Introduction

- An active magnetic bearing (AMB) system supports a rotating shaft, without any physical contact by suspending the rotor in the air, with an electrically controlled (or/and permanent magnet) magnetic force.
- It is a mechatronic product which involves different fields of engineering such as Mechanical, Electrical, Control Systems, and Computer Science etc.
Introduction to Active Magnetic Bearings

Working principle of magnetic bearing

Advantages of Magnetic Bearings

- Magnetic Bearings are free of contact and can be utilized in vacuum techniques, clean and sterile rooms, transportation of aggressive media or pure media
- Highest speeds are possible even till the ultimate strength of the rotor
- Absence of lubrication seals allows the larger and stiffer rotor shafts
- Absence of mechanical wear results in lower maintenance costs and longer life of the system
- Adaptable stiffness can be used in vibration isolation, passing critical speeds, robust to external disturbances

Classification of Magnetic Bearings

According to
- control action
  - Active
  - Passive
  - Hybrid
- Forcing action
  - Repulsive
  - Attractive
- Sensing action
  - Sensor sensing
  - Self sensing

Magnetic effect
- Electro magnetic
- Electro dynamic

Application
- Precision flotors
- Linear motors
- Levitated rotors
- Bearingless motors
- Contactless Geartrains

Applications of Magnetic Bearings

- Turbo molecular pumps
- Blood pumps
- Molecular beam choppers
- Epitaxy centrifuges
- Contact free linear guides
- Variable speed spindles
- Pipeline compressor
- Elastic rotor control
- Test rig for high speed tires
- Magnarails and maglev systems
- Gears, Chains, Conveyors, etc
- Energy Storage Flywheels
- High precision position stages
- Active magnetic dampers
- Smart Aero Engines
- Turbo machines

Fields of Applications of Magnetic Bearings

- Semiconductor Industry
- Bio-medical Engineering
- Vacuum Technology
- Structural Isolation
- Rotor Dynamics
- Maglev Transportation
- Precision Engineering
- Energy Storage
- Aero Space
- Turbo Machines

Electromagnetism

- Electromagnetic field
- Lorenz force
**Electromagnetism**

- When a charged particle is at rest it won’t emit electromagnetic waves rather it is surrounded by electrostatic field.

- When the charged particle is in uniform motion (i.e. the motion with uniform velocity in a direction) the electrostatic field is associated with magnetostatic field.

**Feed back loop of electromagnetism**

- The electric and magnetic fields are generated by electric charges.

- Charges generate electric fields.

- Movement of charges generate magnetic fields.

- The electric and magnetic fields interact only with each other.
  - Changing electric field acts like a current, generating vortex of magnetic field.
  - Changing magnetic field induces (negative) vortex of electric field.

- The electric and magnetic fields produce forces on electric charges.
  - Electric force which is generated by the electric field and is in same direction as electric field.
  - Magnetic force which is generated by the magnetic field and is perpendicular both to magnetic field and to velocity of charge.

- The electric charges move in space.
  - The electric charges move in space when they are acted upon by field forces.

**The four fundamental forces**

- Strong nuclear force: which holds atomic nuclei together.

- Weak nuclear force: which causes certain forms of radioactive decay.

**The four fundamental forces**

- Gravitational force: which causes the masses to attract each other.

- Electromagnetic force: which is caused by electromagnetic fields on electrically charged particles.
The four fundamental forces

- All the other forces are derived from these four fundamental forces
- Electro-magnetic force is one of these four fundamental forces

Force between two electrically charged particles

- Coulomb force (Static)
  \[ f_c = \frac{q_1q_2}{4\pi\varepsilon_0 r^2} \]
  \[ q_1 \quad f_c \quad q_2 \]

- Lorenz force (Dynamic)
  \[ f_l = \left( \frac{\gamma q_1 q_2 r}{4\pi\varepsilon_0 c^2} \right) + \left( \frac{r}{4\pi\varepsilon_0 c^2} \right) \]

