Instability of Rotor-Bearing Systems

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Introduction

Rotor dynamics is the study of rotating machines and has a very important part to play throughout the modern industrial world. Rotating machinery is used in many applications, *eg*, Turbomachines, Power Stations , Machine tools, Automobiles, Household machines, Aerospace applications, Marine propulsion, Medical equipment. The interaction these machines have in there surroundings is of great importance as if these machines are not operating at the correct speed ranges, vibration can occur which may ultimately cause failure. Failure of machinery in applications such as aeroengines, turbomachines, space vehicles, etc. creates enormous repair costs and more importantly may put human life in danger. This means governments and industries put a great deal of resources into the study of rotor dynamics to calculate the 'safe' operating ranges before the machine goes into service and also methods of detecting imminent failure. Rotor dynamics is a collective term for rotating machines and can be split into the sub-groups that make it up. These are rotating shafts, bearings, seals, out of balance systems, instability and condition monitoring.

Rotating Shaft

Single-mass rotor

The simplest case in rotor dynamics is the de Lavel rotor. The de Lavel rotor consists of a long, light shaft carrying a mid-span disk.

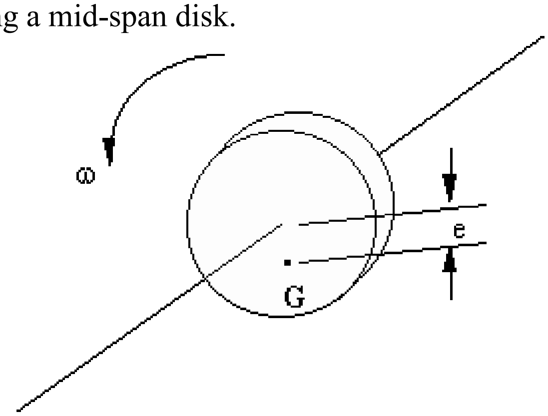

Fig.1: Single Mass rotor

The mid-span disk has a centre of mass that due to unbalance is at a point '*G*' a distance '*e'* from the geometrical centre; this distance is known as the eccentricity.

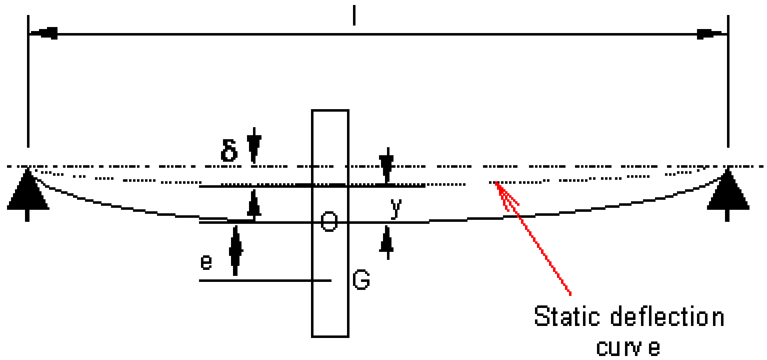

Fig.2: Deflection of Rotor

In a rotating shaft the speed at which instability occurs is called whirling or critical speed. The frequency when the shaft reaches its critical speed can be found by calculating the frequency at which transverse vibration occurs. For Example in the case shown below: -

The centripetal force exerted on the shaft = $m\omega^2\left(y+e\right)$ [1]

The inwards pull of the shaft = $\dfrac{kWIy}{l^3} = mf$ [2]

Where *k* depends on the load and fixing conditions, rearranging equation [2]:

$$\frac{f}{y} = \frac{kEI}{ml^3} = \frac{g}{\delta} \qquad [3]$$

Equating equations [1] and [3] gives

$$m\omega^2\left(y+e\right) = \frac{kEIy}{l^3} \qquad [4]$$

Rearranging equation [4],

$$y = \frac{e}{\left(\dfrac{KEI}{m\omega^2 l^3} - 1\right)} \qquad [5]$$

Whirling occurs when the deflection of the shaft becomes infinite

When, $\dfrac{KEI}{m\omega^2 l^3} = 1$

So the critical speed is given by: $\omega_c = \sqrt{\dfrac{KEI}{ml^3}} = \sqrt{\dfrac{g}{\delta}}$ [6]

However in reality, the above analysis is a simplification, as the rotor will rotate at an angle due to the deflection around the static deflection curve.

