Tutorial 2: Hybrid Automata & Reachability. 1

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Stochastic HS

Switched System

Hybrid Dynamical System (Teel et al.)

$H = \langle \underline{Q, Q_0, A}, D, \mathcal{T} \rangle$ = Automaton + ODE

$\boxed{\text{trajectories}}$

Variables : Set of vars X, U, V : $X = \{x_1, x_2, s, u\}$

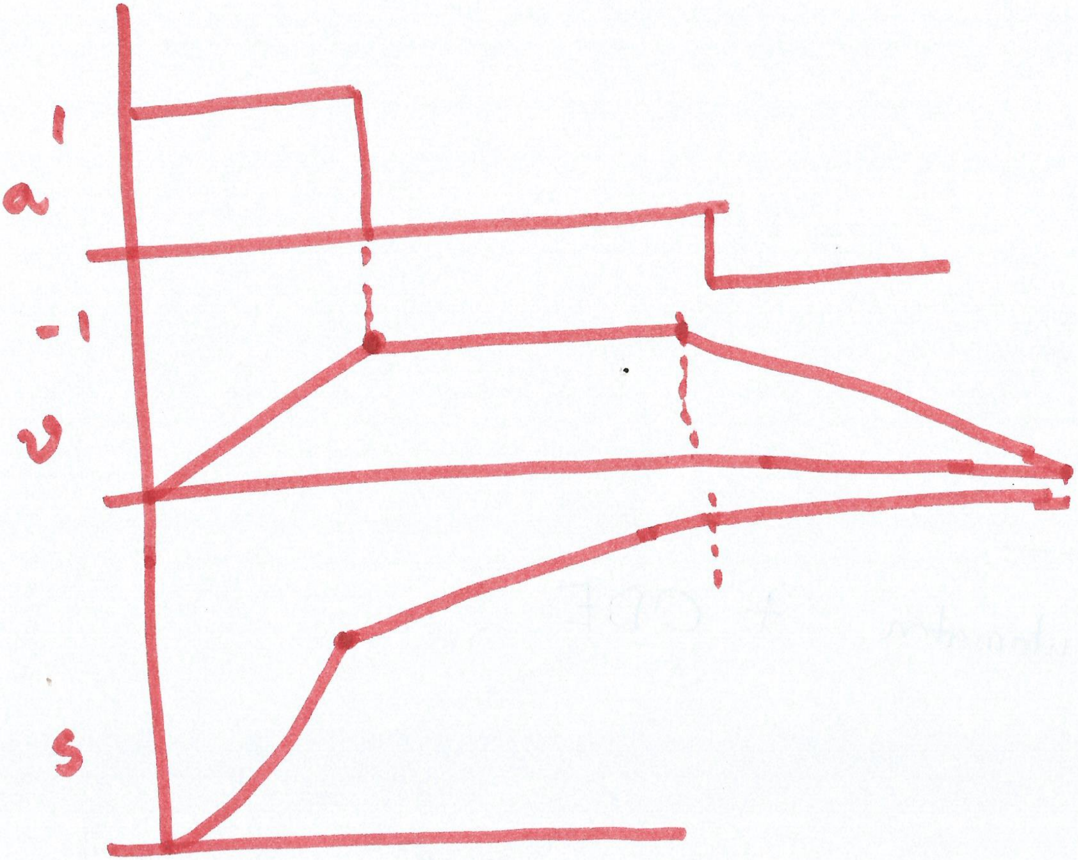
$\forall x \in X$ variable type$(x) = \mathbb{R}$ $\mathbb{R}$, $\mathbb{N}$, $\{0,1\}$

type$(s) = \mathbb{N}$

Valuation of X maps each $x \in X$ to type(x)

$V = \{x, m\}$ type$(x) = \mathbb{R}$ type$(m) = \{0,1\}$ valation of V $v\langle x \to 5.5, m \to 0 \rangle$

val(X) : Set of all possible valuations $\mathbb{R} \times \{0,1\}$



$$\frac{ds}{dt} = v$$

$$\frac{dv}{dt} = a$$

$$a \in \{0, 1, -1\}$$

$$\uparrow$$

v : valuation for V

$v\lceil x$: restriction of v to x $\qquad$ $v\lceil x : \{x\} \to type(x)$ $\qquad$ (2)