Electric and magnetic components of Lorenz force

- If \( q_1 = q \) then
  \[ F = q\left( E + v \times B \right) \]
  \[ E = \frac{\gamma p r}{4\pi\varepsilon_0 c^2} \] Electric flux; \[ B = \frac{\gamma p v \times r}{4\pi\varepsilon_0 c^2} \] Magnetic flux;
  \[ r = \frac{m}{c^2} \] Lorenz factor;
  \[ \varepsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{J} \cdot \text{m} \] Electric permeability of vacuum;
  \[ \frac{1}{\varepsilon_0} = \mu_0 = 4\pi \times 10^{-7} \text{ N} \cdot \text{A}^{-2} \] Magnetic permeability of vacuum;

Effective Lorenz force in macro calculations

- For macro calculations Lorenz force is reduced to the form
  \[ F = q(v \times B) \]
  \[ B \quad v \quad F \]
- Lorenz force acts perpendicular to both velocity of charged particle and magnetic flux

Comparison Electric and magnetic components of Lorenz force

\[ \frac{|\mathbf{B}|}{|E|} \leq \frac{v}{c} \leq \frac{1}{10^6} \]

Three conclusions:
- Magnetic component of Lorenz force is at least smaller by a factor of \( 10^6 \) but we don’t face the effect of electric field in conductors because protons and electrons are equal in number and generate equal and opposite electric fields cancelling each other.
- Protons have no motion with reference to conductor and there won’t be magnetic component from them. Thus the magnetic component observed is the relativistic effect of electrons only.
- When the conductor is moving with reference to another frame both the protons and electrons will move with the same velocity thus the relativistic effects due to the velocity of conductor will be cancelled out.

Relations between \( \mathbf{E} \) and \( \mathbf{B} \)

- Gauss’ Law for linear materials
  \[ \nabla \cdot \mathbf{E} = \frac{q}{\varepsilon_0} \quad \int_S \mathbf{E} \cdot d\mathbf{s} = \frac{1}{\varepsilon_0} \int_E q dV \]
- Gauss’ Law for magnetism
  \[ \nabla \cdot \mathbf{B} = 0 \quad \int_S \mathbf{B} \cdot d\mathbf{s} = 0 \]
- Faraday’s law of magnetic induction
  \[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{s} \]
- Ampere’s law and Maxwell’s extension
  \[ \nabla \times \mathbf{B} = \mu_0 \left( \mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \quad \int_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_C \left( \mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) dS \]

These relations are called simplified Maxwell’s relations who formulated the original relations from previous works.
Design of magnetic actuator

- Bearing magnet
- Magnetic circuit
- Coil

Areas involved in the design of magnetic bearing systems

Magneto mechanical systems

According to the known technology till now, magnetic bearings can be classified for their design according to the purpose of the levitated object as

- Precision flotors (precision stages, isolation bases, isolation springs)
  - Levitation force
  - Propulsion force

A magnetic Precision Stage

Linear motors
(Contactless sliders, maglev trains and conveyors)
- Levitation force
- Propulsion force

Principle of a linear motor
Levitated rotors (gas turbines, energy storage flywheels, high speed spindles, balancing and vibration control of rotors)
- Radial load
- Thrust load

Rotor levitated by Radial and Axial Active Magnetic Bearings

Bearingless motors (canned pumps, compact pumps, blood pumps, spindle drives, semiconductor process)
- Radial load
- Thrust load
- Torque

Bearingless Motor

Contactless Gears and Couplers
- Regulated torque transmission

Regulated torque transmission

Linear systems from rotary systems

Macro Geometry of Thrust Magnetic Bearing

Design of a thrust magnetic bearing

Figure 1: Parts of Thrust Magnetic Bearing
Optimal design is carried out in two steps:

- **Modeling the magnetic circuit**
  - Determines the accuracy of achieving the objective.

- **Optimization of the parameters**
  - Determines the efficiency of achieving the objective.