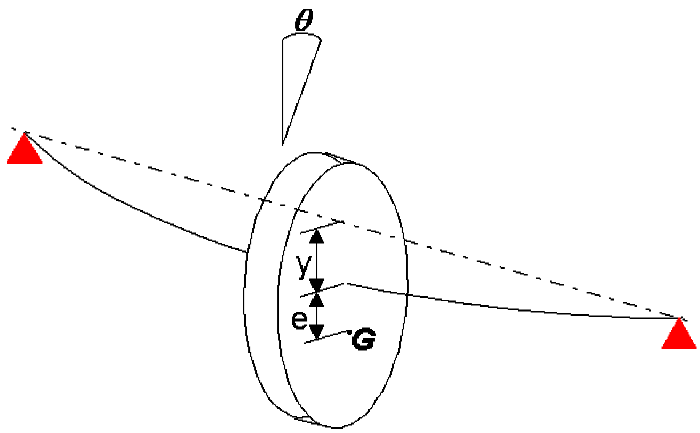

Fig.3: Rotating Rotor

The above situation may be expressed using matrices as:

$$\begin{Bmatrix} y \\ \theta \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{Bmatrix} F_y \\ M_{yz} \end{Bmatrix} \qquad [7]$$

Where the coefficients α_{nm} are the influence coefficients due to the deflection of the shaft due to the loading conditions. The influence coefficients may be found in by simple beam deflection theory:

$$\alpha_{11} = a^2b^2 / 3EIl$$
$$\alpha_{12} = -\left(3a^2 - 2a^3 - al^2\right) / 3EIl \qquad [8]$$
$$\alpha_{21} = ab\left(b-a\right) / 3EIl$$
$$\alpha_{22} = -\left(3al - 3a^2 - l^2\right) / 3EIl$$

The influence coefficient matrix needs to be inverted to give:

$$\begin{Bmatrix} F_y \\ M_{yz} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} y \\ \theta \end{Bmatrix} \qquad [9]$$

The equations of motion for the mass of the rotor are

$$me\omega^2 - F_y = m\frac{d^2y}{dt^2}$$
$$M_{yz} = I_d\frac{d^2\theta}{dt^2} \qquad [10]$$

If the vibrations are sinusoidal then, $\frac{d^2y}{dt^2} = -\omega^2 y$ and $\frac{d^2\theta}{dt^2} = -\omega^2\theta$

Equations [10] may then be inserted back into the matrix [9]

$$\begin{Bmatrix} me\omega^2 \\ 0 \end{Bmatrix} = \begin{bmatrix} \left(k_{11} - m\omega^2\right) & k_{12} \\ k_{21} & \left(k_{22} - I_d\omega^2\right) \end{bmatrix} \qquad [11]$$

If the values for the k matrix are known then the matrix may be solved and values of y and θ may be found.

Bearing Systems

Bearings are designed to transfer pure radial loads, pure thrust loads, or a combination of the two between two objects one rotating relative to the other.

Rolling element Bearings

In engineering, the most common type of bearing system used is rolling element bearings. Separated into sub-types, Deep grove ball bearing, Angular contact bearings, Four-point (duplex bearing), Self-aligning bearings, Cylindrical roller bearings, Spherical roller bearings, Tapered roller bearings, Needle roller bearings, Thrust ball bearings. The bearing stiffness of rolling element bearings is an important part of the overall characteristics of the dynamic operation of the system. Damping in rolling element bearings is usually very small but can be increased by the use of a squeeze film.

Journal Bearings

Journal bearings are made up of a circular section of shaft, the journal, rotating inside a bearing bush. The 0.1-0.2 % space between the two is partially filled by the lubricating fluid. When at rest the journal rests at the base bottom of the bush. When rotating, the lubricant is drawn up around the journal due to the effect of elastrohydrodynamic lubrication. Elastrohydrodynamic lubrication is when lubricant is introduced between two surfaces in rolling contact, which creates a huge increase in pressure. As viscosity is exponentially related to pressure, a large increase in viscosity occurs between the journal and bush creating a thin film preventing contact.

