2
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SM Receiver

- MRC diversity scheme
- Two step detection of transmitted symbol
- Antenna index estimation \((\mathbf{h}_j)^H \mathbf{y}\) is noise except for actual transmit antenna

\[
\hat{j} = \arg \max_j \frac{|\mathbf{h}_j^H \mathbf{y}|}{\|\mathbf{h}_j\|^2}
\]

- ML symbol detection \(\mathbf{y} = \sqrt{\rho} \mathbf{h}_j \mathbf{x}_q + \mathbf{n}\) transmit antenna [6]

\[
\hat{x}_q = \arg \min_j \left\| \mathbf{h}_j \mathbf{x}_q \right\|^2 - 2 \text{Re}\left\{\mathbf{h}_j^H \mathbf{y} \mathbf{x}_q^*\right\}
\]
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- Assume two estimation processes are independent
  - Transmit antenna index estimate
  - Estimation of the transmit symbol

- Notation
  - $P_a$ is probability that the antenna index estimation is incorrect
  - $P_s$ is probability that the transmitted symbol estimation is incorrect
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- Probability of correct estimation
  \[ P_c = \left(1 - P_a\right)\left(1 - P_s\right) \]

- Probability of error
  \[ P_e = 1 - P_c = 1 - \left(1 - P_a\right)\left(1 - P_s\right) = P_e \]
  \[ = P_a + P_s - P_a P_s \]

- Assume QAM

- CEP
  \[ P_e(\gamma) = aQ\left(\sqrt{b\gamma}\right) - cQ^2\left(\sqrt{b\gamma}\right) \]
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- Table 2 MODULATION PARAMETERS FOR VARIOUS MODULATION SCHEMES [8]

<table>
<thead>
<tr>
<th>Modulation Scheme</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>BFSK</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>MPSK</td>
<td>2</td>
<td>$2\sin\left(\frac{\pi}{M}\right)$</td>
<td>0</td>
</tr>
<tr>
<td>MPAM</td>
<td>$\frac{2(M - 1)}{M}$</td>
<td>$\frac{6}{M^2 - 1}$</td>
<td>0</td>
</tr>
<tr>
<td>QPSK or MSK</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Coherent DPSK</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>MQAM</td>
<td>$\frac{4\sqrt{M - 1}}{\sqrt{M}}$</td>
<td>$\frac{3}{M - 1}$</td>
<td>$4\left(\frac{\sqrt{M} - 1}{\sqrt{M}}\right)^2$</td>
</tr>
</tbody>
</table>
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• Q-function as sum of exponentials [7]

\[ Q(x) \approx \frac{1}{12} e^{-\frac{x^2}{2}} + \frac{1}{4} e^{-\frac{2x^2}{3}} \]

• CEP becomes

\[ P_e(\gamma) \approx \frac{a}{12} e^{-\frac{b\gamma}{2}} + \frac{a}{4} e^{-\frac{2b\gamma}{3}} - \frac{c}{144} e^{-b\gamma} - \frac{c}{16} e^{-\frac{4b\gamma}{3}} - \frac{c}{24} e^{-\frac{7b\gamma}{6}} \]

• where a=2, b=1, and c=1 for 4-QAM
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• the integral of fits the definition of MGF [9] of the received SNR, $MGF(\gamma)$.

$$MGF(\gamma) = \int_{0}^{\infty} \exp(-\gamma x) p_x(x) dx$$

• Probability of error in symbol detection

$$P_s \approx \frac{1}{6} MGF\left(\frac{\gamma}{2}\right) + \frac{1}{2} MGF\left(\frac{2\gamma}{3}\right) - \frac{1}{4} \left[ \frac{1}{36} MGF\left(\gamma\right) + \frac{1}{4} MGF\left(\frac{4\gamma}{3}\right) + \frac{1}{6} MGF\left(\frac{7\gamma}{6}\right) \right]$$
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- Probability of error in transmit antenna index estimation
  \[ P_a = Q\left(\sqrt{\gamma_{\text{eff}}}\right) \approx \frac{1}{12} \text{MGF}\left(\frac{\gamma}{4}\right) + \frac{1}{4} \text{MGF}\left(\frac{\gamma}{3}\right) \]

- where
  \[ \gamma_{\text{eff}} = \frac{\gamma}{2} \left\| h_j - h_j \right\|^2 \]

- MGF for Rice fading case
  \[ \text{MGF}_{\text{Rice}}(\gamma) = \frac{1 + K}{1 + K + \gamma} \exp\left(-\frac{K\gamma}{1 + K + \gamma}\right) \]
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• Fig. 15 SER vs SNR (dB) for 2×2 SM MIMO system considering Rician fading
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- SM with antenna selection [10]
- Some of the selected antennas for SM may be completely down
- SM systems combined with antenna selection
  - Antennas selected at transmitter
  - SM applied over selected antennas with better links
- CSI assumed to be available at transmitter
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• SM with antenna selection

Apply SM on selected antennas which give the best links

Fig. 16 4 × 4 TAS SM MIMO system
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- Order statistics
- Antenna Selection Parameter

\[ A_j = \sum_{i=1}^{N_r} |h_{i,j}|^2 \]

\[ f_{A_j}(x) = \frac{x^{N_r-1}e^{-x}}{\Gamma(N_r)}, x \geq 0 \]

\[ F_{A_j}(x) = 1 - e^{-x} \sum_{i=0}^{N_r-1} \frac{x^i}{i!}, x \geq 0 \]
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• $A_j$s are arranged in ascending order
• $S$ antennas corresponding to highest $A_j$s are selected
• PDF of $A(r)$ such that $A_{(1)} \leq A_{(2)} \leq \ldots \leq A_{(N_t)}$ can be given as

$$f_{A(r)}(x) = \frac{1}{B(r, N_t - r + 1)} \{F_{A_j}(x)\}^{r-1} \{1 - F_{A_j}(x)\}^{N_t-r} f_{A_j}(x)$$

• where $r = N_t - S + 1$
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• The PDF of received SNR can be given as

\[ f_{\gamma}^r(x) = \frac{1}{(N_t-r+1)\Gamma(N_r)} \sum_{i=r}^{N_t} \sum_{j=0}^{i-1} \sum_{t=0}^{M} \frac{1}{B(i,N_t-i+1)} \left( \begin{array}{c} i-1 \\ j \end{array} \right) (-1)^j C_t(j,N_r) x^{N_r+t-1} e^{-x(N_t-i+j+1)} \]

• where \( M = (N_r-1)(N_t-i+j) \) and \( C_t(j,N_r) \) is coefficient of \( x^t \) in

\[ \left( \sum_{i=0}^{N_r-1} \frac{x^i}{i!} \right)^{N_t-i+j} \]
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- Outage probability

\[
P_{out}(\gamma, R) = P_r \left\{ A_{(r)} < \frac{2^R - 1}{\gamma} \right\}
\]

\[
P_{out}(\gamma, R) = \frac{1}{(N_t - r + 1) \Gamma(N_r)} \sum_{i=r}^{N_t} \frac{1}{B(i, N_t - i + 1)}
\]

\[
\times \sum_{j=0}^{i-1} \binom{i-1}{j} (-1)^j \sum_{t=0}^{M} C_t(j, N_r) \frac{\gamma(N_r + t, \gamma_{th}(N_t - i + j + 1))}{(N_t - i + j + 1)^{(N_r + t)}}
\]

- where \( \gamma_{th} = \frac{2^R - 1}{\gamma} \)
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Fig. 17: Outage Probability Vs. SNR curve for TAS SM MIMO systems with antenna selection (2 bits/s/Hz)
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