### Magnetic circuit analogy with electric circuit

<table>
<thead>
<tr>
<th>Magnetic circuit</th>
<th>Electric circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magneto Motive Force (MMF)</td>
<td>Electro Motive Force (EMF) or Voltage (V)</td>
</tr>
<tr>
<td>Magnetic Flux ($\phi$)</td>
<td>Electric Current (i)</td>
</tr>
<tr>
<td>Reluctance (R)</td>
<td>Resistance (R)</td>
</tr>
</tbody>
</table>

### Ideal magnetic circuit model

\[
\oint H \cdot dl = \int J \cdot nda \quad \text{(Ampere's law)}
\]

\[
2H_1 l_1 + H_2 l_2 + H_3 l_3 = ni
\]

\[
B = \mu H \quad \text{or} \quad H = B / \mu
\]

\[
2B_1 l_1 + \left( \frac{B}{\mu_s} l_1 + \frac{B}{\mu_r} l_1 \right) = \mu_r ni
\]

If \( \frac{B}{\mu_s} l_1 + \frac{B}{\mu_r} l_1 \) is neglected,

\[
B_1 = \frac{\mu_r ni}{2}\]

Flux density is used to find the force exerted.

### Extension of the ideal model

If \( K_s \) is added for \( \mu \left( \frac{B_s}{\mu_s} l_s + \frac{B_s}{\mu_r} l_s \right) \) as core loss factor and \( K_i \) is added as coil loss factor, then the model reduces to:

\[
2K_s B_1 l_1 = \mu_s K_s ni
\]

\[
B_s = \frac{\mu_s K_s ni}{2K_s l_s}
\]

### Force by using flux density

\[
B = \frac{N_{lu}}{2\mu_0 A_s}
\]

\[
F = \frac{B_s^2}{2\mu_0} A_s
\]

\[
\Delta F = \frac{3A_s}{\mu_s} B_0 \Delta B
\]
Linear range of flux density

Hysteresis is assumed to be negligible while setting the linear range

Different quantities used in magnetic circuit

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Formula</th>
<th>Units</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current density</td>
<td>$j$</td>
<td>$\frac{\mu_0 A}{\varepsilon_0}$</td>
<td>A/m²</td>
<td>1604</td>
</tr>
<tr>
<td>Magnetic inductance</td>
<td>$L$</td>
<td>$\frac{\mu_0 N^2 A}{\varepsilon_0}$</td>
<td>H-m</td>
<td>0.0063</td>
</tr>
<tr>
<td>Nominal inductance</td>
<td>$L_n$</td>
<td>$\frac{\mu_0 N^2 A}{\varepsilon_0}$</td>
<td>H</td>
<td>804.2</td>
</tr>
<tr>
<td>Magnetic force by inductance</td>
<td>$F$</td>
<td>$\frac{\mu_0 N^2 A}{\varepsilon_0}$</td>
<td>N</td>
<td>804.2</td>
</tr>
<tr>
<td>Magnetic force by flux density</td>
<td>$F$</td>
<td>$\frac{\mu_0 N^2 A}{\varepsilon_0}$</td>
<td>N</td>
<td>804.2</td>
</tr>
<tr>
<td>Magnetic force for diff actuator</td>
<td>$F$</td>
<td>$\frac{\mu_0 N^2 A}{\varepsilon_0}$</td>
<td>N</td>
<td>19.84</td>
</tr>
</tbody>
</table>

Design vector for optimal design

Known parameters are:
- Gap
- Inner radius of the bearing
- Outer radius of the bearing

Free parameters:
- Inner radius of the coil space
- Outer radius of the coil space
- Height of the coil space
- Current density supplied

