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Application of Queuing Theory to Libraries and Information Centres

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[*In this paper, an attempt has been made to discuss the application of Queuing Theory as applicable to library and information fields. Examples related to circulating of books are discussed.*]

1. INTRODUCTION

Queuing problem is identified by the presence of group of customers who arrive randomly to receive some service. The customer, upon arrival may be attended to, immediately or may have to wait until the server is free. This methodology is applicable in the field of Business, Industries, Government, Transportation, Restaurants, Library etc. Queuing models are basically relevant to service oriented organisations and suggest ways and means to improve the efficiency of the service.

2. SCOPE OF APPLICATION IN LIBRARIES

It is applicable to the following services provided in the Library.

-- circulation of books
-- counter service
-- allied services like reprography

3. CHARACTERISTICS OF A QUEUING MODEL

A general queuing system is denoted by (a/b/c) : (d/e)

where
- a: probability distribution of inter arrival time
- b: probability distribution of service time
- c: number of counters in the system
- d: queue size
- e: queue discipline

Efficiency indices for queuing system are

$\rho = \lambda / \mu$ = traffic density

W_q = expected waiting time in queue

L_q = expected number of customers in queue

Where λ = mean arrival rate and μ = mean service rate.

The time spent by the customer in the queue is of interest to the decision maker. One of the objectives of study of queuing is to find out the optimum service rate and the number of servers (counters) so that, average cost of waiting in queuing system and the cost of service are minimised.

4. METHODOLOGY OF APPLICATION

Data on arrival rate and service time have to be collected for a sustained period to fit a suitable statistical distribution. Simulation models can also be used for this purpose. Having fitted satisfactory statistical distributions, the next task is to evaluate ρ, W_q and L_q. If these indices are within the specified performance measures, present queuing system is a satisfactory one. If not, studies in terms of increasing the number of counters of service and the economics thereof, should be carried out.

5. ILLUSTRATION

5.1 Counter Service Example

Suppose the service mechanism of the library has one counter for issue/return of books. Let us assume the service time is exponential with mean $\mu = 20$ customers/hr. The arrival rate of customers (students as well as faculty) at the counter be approximated by Poisson distribution which is a satisfactory model in most cases with arrival rate of one in 5 minutes which means average number of customers arriving is 12/hr .

Let us calculate the various performance measures:

$$\rho = \lambda / \mu = 12/20 = 3/5 < 1$$

which means the 'traffic' is in control.

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{12}{20(8)} = \frac{3}{40} \text{ hr}$$

$$= 4.5 \text{ minutes}$$

$$L_q = 12 \times \frac{3}{40} = \frac{36}{40} \simeq 1$$

Now supposing the customers demand introduction of another "counter service"; Is it justifiable? When should the management go for it? What should be the increase in the arrival data warranting such action? Are the kinds of analysis possible for arriving at optimum queuing system?

5.2 Circulation of Books

In this case, the customer behaviour is vastly different from other situations. If on arrival, the customer does not find the book on the shelf, he may give up totally and does not try again. Assuming R is the circulation rate per year and $1/\mu$ is the mean circulation time (loan period) the book will not be on the shelf a fraction R/μ of the year. (Assume duplicate books are not available). During this time, $\lambda(R/\mu)$ persons come looking for the book and having found it missing from the shelves, give up and leave. (λ is the arrival rate, the number of persons per year who would like to borrow the book). It can be seen that the number of persons who have found the book on the shelf and have been able to borrow the book is $\lambda - \lambda\,(R/\mu)$. But by definition it is equal to R, the number of times, the book is borrowed per year.

$$\lambda(1 - R/\mu) = R$$

$$\Rightarrow \qquad R = \frac{\lambda\mu}{(\lambda + \mu)}$$

giving the expected circulation rate in terms of demand rate and 'return ratio'. It can also be shown that the probability that a borrower will find the book on the shelf (P_1), the probability that the book is in circulation, when he comes by (P_0), expected circulation rate (R) and the mean unsatisfied demand (U) are given by

$$P_0 = \frac{\lambda\mu}{(\lambda + \mu)} \qquad P_1 = \frac{\mu}{(\lambda + \mu)} \qquad R = \frac{\lambda\mu}{(\lambda + \mu)} \qquad U = \frac{\lambda^2}{(\lambda + \mu)}$$

The management can fix the limitations on the value of U, say 2 or 3 or 4 beyond which, (duplicate) multiple volumes have to be bought. Criterion for multiple copies can be derived in terms of the demand rate λ also.

Alternative situation is when everyone who desires the book, either hands in a reservation card or otherwise pesters the librarian until he eventually gets the book. This is a simple case of general queuing system. All arrivals are eventually served but on the average L_q

of them are waiting to get the book. For this model, the actual rate of circulation R per year is equal to the arrival rate, because all arrivals are eventually able to borrow the book. The various measures of queuing system in the case are

-- Average number of persons and any time, waiting to borrow the book

$$L_q = \frac{R^2/\mu}{\mu - R}$$

-- Average length of time, would be borrower has to wait until, he can borrow himself

$$\frac{L_q}{R} = \frac{R/\mu}{\mu - R}$$

-- Average number of unsatisfied customers per year

$$U = \mu L_q$$

In general the limiting value of U will lie between 2 to 4. It can be shown that whenever the yearly circulation rate R is greater than about 1/3U, too many would be borrowers are either waiting too long to borrow book or else, are giving up entirely. In either case, the library is not servicing them adequately!

6. REMARKS

Queuing model, becomes quite complex when the 'practices' of individual library deviate from the standard -- say, the book will be let out for more than stipulated period or notices are sent for return of the book only when it is known that some one else wants the book!

The theory of Queuing is sufficiently developed so that any of these complications could be included in the model, but the resulting formulae would be complicated and would depend on additional parameters that would be hard to evaluate. In such situations, it is advisable to start off with simple models and introduce complications one by one, until sufficient accuracy is obtained.

7. REFERENCES

1. Wagner H M. Principles of Operations Research

2. P M Morse. Library Effectiveness   - A systems approach