An antifriction bearing lubricant has the following properties:

- Provides a lubricant between the surfaces
- Help distribute and dissipate heat
- Prevents corrosion of the bearing surfaces
- Protects from foreign matter entering the bearing

Journal bearings can have a significant effect of the machines dynamic characteristics as the oil film acts as a complicated set of springs and dampers and is a major factor in the vibration of turbomachinery. The instability caused by the oil film in journal bearings is called non-synchronous whirl or oil whip.

Gas Bearings

Gas lubricated bearings operate similarly to oil lubricated bearings but have different performance characteristics because of the compressible nature of the lubricant creating different pressure distributions. This can create significant sub-ambient pressures, as the lubricant cannot cavitate. Gas bearings of similar size to oil lubricated bearings carry smaller loads and smaller clearances. Due to the lower viscosity the lubricant shear stresses are lower and hence the operational speed can be very high without excessive power being needed and less heat generated. The bearing can operate in environments of high temperatures, as this does not affect the lubricant properties. Gas bearings are separated into three distinct groups:

- Self-acting hydrodynamic bearings
- Externally pressurised aerostatic bearing
- Porous wall bearings

Squeeze film bearings

These are a special case of a hydrodynamic journal bearing where the rotation speed is zero and the hydrodynamic pressure forces are produced by the lubricant being squeezed between the journal and bearing surface. Squeeze films have extensively been used in conjunction with rolling element bearings as in figure 3.1. Using the two bearings in this type of configuration gives the effect of increasing the effective damping of the rotor, reducing the machine vibrations.

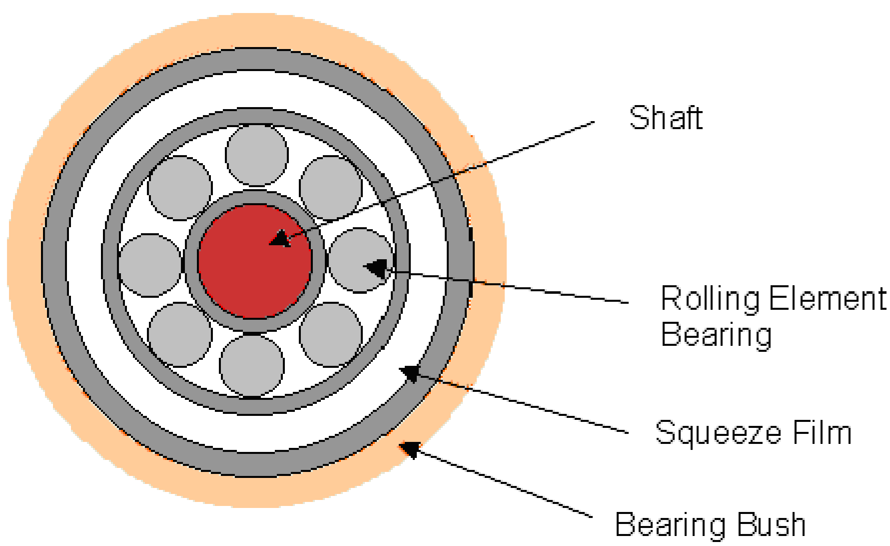

Fig.4: Squeeze film bearing

Hydrostatic bearings

Hydrostatic bearings require the lubricant pressure to be provided from an external source and not from the journal rotation as in the hydrodynamic case. The lubricant is supplied to the bearing under pressure and then enters recesses located in the bearing bush. Here the pressure drops slightly and the lubricant then flows out of the recesses over the bearing and into the drain holes. Hydrostatic bearing can support very large loads even at zero rotational speed, as there is always full film lubrication. The frictional losses of this type of journal are proportional to rotating speed so it has a low start-up resistance.