All the other parameters are dependant

Input parameters taken for the design of thrust magnetic bearing

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner radius of the bearing</td>
<td>25.00mm</td>
<td>Specific gravity of the stator iron</td>
<td>7.77g/cm³</td>
</tr>
<tr>
<td>Operating air gap</td>
<td>4.00mm</td>
<td>Specific gravity of the copper</td>
<td>8.91g/cm³</td>
</tr>
<tr>
<td>Operating load</td>
<td>2025N</td>
<td>Specific gravity of permanent magnet material neodymium-iron-baron</td>
<td>7.5g/cm³</td>
</tr>
<tr>
<td>Variation in the gap</td>
<td>±5%</td>
<td>Coil mmf loss factor</td>
<td>1.39%</td>
</tr>
<tr>
<td>Variation in the load</td>
<td>±10%</td>
<td>Actuator loss factor</td>
<td>1.67%</td>
</tr>
<tr>
<td>Saturation flux density</td>
<td>1.00T</td>
<td>Flux leakage factor</td>
<td>0.84%</td>
</tr>
<tr>
<td>Remnant flux density of bias magnet</td>
<td>1.2T</td>
<td>Packing factor</td>
<td>0.85%</td>
</tr>
<tr>
<td>Saturation current density</td>
<td>4.0A/mm</td>
<td>Maximum allowable coil volume</td>
<td>820mm</td>
</tr>
<tr>
<td>Maximum outer radius of bearing</td>
<td>120mm</td>
<td>Maximum height of bearing</td>
<td>70mm</td>
</tr>
</tbody>
</table>
Radial magnetic bearing

The component of force will be at an angle of half of the angle between two poles

\[ F = \frac{\mu A_i (K_m)^2}{4(K_m^2)} \cos \frac{\alpha}{2} \]

Three pole radial magnetic bearing

Magnetic Circuit for three pole AMB

Coil design

- Admissible coil temperature is determined by the choice of insulation type
- Number of turns are chosen such that it generates maximum admissible magnetomotive force at the maximum current supplied by the power amplifier

Winding scheme

Split flux
- Efficient use of iron
- Simpler current control

Unsplit flux
- Non-split rotating laces

Permanent magnetic bearings

Permanent magnetic bearings
MAGNETIC BEARINGS
CONTROL

Introduction

- Control is the process of bringing a system into desired path when it is going away from it
- Earnshaw (1842) had shown that it is impossible to hover a body in all six degrees of freedom by using permanent magnets
- But it is possible to maintain the body in equilibrium condition by active control

Types of control systems

- Open loop control systems
  - The control in which the output of the system has no effect on input is called open loop control
  - Open loop control is used when the input is known and there are no external disturbances
  - An example of open loop control is washing machine which works on time basis rather than the cleanliness of clothes

- Closed loop control systems
  - If the control maintains a prescribed output and reference input relation by comparing them and uses their difference as controlling quantity, it is called feedback or closed loop control
  - Temperature control of a room or a furnace is an example of closed loop system

Classification of controllers

- According to control action controllers are classified as:
  - Two-position or on-off controllers
  - Proportional controllers
  - Integral controllers
  - Proportional-integral controllers
  - Proportional-differential controllers
  - Proportional-differential-integral controllers

- Two-position or on-off controllers
  - The output of the controller \( y(t) \) will be a maximum or minimum according to the state of error \( e(t) \) as below:
    \[
    y(t) = y_0 \quad \text{for} \quad e(t) < 0
    \]
    \[
    = y_1 \quad \text{for} \quad e(t) > 0
    \]
  - \( y_0 \) and \( y_1 \) are minimum and maximum values of output
Classification of controllers

- **Proportional controllers:**
  - The output of the controller \( y(t) \) is proportional to the magnitude of the actuating error \( e(t) \) signal as \( y(t) = g_p e(t) \)
  - By Laplace transformation
    \[
    \frac{Y(s)}{E(s)} = g_p
    \]
  - \( g_p \) is called proportional gain

- **Integral controllers:**
  - Controller output is changed at a rate \( \frac{dy}{dt} \)
  - By Laplace transformation
    \[
    \frac{Y(s)}{E(s)} = g_i s
    \]
  - \( g_i \) is called integral gain

- **Proportional-Integral (PI) controllers:**
  - Control action is a combination of both proportional and integral action
    \[
    y(t) = g_p e(t) + g_i \int_0^t e(t) dt
    \]
  - By Laplace transformation
    \[
    \frac{Y(s)}{E(s)} = g_i \left( 1 + \frac{1}{Ts} \right)
    \]

