A Queueing Theory-Based Analytic Model of a Distributed Computer Network

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Abstract

This paper describes the development of an analytic model for performance studies of distributed computer networks. The model factors each node of a network into processing and channel components and models each separately using M/D/r and M/M/l queues. The model also includes a correction factor to account for the non-exponential nature of the input to the channel component. Results produced by the model are compared with results previously reported in the literature to estimate the magnitude of improvement which can be expected.

I. INTRODUCTION

Distributed computer network have evolved very rapidly in the last decade. The description of the first such network the ARPANET-appeared in the literature in 1967 .Twelve years later there are literally dozens of such networks either planned or in operation . A distributed network is composed of two distinct components: host computers and a communication subnetwork. A host is the generator and consumer of all network messages and the provider of all user services. In the context of existing distributed networks, they usually correspond to independent computer systems, cluster controllers, or intelligent stand-alone terminals.

The hosts (and ultimately the end-users) communicate with each other.via independent front-end processors called communication interfaces, or abbreviated CI. The CI handles all communications responsibilities, such as routing, error handling, and flow control. The CI, which is typically a minicomputer, is assumed to be composed of a primary message buffer, one or more identical processors, and all necessary line interface (LI) equipment, as shown in Fig. 1. The configuration shown is a fairly standard model and resembles most actual front-end processors for existing distributed communication subnetworks. The CI's, together with the set of connecting point-to-point communication links, form the communication subnetwork (also called the transport subnetwork) which is responsible for all communication management from source host to destination host. The performance of a communication subnetwork is primarily characterized by two critical variables:

- End-to-end delay-the delay a message observes from the time it initially enters the subnetwork at the source host until it is correctly delivered to the destination host and leavesthe communication system,
- Throughput the volume of traffic that the entire communication subnetwork handles per unit of time.

Two quite distinct approaches have been used to analyze their performance. The first method is discrete event simulation ,the second is **queueing theory** . While discrete event simulation can provide interesting insight into subnetwork behavior, it is a totally impractical method for studying the behavior of large complex distributed networks. It is useful primarily when there does not exist any known analytical methods and it becomes the only tool to investigate system performance.

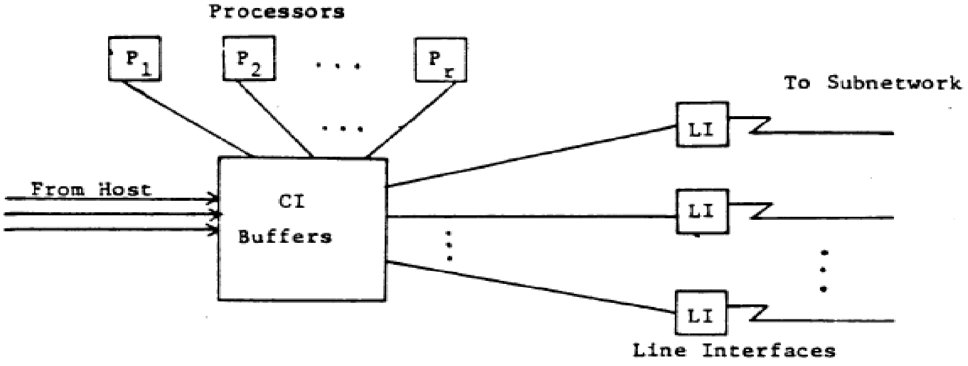

Fig .1 Communication Interface(CI)

A theoretical approach using queueing theory is a cost effective and practical technique to analyze the performance of large distributed communication subnetworks.A very well-known model of a communication subnetwork which considers each CI as a collection of channels is shown in Fig.2. The model is an open network of N queues, where N is the total number of channels in the communication subnetwork. The diagram in Fig. 2 shows queueing model of a CI node. This decomposition of the network into single channels, along with the independence assumption for message length (namely message length L is drawn from the probability density function p(L) = u*e**(-u*L) independently each time it enters a node [7], [8]) makes it possible to treat each channel as an M/M/ 1 queueing system and compute the average delay for each channel. Overall average end-to-end delay is simply the sum of individual delay contributions of each channel in the communication subnetwork.

