Paper Review:
By: Rajinder Arora
Roll No: 04020517

**TIME-DEPENDENT QUEUEING APPROACH TO HELICOPTER ALLOCATION FOR FOREST FIRE INITIAL-ATTACK***

JAMES H. BOOKBINDER
*Ministry of Natural Resources, Queen's Park, Toronto, Ontario*
DAVID L. MARTELL
*University of Toronto, Toronto, Ontario*

Helicopters are used extensively to transport initial-attack crews to forest fires in the province of Ontario. Each day fire managers must decide how to allocate the available helicopters to initial-attack bases. The helitack transport system at each base can be viewed as a multi-channel queue with customers (fires) and servers (helicopters). The authors describe a time-dependent queueing model of the helitack system and use numerical methods to estimate some of its operating characteristics. A dynamic programming model is then used to specify an optimal allocation of the available helicopters to helitack bases. The importance of early initial-attack has long been recognized by forest fire managers. The sooner an initial-attack crew begins control action on a fire, the more likely it will be extinguished at a small size.

In this paper, authors explore the possibility of using queueing theory and dynamic programming to develop decision-making aids to assist fire managers who must decide where to locate their helicopters each day. Authors focus on the transportation aspect of initial-attack using helicopters, upon a "queue" of fires whose arrival rates are described by time-dependent probabilities. Following Koopman,'"' they present a set of differential equations, the solution of which gives the probability that at a time there are n fires in the queueing system. Numerical solution yields the time-dependent distribution of queue sizes.
A fire which occurs in a given sector is presumed to be served by a helicopter stationed at that sector's helitack base. Although this dispatching rule may seem overly restrictive, the distances between helitack bases are usually such that authors believe this simplifying assumption to be reasonable.

Each morning the regional fire manager must decide how to allocate the available helicopters among the helitack bases within the region. Authors assume that the number of available helicopters is such that at least one helicopter can be allocated to each base. They further assume that the helitack aircraft will be used solely for initial-attack transport purposes and that the allocation made in the morning remains in effect for the entire day. Given these assumptions, the helitack transport system can be viewed as being comprised of a queueing system in each sector which operates independently of the other sectors in the region. The queueing system in each sector can be envisaged as follows.

Each time a fire is reported to the district dispatcher (customer arrives), the following sequence of events occurs:

1 The fire enters the initial-attack queue and awaits service.
2 As soon as a helicopter is available, it is used to service the fire which is waiting at the front of the initial attack queue. Service includes loading the crew, taking off, flying to the fire, scouting the fire, landing the crew, taking off again, flying back to the initial-attack base, landing and refuelling.
3 The helicopter is then available to transport another initial-attack crew to the next fire in the queue.

**THE FIRE ARRIVAL PROCESS**

Since the Poisson distribution is preserved under random selection,' it is reasonable to assume that the probability distribution of the number of fires that arrive in a sector each day is Poisson

Let $F$ denote the expected number of forest fires that arrive in a particular sector on a given day. Because of diurnal weather variations and the characteristics of the detection system, the rate at which fires arrive varies during the day. We will assume that fires arrive according to a non-stationary Poisson process. Suppose the day is divided into $T$ time periods and the length of period $j$ is $t_j$ hours. Let $p_j$ denote the probability that a fire which arrives in the sector does so during period $j$. Let $\lambda_j$ denote that fire arrival rate in the sector during period $j$, expressed in terms of fires per hour. We will suppose that $\lambda_j$ can be estimated using the following equation:

$$\lambda_i = (P_J F)/T_J$$

**THE INITIAL-ATTACK TRANSPORT SERVICE PROCESS**

The helitack transport service time for each fire includes the time required to load the helicopter, transport the initial-attack crew to the fire, return to the helitack base and refuel. Although the actual service time distribution should be truncated above and below, authors assume that a standard negative exponential distribution is a satisfactory approximation. The mean service time will be denoted by $1/\mu$ where the service rate of each helicopter is M fires per hour. The number of service channels in each sector corresponds to the number of helicopters allocated to it for the day. However, only one queue will operate at each base, with service provided by the first available helicopter.

**THE DISTRICT HELITACK QUEUEING SYSTEM**

Consider a queueing system for which the arrival rate $\lambda(t)$ and service rate $\mu(t)$ are time-dependent. That is, in the case of Poisson arrivals, $\lambda(t)\Delta t$ is the probability that between times $t$ and $(t + \Delta t)$ a customer joins the queue. Similarly, if the service distribution is negative exponential, $\mu(t)\Delta t$ is the probability that a customer has finished service by time $(t + \Delta t)$, given that at time $t$ service has already begun.