- **Proportional-Differential (PD) controllers:**
  - The control action is defined by
    \[
    y(t) = g_p e(t) + g_d \frac{de(t)}{dt}
    \]
  - By Laplace transformation
    \[
    \frac{Y(s)}{E(s)} = g_d (1 + Ts)
    \]

- **Proportional-Integral-Differential (PID) controllers:**
  - It has the advantages of all three actions. So this is the most common type of industrial controllers
  - Mathematical form of PID action is
    \[
    y(t) = g_p e(t) + g_i \int_0^t e(t) dt + g_d \frac{de(t)}{dt}
    \]
  - By Laplace transformation
    \[
    \frac{Y(s)}{E(s)} = g_i \left( 1 + \frac{1}{Ts} + Ts \right)
    \]
Control Design

An overall system

Input

System

Output

Studying the behaviour of a system

Known

Known

Unknown

unknown

Studying the characteristics of a system

Known

Unknown

unknown

Designing of a control system of required behaviour

Methods of design and analysis of controllers

Transfer-function method

State-variable method

Frequency response analysis

Pole-placement analysis

(Polar and steady state Response analysis)

(Classical control)

(Pole-placement method and Linear-quadratic optimization)

(Modern control)

Methods of design and analysis

Transfer-function method

State-variable method

Root locus analysis

Frequency response analysis

Pole-placement analysis

Linear-quadratic optimization

Transfer-function method

State-variable method

Root locus analysis

Frequency response analysis

Pole-placement analysis

Linear-quadratic optimization

(Classical control)

(Pole-placement method and Linear-quadratic optimization)

(Modern control)

Steady state and transient response analysis, Root locus analysis and Frequency response analysis are the main methods of design and analysis.

Analysis consists of system of n first order differential equations.

It is useful for linear and simple systems only

Frequency domain method

Time domain method

Analysis consists of system of n first order differential equations.

It is useful for non-linear and complex systems also.

Frequency domain method

Time domain method

Analysis consists of system of n first order differential equations.

Transfer-function method

State-space method

Classical control method

Modern control method

Used for single input single output (SISO) systems

Used for multi input multi output (MIMO) systems can be used for SISO also

It is useful for linear and simple systems only

It is useful for non-linear and complex systems also.

Steady state and transient response analysis, Root locus analysis and Frequency response analysis are the main methods of design and analysis.

Mechanical and electro-magnetic stiffness

Mechanical spring

Magnetic spring

Operating current

Instantaneous current

Operating current

mg

Operating position

mg*

Equilibrium position

Operating current

Instantaneous current

Operating position

mg*

Equilibrium position

Mechanical spring stiffness

Magnetic spring stiffness

Magnetic Bearing Control

• Equilibrium and Operating points
  • For a mechanical spring there will be an equilibrium point where the force resisted by the spring is equal to the force applied on the spring
  • For electro magnets there will be a quantity of current corresponding to position of the object and force applied. At this point the gravity force and magnetic force will be equal. A slight movement form this point will cause indefinite movement of the body. This point is called operating point
Linearization of current

\[ i = i_m - i_0 \]
\[ f = k_i j \]

\(i_m\) is the instantaneous current
\(i_0\) is the deviation of current from operating current
\(f\) is instantaneous force

Linearization of displacement

\[ x = x_0 - x_m \]
\[ f = k_x x \]

\(x_0\) is the instantaneous position
\(x_m\) is the deviation of position from operating position
\(f\) is instantaneous force

Linearized formula around the operating point will be

\[ f(x, i) = k_x x + k_i j \]

where
- \(x\) is the displacement from the operating position
- \(i\) is the deviation of current from operating current
- \(k_x\) is displacement stiffness
- \(k_i\) is current stiffness
- \(f\) is instantaneous force

- Linearized equation is suitable for most of the applications of magnetic bearings
- It is not valid in three occasions
  - When \(x = x_0\) the rotor touches the bearing magnet
  - When there are strong currents such that magnetic saturation of the material occurs
  - When \(i = -k_i\) or very small currents there won’t be levitation of the rotor because of very small magnetic forces.