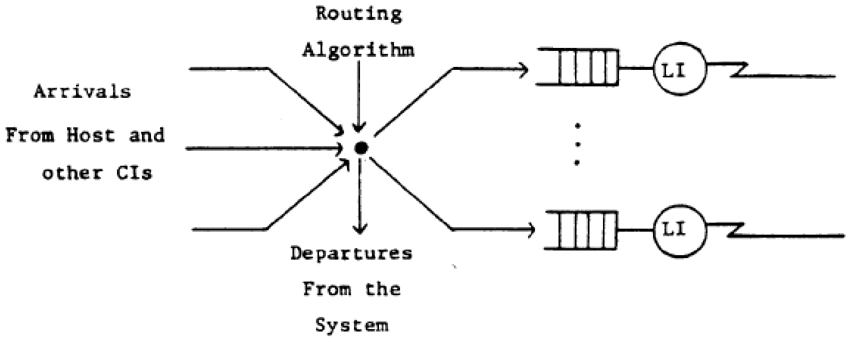

Fig .2 Queueing model of a communication interface node.

II. DESIGN AND DEVELOPMENT OF THE MODEL

A communication sub-network (Fig. 1) consists of a number of interconnected communication interfaces (CI). Each CI is modeled as an M/D/r and M/M/1 queueing system , as shown in fig.3 shown below. Input to a CI from one or more of its associated hosts is called external input since it is external to the subnetwork. Input to a CI from one or more of the other CI's in the subnetwork is called internal input. When a message arrives at a CI, either from other CI's or from a host, it will enter a queue of messages waiting for service from a processor. This queue is termed as message queue (MQ).MQ is assumed to have unlimited number of messages and is serviced by one or more servers(processors).

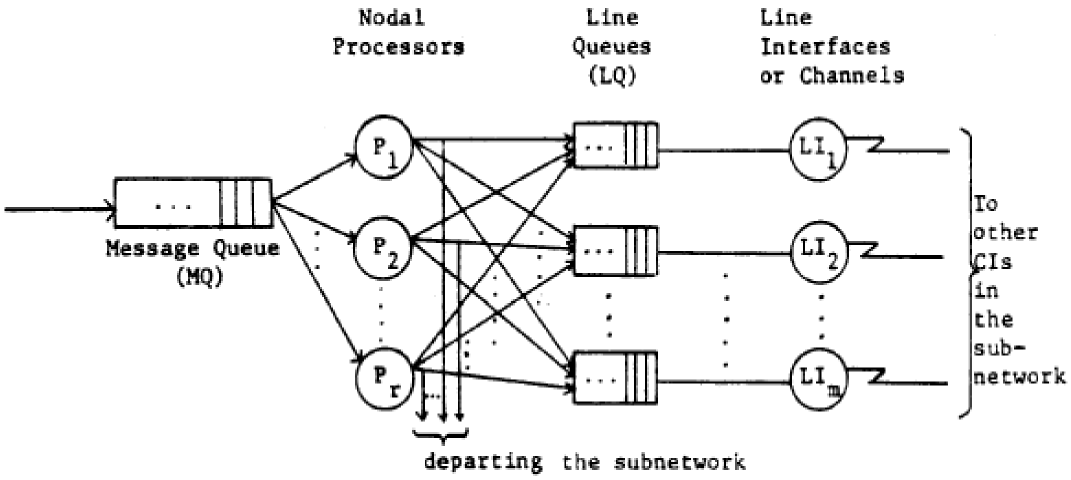

Fig.3 Analytic model of a communication interface.