Seals

Seals are used to protect bearings from dirt and foreign matter and retain lubricants. Seals broadly fall into two categories: -

- Static seals where the two surfaces do not move relative to one another.
- Dynamic seals where sealing takes place between two surfaces, which have relative movement.

Rotor Unbalance

There are two types of rotating unbalance:

1. Static Unbalance

All the unbalanced masses lie in a single plane resulting in an unbalance of single a radial force. Static unbalance can be detected by placing the shaft between two horizontal rails and allowing the shaft to naturally roll to the position at which the unbalance is below the shaft axis.

2. Dynamic Unbalance

This is when the unbalanced masses lie in more than one plane. The static test will only detect the resultant force. The unbalance has to be detected by rotating the shaft and measuring the unbalance. The machine for carrying out this detection are called 'balancing machines' and consist of spring mounted bearings that support the shaft. By obtaining the amplitude and relative phase it is possible to calculate the unbalance and correct for it. This is a problem of two degrees of freedom as transitional and angular motions take place.

Instability in Rotating Machinery

Synchronous whirl and critical speeds

Synchronous whirl is the natural frequency of the system at which self-exited vibration occurs. Often in engineering it is required to obtain a quick approximation of the solution. Methods of obtaining simple solution to obtain the natural frequency:

- Dunkerley's formula,.
- Energy method (Rayleigh's principle)

Non-synchronous whirl and oil whip

To understand the behaviour of journal bearings under dynamic conditions it is important to know what is happening to the oil-film. If the amplitude of vibration of the journal becomes sufficiently large the oil film regime changes depending on the character of vibration. This is one of the sources of complication of the dynamic phenomena in turbine bearings. To show the different oil film characteristics an idealised bearing is used, shown in Fig. 5 (a), displaying the steady running film regime and the oil film regimes which may hold for a pressure bearing film with the journal in vibration.