The first treatment of the M/M/1 queueing system with time-varying parameters *λ(t)* and *μ (t)* was by Clarke. The equations for a birth and death process were solved analytically to yield complicated expressions for $P_n(t)$, the probability that at time *t* there are *n* customers in the queueing system. This approach, based upon an integral equation for the generating function of $P_n(t)$, is a suitable starting point for a calculation if analytical expressions are available for both *λ(t)* and *μ (t).*

It should be noted that in the preceding references and in the approach of the present paper, a time-dependent approach was used because the arrival rate was not constant, but varied significantly throughout the period of interest. Consequently, there is no question of using a steadystate solution; the latter may not exist and in any case it is not meaningful. Numerical solution of the birth-and-death equations is required to find $P_n(t)$

**TIME-DEPENDENT QUATIONS FOR AN S-SERVER QUEUE**

$$\begin{aligned}
n = 0 \quad & dP_0/dt = -\lambda(t)P_0(t) + \mu P_1(t), \\
1 \leqslant n < S < N \quad & dP_n/dt = -(\lambda(t) + n\mu)P_n(t) + \lambda(t)P_{n-1}(t) \\
& \qquad + (n+1)\mu P_{n+1}(t), \\
1 \leqslant S \leqslant n < N \quad & dP_n/dt = -(\lambda(t) + S\mu)P_n(t) + \lambda(t)P_{n-1}(t) \\
& \qquad + S\mu P_{n+1}(t), \\
n = N \quad & dP_N/dt = -S\mu P_N(t) + \lambda(t)P_{N-1}(t),
\end{aligned}$$

where $P_n(t)$ is the probability that at time *t,* there are *n* customers in the sector queueing system, including, of course, any that are presently being served.

## 6 Numerical Solution in the Transient Case

### 6.1 *Numerical values of parameters*

The system of differential equations governing the evolution of the queueing system (hereafter referred to as (I)) requires the specification of $\lambda(t)$, $\mu$, $N$ and the initial conditions $P_n(t_0)$, $n = 0, 1, 2, \ldots$ N. Note that we have specialized to the case $\mu(t) = \mu$ for all $t$.

**SERVICE RATES**

It should be noted that for the 35 sorties in question, the standard deviation *a* of service times is only 0.74 hours, *a/n* is thus approximately 1/2, whereas for a negative exponential distribution, *a* == *n.* Moreover, it would be unusual to observe a service time less than 0.2 hr or greater than about 4.0 hr. (The helitack "zone of infiuence" for most helicopters is usually less than 100 miles.) These remarks concerning *a* and yu cast doubt upon the service times as a sample from a negative exponential distribution. Our suspicion that the service time distribution is not negative exponential was.confirmed by the results of goodness-of-fit tests.. In all three cases, the null hypothesis that the service time distribution is negative exponential was rejected at the *a* = .01 level of significance.

**NUMERICAL PROBLEMS**

Initiation of numerical solution of the differential equations requires specification of the probabilities *Pn(t)*- Since the integration begins at *to = 5* A.M., the first thought is to take Po(5) = 1, (6.4.1)
with $P_0(5) = 0$ for n > 1

With (6.4.1) as initial conditions and the above modification for M.numerical integration proceeded by first smoothing the arrival rate X(i). That is, to remove the "artificial" divisions caused by hourly boundaries, we took for the discrete units of time j = 1, 2, 3 the moving average

$$\bar{\lambda}_j = (\lambda_{j-1} + \lambda_j + \lambda_{j+1})/3.$$

A variable step size *M* was used, in order to keep the local error per unit step within a tolerance of 0.1%. The result of the integration is thus, for each value of S and *F,* the distribution *Pn(t)* throughout the day. From this distribution, we calculated

$$\max_t L_q(S, t),$$

where *Lq* is the expected number of fires actually awaiting service. This number is used as input to the dynamic programming algorithm for helicopter allocation, which is discussed in the following section.

**THE HELICOPTER ALLOCATION MODEL**

Suppose the parameter $v_k$ denotes the relative fire damage that might be incurred if an acre of forest in sector *k* is burned, compared to an acre in another sector. Without loss of generality, we can assume that $v_k$ ranges from 0 to 1. In other words, if $v_k$, equals .2 and $v_j$ equals .4, we are assuming that the penalty for delaying initial attack on a fire in sector *j* is twice that in sector *k.*

Assuming that at least one helicopter is allocated to each of *B* helitack bases and letting $X_k$ denote the number of helicopters allocated to base *k,* the allocation problem can be formulated as a simple dynamic programming problem with the following recursion relationship.

$$f_k^*(S_k) = \underset{1 \leqslant X_k \leqslant S_k-(B-k)}{\text{Min}} \left( V_k \underset{t}{\text{Max}}\, L_q(X_k, t) + f_{k+1}^*(S_k - X_k) \right),$$

where $S_k$ denotes the number of helicopters available at the start of stage *k.*

DISCUSSION

The service time is therefore independent of the length of time the fire waits in the queue.
Although a FIFO queue discipline was assumed, some fires, particularly those threatening high value areas, are dealt with on a priority basis. Further research should be undertaken to develop an expanded model that incorporates more of the complexity of the helitack planning problem.
This would of course entail a detailed analysis of dispatching rules, an important aspect of fire management which is beyond the scope of our present model. Our hope is that future developments in queueing theory will miake it possible to incorporate these and other aspects of initial attack planning into an improved fire management decision-making aid.

Dear Sir,
I have reviewed the part one of this paper , the second part deals with dynamic programming that I

haven't reviewed. I shall welcome any comments and remarks on this review. I will be happy to make the changes required and send it back after doing required corrections.

Regards,
Rajinder Arora