Initially, we will assume that messages arrive at the MQ with an exponential distribution function and the queue discipline is first-come-first-serve. When a message is serviced by a processor there are two possibilities.It either sinks into the server as it is its destination or it is routed to another CI in the subnetwork and gets placed in one of the *m* line queues. Each line queue is serviced by a channel with capacity S11 (in bits per second). Since message lengths are assumed to be exponentially distributed, the service time for a line queue server is exponentially distributed with mean service time $L/S_{i,j}$ where L is the mean length of messages and S_{ij} is the capacity of channel j at node i.

Let N be the number of nodes in the network. For each node i in the network let us define the following terms:

x_i the total arrival rate to the message queue at node
i-including both internal and external traffic,

,y_j the total external input to node i,

X_{ij} the total input rate for line queue j at node i,

m_i the number of channels at node i,

T_i the total average delay for the message queue (queue waiting time plus processing time) at node i,

T_{ij} the total average delay for line queue j (queue waiting time plus transmission delay) at node i.

Then the total external traffic y for the subnetwork is given by

$$Y = \sum_{i=1}^{N} Yi$$

and the average end-to-end delay is given by

$$T = \frac{1}{Y} [\sum_{i=1}^{N} \{\lambda i Ti \sum_{i=1}^{mi} \lambda ij.Tij\}]$$

The internal input can be found by using two data structures called the routing table (RT) and the destination probability table (DPT).

The destination probability table specifies for each i, j pair, the probability that a message generated at node i will be ultimately destined for node j. DPT is an n*n table and entry DPT(i, j) specifies what fraction of Yi, the external input, will have destination j, namely Yi *DPT(i, j). Let us define the notation path (j -> k, i) to mean an ordered sequence of nodes connecting node j to node k, which passes through intermediate node i. That is, X1 = j, Xm = k, Xp = i,1 < p < m. Let us also define

$$\sum_{i=1}^{N}$$

k € path(j -> k,i)

as a summation over the set of all destination nodes k, such that node i lies on the path from source node j.

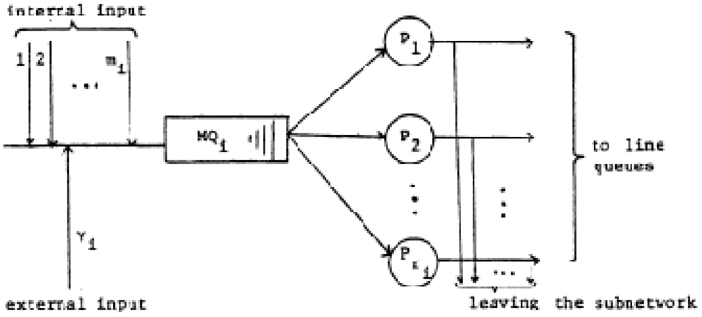

Fig 4.The MQ and processor section of a communication interface.

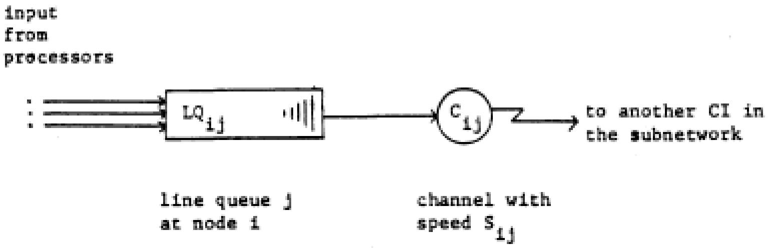

Fig 5.The line queue and its channel as an M/M/1 system

To find the average message queue delay we will use the approximation formula for M/D/r systems described as :

$$W(\mathrm{M/D/r}_i) = \left[1\ (1-\rho_i)(r_i-1) * \frac{\sqrt{4\ 5r_i}-2}{16\rho_i r_i}\right] \times \frac{W(\mathrm{M/D/1})}{W(\mathrm{M/M/1})} * W(\mathrm{M/M/r}_i)$$

Where pi = the processor utilization at node i for all processors

ri = the number of processors at node i

and W(M/D/1), W(M/M/1) and W(M/M/r1) are the average queue delay for M/D/1, M/M/1 and M/M/r1 queueing systems, respectively.