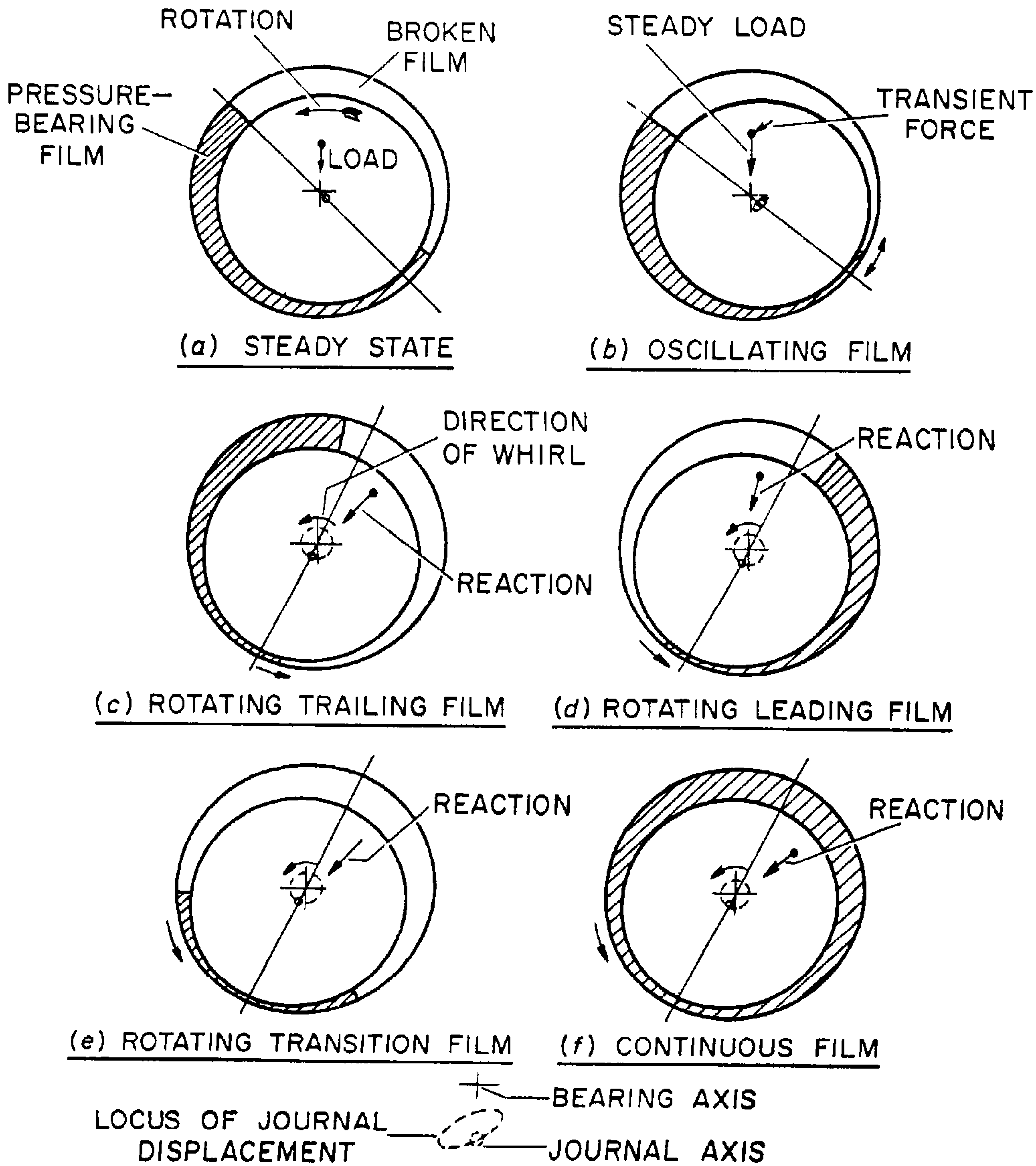

Fig. 5: Bearing film status

Closely related to the steady running condition is the oscillating film condition, shown in Fig. 5 (b), and this occurs when a small amplitude is sustained by a fluctuating force superimposed on the steady running condition. Resulting in both squeeze and angular swing of the film and is a frequent occurrence of bearings in service. If journal motion becomes a whirl of large amplitude due to severe vibration the minimum film thickness may rotate around the bearing with the whirl and the oil-film no longer related to the steady-running condition. If the whirl has substantial magnitude at constant eccentricity and the idealised bearing is in the condition of steady running relative to the axes rotating with the whirl. The effective speed of rotation is given by, *N—2F*.

Where, N – journal rotation speed
F – frequency of whirl in same direction as N

The position of the pressure-bearing film depends on the direction of the mean peripheral speed of the bearing relative to the journal (if N is greater or smaller than 2F). When $2F < N$ the pressure bearing film is built up on the trailing side of the minimum film thickness, Fig. 5 (c). The bearing reaction has a component normal to the attitude radius in the direction of the whirl. Resulting in the amount of whirl increasing and tendency for instability. As reaction force increases and the minimum film thickness decreases, the direction of the reaction swings closer to the attitude radius. With damping elsewhere in the system a steady whirling action may be attained. When $2F > N$ the pressure bearing film is built up on the leading side of the minimum film thickness, Fig. 5 (d). In this case the corresponding component tends to damp the whirl. Resulting in a bearing that does not tend to excite instability. When $2F \approx N$, a rotating transition film may be formed, Fig. 5 (e). Conventional hydrodynamic theory suggests that at the transitional speed a pressure bearing film cannot be sustained however in practice a transition film sometimes forms. This may possibly explained by either the oil being dragged away by the relative movement either side of the minimum film thickness or the bearing being warmer at this position and oil being dragged in due to differences in viscosity. The transition condition is important as it may exhibit a resonance to maintain rotor whirl when there is a natural frequency at half the running speed.

Additional transitional films have occasionally been experienced such as causing whirl at comparatively low forces, even though the frequency is not near half the running speed. This seems to be as if the journal looses its 'grip' on the bearing.

In real bearings in turbomachinery the oil-film regimes are not identical to the idealised circular bearings. As the bearings always have at least one hole or grove for lubrication purposes and sometimes drainage points. These bearings usually have a substantial arc in the bottom half with a narrow or discontinuous bearing surface in the top half (provided as a safeguard against vibration). However, the idealised condition provides a good first estimate when vibration is small in real turbomachinery bearing.

