This paper discusses of what is called as a synchronization node. Let us first define what a synchronization node is. A synchronization node consists of M infinite capacity buckets or buffers each having independent inputs. There is no server as is the classical M/M/1 case but the size of each of the buffers instantly reduces by one as soon as all buffers become non-empty. The analysis of systems of this type will be applicable to several practical situations. For e.g. let us consider the production of a particular device which has several (M) independently made subcomponents. Each component has its own manufacture rate and one unit of the device is manufactured in as soon as one has all at least one unit of all its components. It is quite intuitive that if the rates of production of various devices vary significantly then the output process or rather departure process will solely depend on the slowest input process. However, the most interesting cases and also practical scenarios will occur when the input processes will all have identical distributions with same parameters.

The author makes a smart observation that at least **one of the buffers is sure to be empty at all points of time and the output process behaves exactly as the input process of this buffer**.

2 buffer synchronization node with unequal rates:
If two process have input rates λ_1 and λ_2 respectively where, $\lambda_1 < \lambda_2$ .Then there exist a finite time τ after which N_1 (t) < N_2 (t) for all t > τ. Thus, after this time the output process behaves exactly like the slower input process, the first process in this case.

2 buffer synchronization node with equal rates (Poisson process with parameter λ)
The more interesting case comes now. It is easy to easy that $Q(t) = Q_1(t) – Q_2(t) = N_1(t) – N_2(t)$ completely describes the system at any point of time. Now, looking about each arrival in the first process as head and the second process as a tail, we can compare the situation with the problem in which the state of the system is defined by the difference of number of heads and tails. It is well established and easily shown that all states are null recurrent in such a situation. Getting back to our original problem, we exploit the null-recurrence to conclude that as t tends to infinity P[Q(t) = 0] is arbitrarily small.
Thus, the chances of both buffers being empty together are arbitrarily small. Hence, the output process will behave exactly like the input process of the empty buffers. Since, both processes have same input rate λ the output process **weakly converges** to a Poisson process with rate λ.

The same ideas are extended to M-buffer nodes using similar arguments.(theorem 2)

Now the author shows another interesting fact (in theorem 3). That though the output process weakly converges to a Poisson process with rate λ it can not be coupled tightly with any other Poisson process. The idea here is also simple and interesting. The author exploits that there is a point of time where 2 or more buffers are empty at the same time. Hence the inter-event time of the output process stochastically dominates a Poisson process with rate λ.

The author talks about SM/M/1 queues and concludes the intuitive results about the same. In most case we find that synchronization nodes preserve the results obtained for simple Poisson processes.

The later part of the paper talks about Jackson networks with synchronization nodes. It is observed that Jackson nodes preserve the product form steady state distribution as in case of simple service nodes. However, due to null-recurrence no equilibrium state is reached but the product form distribution is obtained by introduction of “ghost tokens” at the synchronization nodes.

To conclude I would say that this paper discussed and explained an interesting variant of the queuing systems we have studied. Most results were intuitive but interesting nevertheless. The proofs exploited a wide range of simple ideas (the idea of null-recurrence to state an example).

The widespread applicability of synchronization nodes in practical situations enhances the importance of this paper.

Vaibhav Mehrotra
04010149