The total input traffic rate to channel j at node i, Xij can be calculated using the routing table and the destination probability table in the same way we found Xi. Let Wij(LQ) be the average waiting time for line queuej at node i. Then

$$W_{ij}(\mathrm{LQ}) = \frac{\rho_{ij} * \overline{X}_{ij}}{(1 - \rho_{ij})}$$

where ρij is the utilization of channel j at node i and is given by

$$\rho_{ij} = \lambda_{ij} * \overline{X}_{ij} = \lambda_{ij} * \frac{\overline{L}}{S_{ij}}$$

Where

$$\overline{X}_{ij} = \frac{\overline{L}}{S_{ij}}$$

(L is the mean message length which are exponentially distributed).

III.DEVELOPMENT OF THE CORRECTION FACTOR FOR THE CHANNELS

An approximation of the fraction of average line queue waiting time that the model overestimates for line j at node i because of the violation of the exponential interarrival time distribution assumption can be taken as

$$\frac{\rho i * (1.0 - \rho ij) * \lambda ij}{\lambda i}$$

Therefore, the correction factor for line j at node i, Cij, is defined as

$$Cij = 1.0 - \frac{\rho i * (1.0 - \rho ij) * \lambda ij}{\lambda i}$$

We can define W'(LQ), the corrected average line queue waiting time, as

W'ij.(LQ) = Cij * Wij(LQ).

IV.RESULTS

The corrected analytic model was extensively tested against the simulator results. We modified a wide range of parameters involved with computer communication network design number of nodes, network topology, number of processor at each node, network load and channel speed-to verify the stability and the accuracy of the corrected analytic model under all conditions. In the following tables a small sample of the results from these tests is presented. For each case the percentage of relative error between the analytic results and simulator results is computed.

For each node of the test network, the number of processors and channels were identical. The processor utilization was changed by modifying the number of processors at each node. Since the average message queue waiting time is directly proportional to the processor utilization, as the utilization decreases so does the average message queue delay. This, in turn, decreases the average end-to-end message delay. Note that, at high processor utilization, the contribution of the message queue waiting time is quite considerable and cannot be ignored. However, with this model and the correction factor, excellent results were obtained under all processor loads.

The results for average end-to-end message delay in a 5-node distributed network are represented. Again, all nodes in the network have identical processor and channel characteristics, so it was possible to achieve uniform values for processor and channel utilization at every node. For this network ther-e existed one processor at each node, and the utilization of this processor was changed by varying the processing time of a message. as processor utilization decreased, the average message queue delay decreased. This means messages spent less time in the message queue and the resulting average end-to-end message delay is smaller. However, lowering the channel speed will obviously increase the channel utilization. This means that messages must wait longer in the channel queue for transmission, and, in effect, will cause a higher average end-to-end message delay.

Practically it should be pointed out that the increase in accuracy gained by this new analytical model is achieved with virtually no increase in computer processing time compared to other existing models. It also clearly shows the gross inefficiency of discrete event simulation compared to analytic modeling, and why simulation could never be used to investigate the behavior of large real-world computer networks. The simulation model required over three orders of magnitude more CP time for the investigation of identical test cases. Time complexity analysis has shown that the analytic model should work well for very large networks and computing time will be virtually insignificant. (Tests have been run on distributed topologies with 30 nodes with CP time on the order of 7 ms.) The limiting factor in our program is not time but space. The DPT and RT data structures mentioned earlier all require 0(n2) memory cells, thus limiting the networks which can be studied to a realistic maximum of

N ~ 100 -1 50 nodes

without modification of the program. However, that value should be adequate to handle most existing communication subnetworks.

V.CONCLUSION

In this paper the development and testing of an analytic model for evaluating the performance of a communication subnetwork has been described. The model was extensively tested and the results were compared with both another analytic model and simulation results for an identical system. The results clearly showed that corrected analytic model produced improved results with no increase in processing time.