If the bearing has a large amplitude of vibration or in operation at low mean eccentricity, one or more films on the bearing surfaces in the top half which are discontinuous in angle and intermittent in time. When the oil-film is restricted both the steady running and dynamic characteristics are affected. This results in a high oil-temperature rise in the bearing, lowering the effective viscosity. Directly changing the oil-film regime and dynamic characteristics, severe oil restriction may set up film cavitation causing bearing damage without appreciable journal vibration.

Non-Circular Bearings and stability analysis
In some specialized applications, plain circular bearing is mostly replaced by some other bearings, as plain bearing does not suit the stability requirements of high-speed machines and precision machine tools. Grooved circular bearings and multi-lobe bearings with two lobes, three lobes and four lobes are commonly used. The text follows gives insight into nonlinear transient analysis of multi lobe journal bearing systems. The critical mass parameter (a measure of stability) is estimated for various values of aspect ratios besides finding out the steady state characteristics of multilobe journal bearings (two grooved, two lobe, three lobe and four lobe) such as load bearing capacity, Sommerfeld number and attitude angle

In the present analysis four different types of bearings, namely 2-groove bearings, two-lobe bearings, three-lobe and four-lobe bearings are considered. The grooved circular bearings with two axial grooves of 20^0 circumferential extents each are considered for the provision of oil inlet in the journal bearing clearance area. In case of two-lobe bearings, lobes are separated by axial grooves of 20º circumferential extensions. On the other hand three-lobe bearings having lobe of maximum span 120º and four-lobe bearings having lobe of maximum span 90º is considered. The lobes are separated by 20º axial grooves for three-lobe bearings and 10º axial grooves for four-lobe bearings.In this present work, critical mass parameter (measure of stability) of two-groove, two-lobe, three-lobe and four-lobe is estimated by using non-linear time transient analysis. Fixed coordinate system is employed in calculating fluid film pressure distribution by using successive overrelaxation technique in a finite difference grid. The present analysis is carried out and results are obtained for ellipticity ratio equal to 0.5.

Formulation of the problem
The dimensionless Reynolds equation is given by,

$$\frac{\partial}{\partial\theta}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\bar{z}}\right)=6\frac{\partial\bar{h}}{\partial\theta}+12\frac{\partial\bar{h}}{\partial\tau} \qquad [12]$$

where, $\theta=\frac{x}{R},\bar{z}=\frac{z}{L/2},\bar{h}=\frac{h}{C},\bar{p}=\frac{p}{\mu\omega}\left(\frac{C}{R}\right)^2$

The Reynolds boundary conditions are used in the present analysis.

The equation of motion of the rotor-bearing system for a rigid rotor of mass *2M*, supporting a static load *($2W_0$)*, along *X* axis mounted on two identical hydrodynamic journal bearings given in non-dimensional form as follows:

$$\begin{aligned}\bar{M}\bar{W}_0[\ddot{\varepsilon}-\varepsilon\left(\dot{\phi}\right)^2]&=\bar{F}_\varepsilon+\bar{W}_0\cos\phi\\ \bar{M}\bar{W}_0\left[\varepsilon\ddot{\phi}+2\dot{\varepsilon}\dot{\phi}\right]&=\bar{F}_\phi-\bar{W}_0\sin\phi\end{aligned} \qquad [13]$$

Time Transient Analysis
Fourth order Runge-Kutta method is employed to solve the set of equations of motion. Fluid film forces are estimated at every time step by solving the Reynolds equation for every new set of eccentricity ratio, attitude angle and their time derivatives. Motion trajectories are obtained by plotting eccentricity ratio and attitude angle at every time step showing position of journal centre. Observing these trajectories one can find out whether

the system is stable, unstable or at critical condition. Critical mass parameter of particular eccentricity ratio is found when the trajectory of journal centre ends in a limit cycle or it changes its trend from stable to unstable. A few such motion trajectories are shown in Fig. 1. Non-linear time transient stability analysis is carried out for four different types of bearings with preload factor *0.5*. Effects of axial grooves were taken into account during pressure calculation. Stability curves for all four bearings are plotted together (Fig.2) for comparison.