Magnetic Bearing Control

\[ f = k_x x + k_i j \]

By Newton’s law

\[ f = m \ddot{x} \]

Combing above two equations we get

\[ m \ddot{x} - k_x x = k_i j \]

If controlling current \(i\) is zero then

\[ m \ddot{x} - k_x x = 0 \]

And the response grows exponentially thus the rotor may fall down or touch the magnet

If we supply controlling current \(i\) such that

\[ i(x) = \frac{k + k_i}{k_i} x + \frac{c}{k_i} \ddot{x} \]

then it becomes

\[ m \ddot{x} + c \dot{x} + k_x x = 0 \]

And the response is imitated to a spring mass damper system by the magnetic bearing system
PD controller model

- The model is PD-controller with proportional and differential feedback
  \[ P = \frac{k + k_i}{k_i} \quad D = \frac{C}{k_i} \]
- In design of controller we choose the stiffness and damping to ensure the system come to steady state in optimum time.
- The optimal stiffness suggested is
- The range of damping ratio for better systems suggested is 0.1 to 1

Control of rotors by using magnetic bearings

- Stiffness is very high thus the vibration of the rotor will be transmitted to foundation
- Stiffness is very low thus the rotor can rotate freely about the principal axes of inertia which results in a vibration isolation system.
- Damping is directly observed due to hydrodynamic effects
- As the rotor is free in the air there is no coulomb damping acting on the system. The control law will have damping term.

Topics to be covered

- Rigid rotor model
- Flexible rotor model

Differences between mechanical and magnetic bearing models
Rigid rotor model

Angular velocity of shaft
\[ \alpha = \frac{d\alpha}{dt} \quad \beta = \frac{d\beta}{dt} \]

Rigid rotor model

Kinetic energy is expressed as
\[ T = \frac{1}{2} m \left( \dot{x}^2 + \dot{y}^2 + \dot{z}^2 \right) + \frac{1}{2} \left( J_{xx} \dot{\alpha}^2 + J_{yy} \dot{\beta}^2 \right) \]

Equations of equilibrium can be obtained as by using Lagrange's principle
\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{x}_i} \right) - \frac{\partial T}{\partial x_i} = F_i \]

\( F_i \) is the generalized force corresponding to \( i \)\(^{th} \) variable

Rigid rotor model

If the variable vector is chosen as
\[ x = [x_1 \ x_2 \ x_3 \ x_4]^T = [\beta \ x \ -\alpha \ y]^T \]

Angular velocity vector can be expressed as
\[ \dot{\omega} = \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = \begin{bmatrix} \dot{\alpha} \cos \Omega + \dot{\beta} \sin \Omega \\ -\dot{\alpha} \sin \Omega \cos \Omega - \dot{\beta} \sin \Omega \cos \Omega \end{bmatrix} \]

Rigid rotor model

Equations (1) can be expressed in matrix form by rearranging
\[ Mx + (G + C)x = F \]

\( M \) is the inertia matrix \( (M = M^T) \)
\( G \) is the gyroscopic matrix \( (G = -G^T) \)
\( C \) is the damping matrix \( (C = C^T) \)

\( F \) can be expressed as
\[ F = -(K + N)x \]

\( K \) is conservative force matrix \( (K = K^T) \)
\( N \) is non-conservative force matrix \( (N = -N^T) \)

Rigid rotor model

• Conservative forces include
  – forces due to stiffness

• Non-conservative or circulatory forces include
  – internal or structural damping
  – steam or gas whirl in turbines
  – seal effects
  – process forces such as in grinding
  – Unbalance, etc

• Damping include
  – Coulomb damping due to hydrodynamic effects

Rigid rotor model

• From Eq. (2) and (3) we get
\[ Mx + (G + C)x + (K + N)x = 0 \]

• If the non-conservative and gyroscopic forces neglected, we have
\[ Mx + Kx = 0 \]
Natural modes

- The solution of the equations (5) gives four modes, for there are four degrees of freedom considered