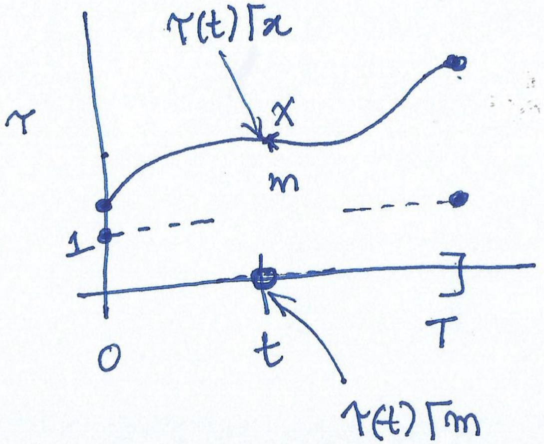
$v\lceil X \quad X \subseteq V$

Trajectories : A trajectory τ of X is a mapping $\tau : J \to val(X)$ where J is left closed with 0 interval of time.

$V = \{x, m\}$

domain $\to$ $J : [0,T]$, $[0,T)$, $[0,\infty)$

closed $\to$ $[0,T]$ $\qquad$ $\underbrace{[0,T)}_{\text{open}}$ $\qquad$ $[0,\infty)$ $\uparrow$ infinite

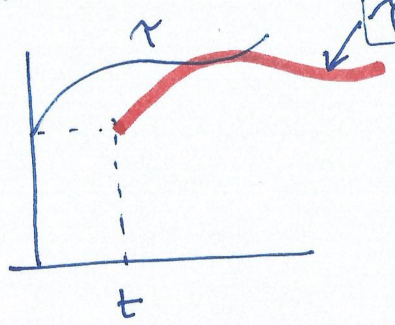


$\tau.fstate$

if τ is closed, then $\tau.lstate$ is well defined.

$\tau(t)\lceil x \qquad \tau(t)\lceil m$

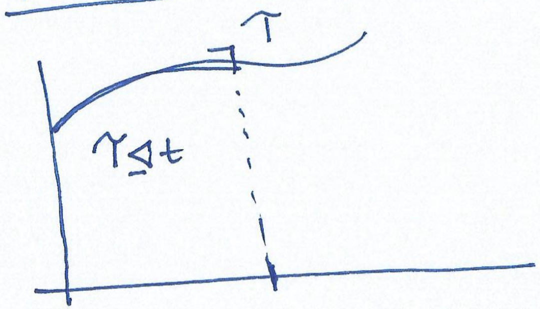
Time Shift $\quad \tau + t$



$\boxed{\tau+t} : [t, \tau.ltime+t] \to val(X)$

$\forall t' \in [\cdot\cdot\,] \quad \tau+t\,(t') = \tau(t'-t)$

Prefix $\quad t \geq 0 \quad \tau \trianglelefteq t$

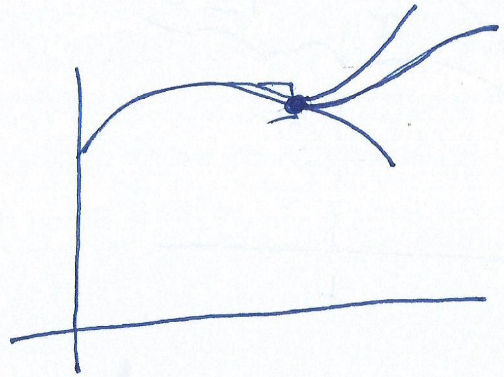


②

X : Variables

$\mathcal{T}$: Set of trajs for X

$\forall x \in X$ either $type(x) = \mathbb{R}$ (Continuous vars)
or $type(x)$ = finite set (discrete vars)



$$\frac{d(x)}{dt} = f(x) \qquad \dot{x} = f(x)$$

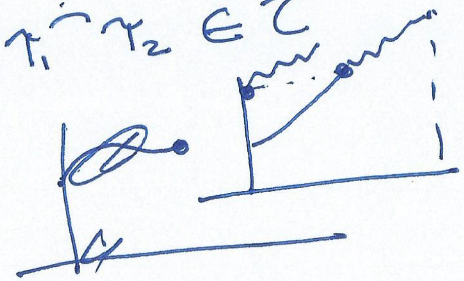
Proposition $\mathcal{T}$ satisfies

(i) Closed under prefix, i.e., $\forall \tau \in \mathcal{T}$ $\forall t \in \tau.dom$ $\tau \trianglelefteq t \in \mathcal{T}$