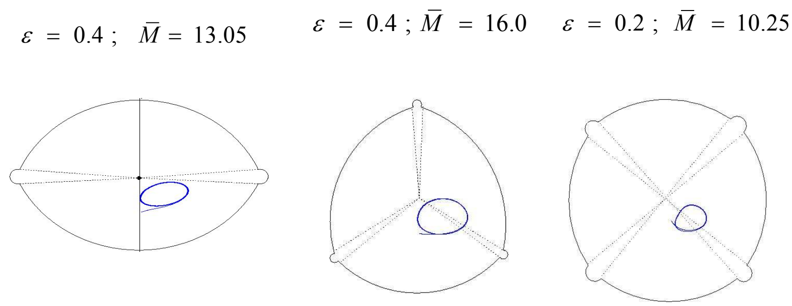

Fig. 6: Trajectory of journal centre (L/D=1.0)

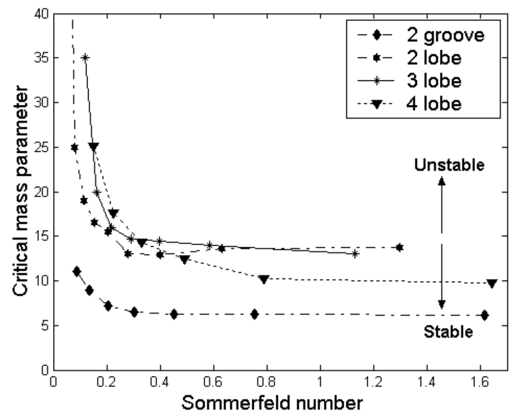

Fig.7: Stability maps

At moderate and heavy load conditions (Sommerfeld number from 0.1 to 0.3), four-lobe bearing is most stable. Three-lobe bearing is the best in the range of Sommerfeld number from 0.3 to 0.8. Two lobe bearing is found to be more stable than other bearings beyond this range. As multi lobe bearings are used widely in industries, the present study and results thereof would be of immense importance to rotor-bearing designers and users.

Conclusion

The discussion put forward here is of introductory nature. Investigations are carried out by different researchers to characterize practical rotor-bearing systems experimentally as well as theoretically. Theoretical analysis can be carried out using FEM even for complicated rotor-bearing models. Readers with interest in the area can definitely explore a lot in this area.

Bibliography

1. Guo, Z., and Kirk, R.G., 2003, “Instability boundary for rotor-hydrodynamic bearing systems, Part 1: Jeffcott rotor with external damping,” ASME Journal of Vibration and Acoustics, Vol. 125, pp.417-422.

2. Lund, J.W., 1974, “Stability and damped critical speeds of a flexible rotor in fluid film bearings,” ASME Journal of Engineering for Industry, Vol. 96, No. 2, pp.509-517.

3. Lund, J.W., 1979, Rotor-Bearing Dynamics, Lecture notes, Technical University of Denmark, ISBN 83-04-00267-1.

4. Lund, J.W., and Nielsen, H.B., 1980, “Instability threshold of an unbalanced rigid rotor in short journal bearings”, Second International Conference on Vibration in rotating machinery, Cambridge.

5. Rao, J.S., 1996, Rotor Dynamics, New Age International Publishers, Third Edition, pp.146-148, pp.258-266.

6. Rao, J.S., 1982, “Synchronous whirl of a flexible rotor in hydrodynamic bearings,” *Mechanism and Machine Theory*, Vol.17, No.2, pp.143.

7. Cheng-Chi Wang, Cheng-Ying Lo and Cha'o-Kuang Chen, 2002, “Non-linear dynamic analysis of a flexible rotor supported by externally pressurized porous gas journal bearings,” *ASME Transaction: Journal of Tribology*, Vol. 124, No.3, pp.553-561.

8. Madhumita Kalita and S K Kakoty, 2005, “Nonlinear Time Transient Stability Analysis of Multilobe Bearings”, Proceedings of WTC2005 World Tribology Congress III September 12-16, 2005, Washington, D.C., USA.