Magnetic bearing model

- In a magnetic bearing if we neglect the conservative, non-conservative, and damping effects, we will have
  \[ M\ddot{x} + C\dot{x} = \mathbf{F} \]
- For small rotations gyroscopic effects can be neglected and the equations in \( x \) and \( y \) directions can be decoupled
  \[ M\ddot{x} = \mathbf{F} \]

Weight considerations

- For weight considerations
  \[ f_g = mg = k_{0} \]
  \[ f_g = \frac{mg}{\cos \alpha} = k_{0} \]

Imbalance considerations

- For imbalance considerations
  \[ f_{im} = \bar{m} \dot{\omega} \Omega \cos(\Omega t + \theta) \]
  \( \bar{m} \) is the imbalance mass
  \( e \) is the eccentricity of imbalance mass
  \( \theta \) is the angular position of imbalance mass

Magnetic bearing model

- It can be written as
  \[ \ddot{m}x = f_x + f_i - f_y + f_{im} \]
where
  \[ f_x = k_x \]
  \[ f_i = k_i \]
  \[ f_y = mg = k_{0} \]
  \[ f_{im} = \bar{m} \dot{\omega} \Omega \cos(\Omega t + \theta) \]
Magnetic bearing model

• It will be
  \[ m\ddot{x} = k_x x + k_i (i - i_i) + \dot{m}e\Omega^2 \cos(\Omega t + \theta) \]

• \( i \) at any instant will be
  \[ i = i_i + \frac{m\ddot{x} - k_x x - \dot{m}e\Omega^2 \cos(\Omega t + \theta)}{k_i} \]

Rigid rotor with magnetic bearing

• Three steps involved:
  – Formulation with respect to centre of gravity
  – Transformation with respect to the bearing coordinates
  – Transformation with respect to the sensor coordinates

Why with respect to sensor coordinates

• Sensors cannot be arranged directly in the magnetic actuator.
• This requires certain gap between the magnet and the sensor.
• The displacements with respect to sensor coordinates will be transformed to bearing coordinates

With respect to centre of gravity

• In slow role \( x \) and \( y \) directions can be decoupled

\[ m\ddot{x} = f \]
\[ I_x\ddot{\beta} = p \]

In matrix form as
\[ Mx = f \]

where
\[ M = \begin{bmatrix} m & 0 \\ 0 & I_x \end{bmatrix} \]
\[ f = \begin{bmatrix} x \\ \beta \end{bmatrix} \]

With respect to bearing coordinates

• Forces are transformed as
  \[ f = f_a + f_b \]
  \[ p = a_1 f_a + b_1 f_b \]
  \[ f = T_f \hat{f} \]

\[ T_f = \begin{bmatrix} 1 & 0 \\ a & b \end{bmatrix} \]
\[ \hat{f} = \begin{bmatrix} f_a \\ f_b \end{bmatrix} \]

With respect to bearing coordinates

• Displacement vector can be transformed as
  \[ x = T_x x \]
  \[ x = \begin{bmatrix} x \\ \beta \end{bmatrix} \]
  \[ T_x = \frac{1}{b-a} \begin{bmatrix} b & -a \\ 1 & 0 \end{bmatrix} \]
  \[ x = \begin{bmatrix} x_a \\ x_b \end{bmatrix} \]
With respect to sensor coordinates

\[
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} = \begin{bmatrix}
    x_p \\
    x_s
\end{bmatrix}
\]

\[
T_r = \begin{bmatrix}
    1 & c \\
    1 & d
\end{bmatrix}
\]

\[
x = \begin{bmatrix}
    x \\
    \beta
\end{bmatrix}
\]

\[
S_s = S_B T T T
\]

\[
S_c = S_B + C \int \frac{dx}{dt}
\]

\[
F = -u x
\]

\[
x_c = C x_s
\]

\[
x_s = T_s x
\]

State feedback

State feedback

- The control vector is found by using control law

\[
u = -f x
\]

- We do not know the velocity components directly from sensors. So a state observer is required to find the velocities

\[
x_s = C x_c
\]

\[
x_s = \text{the full state vector}
\]

\[
x_c = \text{the vector from the sensor}
\]

Model at high speeds

- At high speeds the gyroscopic effects cannot be neglected, thus the model becomes

\[
M S + G \dot{S} = F
\]

- The displacements in x and y directions no longer decoupled, so four forces and four displacements should be taken into consideration simultaneously.