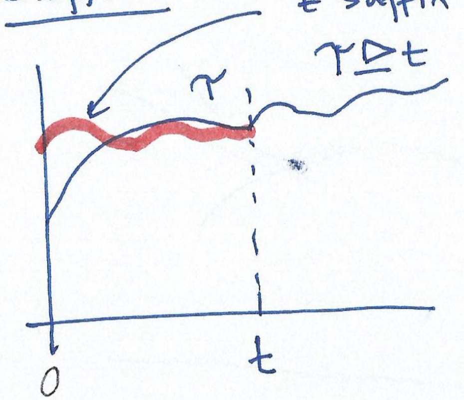
$\tau \trianglerighteq t \in \mathcal{T}$

(ii) . . . Suffix

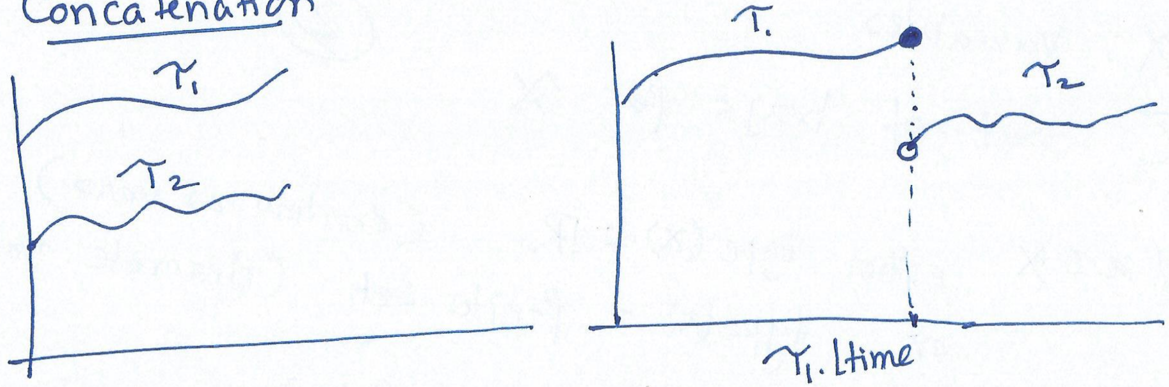
(iii) C . . Concatenation $\forall \tau_1 \tau_2 \in \mathcal{T}$, and τ_1. lstate = τ_2. fstate st. τ_1 is closed
then $\tau_1 {}^\frown \tau_2 \in \mathcal{T}$



Suffix



Concatenation



$$\tau' : [0, \ \tau_1.\mathrm{ltime} + \tau_2.\mathrm{ltime}]$$

$$\tau'(t) = \tau_1(t) \qquad t \le \tau_1.\mathrm{ltime}$$

$$= \tau_2 + \tau_1.\mathrm{ltime} \ (t - \tau_1.\mathrm{ltime})$$

Hybrid Automaton : $\mathcal{A} = \langle X, \Theta, A, \mathcal{D}, \mathcal{T} \rangle$ (4)

$\Theta \subseteq val(X)$ initial States.

A : set of Actions

$\mathcal{D} \subseteq val(X) \times A \times val(X)$

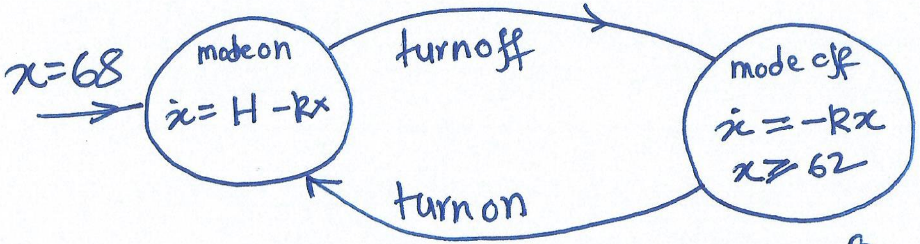
$\mathcal{T}$: set of trajs of ~~cont vars in~~ vars X

Closed under prefix, ~~suffix~~ & concatenation.

