MA-597 **Queueing Theory Assignment**

A paper review by G.Murali Mohan Reddy
(Roll no. 06212310)

Application of Queuing Theory to Libraries and Information Centres

by

U H Acharya, and **G Ravindran,** SQC Unit, Indian Statistical Institute Bangalore-59.

1.INTRODUCTION:

In this paper, applications of queueing theory in the fields of library and information fields have been discussed. Queuing problem is identified by the presence of group of customers who arrive randomly to receive some service. The customer, upon arrival may be attended to, immediately or may have to wait until the server is free. This methodology is applicable in the field of Business, Industries, Government, Transportation, Restaurants, Library etc. Queuing models are basically relevant to service oriented organisations and suggest ways and means to improve the efficiency of the service.

Queueing theory is applicable in libraries for the following services:

(1) circulation of books,
(2) counter service and
(3) allied services like reprography.

2. CHARACTERISTICS OF A QUEUEING MODEL AND THE APPLICATION APPROACH:

A general queueing model is given by (A/B/C) :(D/E)
where,
A: probability distribution of inter arrival time
B: probability distribution of service time
C: number of counters in the system
D: queue size
E: queue discipline

Some important notations in a queuing system are

$\rho = \lambda / \mu$ = traffic density,

Where λ = mean arrival rate and μ = mean service rate,

EW_q = expected waiting time in queue,

EN_q = expected number of customers in queue,

Now, purpose of this paper is to minimize the waiting cost in the queue and to minimize the cost of service. For this purpose, firstly, an optimum service rate and the number of servers (counters) in the system have to be found out.

A suitable statistical distribution has to fit for data on arrival rate and service time in a particular period and then need to evaluate ρ, EW_q and EN_q. If these indices are not within the range of the specified performance measures, then studies in terms of increasing the number of counters of service and the economics thereof, should be carried out.

3. ILLUSTRATION

3.1 Counter Service Example

Suppose the service mechanism of the library has one counter for issue/return of books. Let us assume the service time is exponential with mean μ = 20 customers/hr. The arrival rate of customers (students as well as faculty) at the counter be approximated by Poisson distribution which is a satisfactory model in most cases with arrival rate of one in 5 minutes which means average number of customers arriving is 12/hr .

The traffic control is given by,

$$\rho = \lambda / \mu = 12/20 = 3/5 < 1$$

which means the 'traffic' is in control.

The expected waiting time in the queue is given by,

$$EW_q = \lambda / \mu(\mu-\lambda)$$
$$=12/20(8)$$
$$=3/40 \text{ hour}$$
$$= 4.5 \text{ minutes}$$

The expected number of customers in queue is given by,

$$EN_q = 12 \text{ X } 3/40$$
$$= 36/40 \sim 1$$

Now some questions regarding this queueing system may arise.

Supposing the customers demand introduction of another "counter service"; Is it Justifiable?

When should the management go for it?

What should be the increase in the arrival data warranting such action?

Are the kinds of analysis possible for arriving at optimum queuing system?

3.2 Circulation of Books

If on arrival, the customer does not find the book on the shelf, he may give up totally and does not try again.

Assume

R : circulation rate per year

$1/\mu$: mean circulation time(loan period)

Then, the book will not be on the shelf a fraction R/μ of the year. (Assume duplicate books are not available).

During this time, the number of persons who will come to borrow this book per year is $\lambda(R/\mu)$ and having found it missing from the book-shelves, give up and leave.

Therefore, the number of persons who have found the book on the shelf and have been able to borrow the book is given by, $\lambda - \lambda (R/\mu)$.

But by definition,

$$\lambda(1 - R/\mu) = R.$$

$$\Rightarrow \quad R = \lambda\mu / (\lambda + \mu).$$

giving the expected circulation rate in terms of demand rate and 'return ratio'.
Similarly, the probability that a borrower will find the book on the shelf (P_1), the probability that the book is in circulation, when he comes by (P_0), and the mean unsatisfied demand (U) are given by

$$P_0 = \lambda\mu / (\lambda + \mu).$$

$$P_1 = \mu / (\lambda + \mu).$$

$$U = \lambda.\lambda / (\lambda + \mu).$$

The management can fix the value of U, ranging from 2 to 4 beyond which, (duplicate) multiple volumes have to be bought.

Other case is that, when everyone who desires the book, either hands in a reservation card or otherwise pesters the librarian until he eventually gets the book. This is a simple case of general queuing system. All arrivals are eventually served but on the average EN_q of them are waiting to get the book.

For this model, the actual rate of circulation R per year is equal to the arrival rate, because all arrivals are eventually able to borrow the book.

The various measures of queuing system are,

Average number of persons and any time, waiting to borrow the book

$$EN_q = (R.R/\mu)/(\mu - R).$$

Average length of time, would be borrower has to wait until, he can borrow himself

$$EN_q/R = (R/\mu)/(\mu - R).$$

Average number of unsatisfied customers per year

$$U = \mu.EN_q$$

In limiting case, U will lie between 2 to 4.

If R >1/3U, too many would be borrowers are either waiting too long to borrow book or else, are giving up entirely.

4. CONCLUSION

Queuing model, becomes quite complex in cases, for example when the book will be let out for more than stipulated period or notices are sent for return of the book only when it is known that some one else wants the book. The theory of queuing is sufficiently developed so that any of these complications could be included in the model, but the resulting formulae would be complicated and would depend on additional parameters that would be hard to evaluate. In such situations, it is advisable to start off with simple models and introduce complications one by one, until sufficient accuracy is obtained.

5. REFERENCES

1. Wagner H M. Principles of Operations Research
2. P M Morse. Library Effectiveness - A systems approach.