- The same procedure is to be followed as for the slow rotation

Conclusions on rigid rotor model

- There is an optimal design for each speed
- The optimal design at higher speed may not be stable at lower speeds, for the gyroscopic effects are reduced.
- The optimal design at zero speed may not be the optimal at higher speeds
- The gyroscopic effects will not destabilize the system which is stable at lower speeds.
- Further more the design at lower speeds is decoupled and easier to design. Decentralized designs for lower speeds can be implemented
Conclusions on rigid rotor model

- Thus for stability considerations and other advantages systems are designed for lower speeds and with decentralization

\[ u = -F \cdot x \]
\[ u = -F \cdot x \]
\[ u = -F \cdot x \]
\[ u = -F \cdot x \]

Flexible rotor model

- Rigid rotor can be defined by two points

- Flexible rotor has infinite degrees of freedom. One cannot define uniquely by some of the points

Flexible rotor model

- Equation motion of an Euler-Bernoulli beam is given by

\[ \frac{EI}{m} \frac{d^4y}{dz^4} + m \frac{d^2y}{dt^2} = \frac{1}{2} \]

- The variable separable form is

\[ y(z,t) = Y(z)q(t) \]

Flexible rotor model

- By substituting we get

\[ \frac{EI}{m} \frac{d^4Y(z)}{dz^4} = \frac{d^2q(t)}{dt^2} = \omega^2 \]

- By rewriting we get

\[ \frac{d^4Y(z)}{dz^4} - \beta Y(z) = 0, \quad \beta = \frac{\omega^2}{EI/m} \]

\[ \frac{d^2q(t)}{dt^2} + \omega^2q(t) = 0 \]

Flexible rotor model

- By applying initial conditions and solving we get the natural frequencies \( \omega \)

- By substituting the Eigen values in (29) we get the Eigen functions or model functions \( Y(z) \)

- The mode shapes or modal functions depend on the end conditions
Actuator sensor location
- Sensor should not be set at nodes
- Sensor and actuator should not lie on opposite sides of a node

We can conclude that the sensor can be set at a place where we can get information from each mode under consideration.

Modal reduction
- While designing a flexible rotor system, we cannot consider all the modes of the system for they are infinite.
- Thus we consider first $n$ number of modes corresponding to first $n$ natural frequencies and neglect the remaining modes.
- If we study the effect of the reduced modes, we can find the number of modes which we can consider without destabilizing the system.

Mathematical model of the
- Full system $y = Cx$

Divided system
$$\begin{bmatrix} x_m \\ x_n \\ x_u \end{bmatrix} = \begin{bmatrix} A_m & A_u \\ A_m & A_u \\ B_m \\ B_u \end{bmatrix} \begin{bmatrix} x_m \\ x_n \\ x_u \end{bmatrix}$$
$$y = \begin{bmatrix} C_m \\ C_n \\ C_u \end{bmatrix} \begin{bmatrix} x_m \\ x_n \\ x_u \end{bmatrix}$$

Reduced system
$$x_u = A_u x_u + B_u u$$
$$y = C_u x_u$$

Block diagram of effect of model reduction
- Modeled modes
- Unmodeled modes
- Control spillover (By the input)
- Interconnection spillover (By the parameters of the system)
- Observation spillover (on the estimated output)
Conclusion on flexible rotor control

- Modal reduction is studied to consider the number modes to be taken into consideration for having stable control
- Mechanical design is studied for finding the sensor actuator locations

Conclusions

- Magnetic bearings advantages and applications have been discussed
- Electromagnetism and Control system technologies have been introduced
- Design of thrust and radial magnetic bearings have been studied
- Control of a rotor by rigid rotor and flexible rotor models have been studied

Further References


Thank you