Thermostat

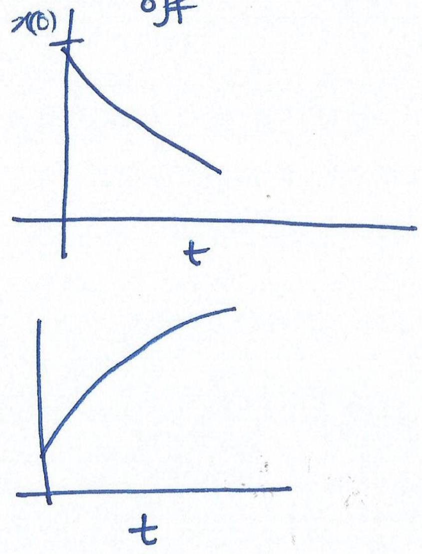
Pre $x \geq 70$ eff mode = off

(5)



$x < 62$ eff mode := on

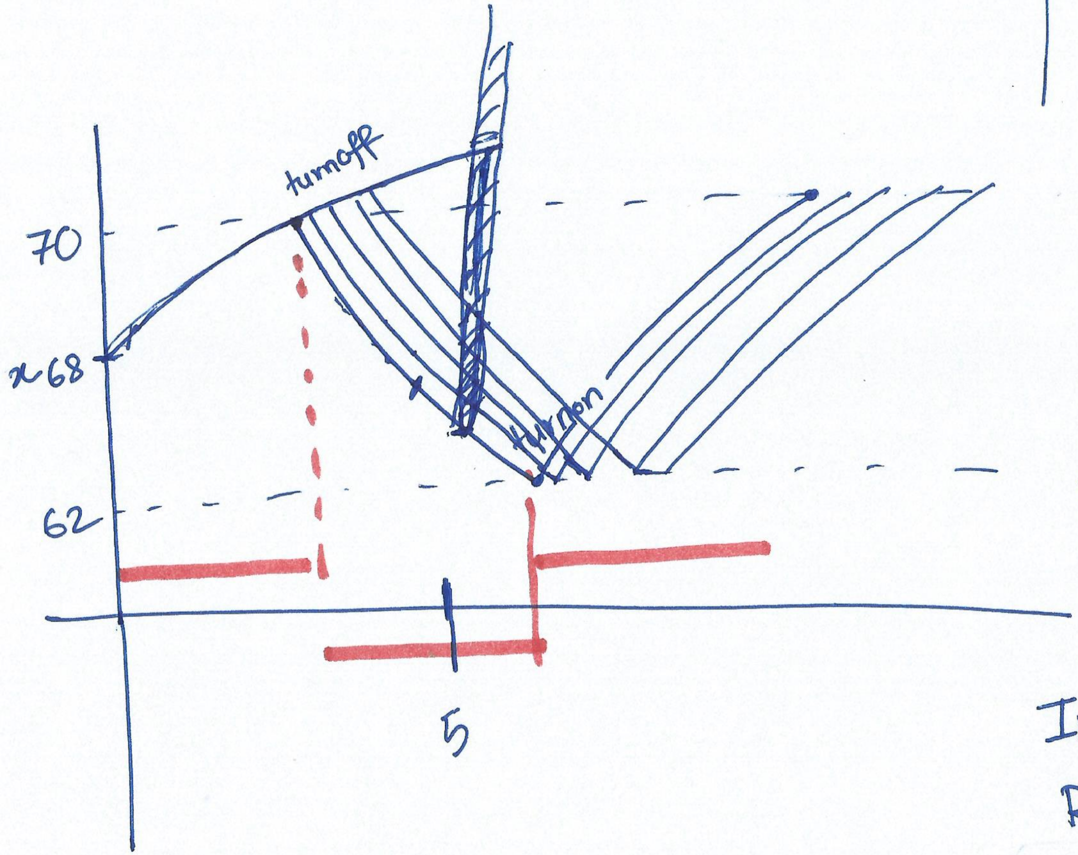
$$x(t) = x(0)e^{-kt}$$



$V = \{x, \text{mode}\}$

$\text{type}(x) = \mathbb{R}$

$\text{type}(\text{mode}) = \{\text{on}, \text{off}\}$



Execs_A

$\boxed{\text{Reach}_A}$

State x is reachable if there exists an closed execution α of A such that

$\alpha.\text{lstate} = x$

Invariant of A $I \subseteq val(X)$ such that

$\text{Reach}_A \subseteq I$.

$\text{Reach}_A(k, T)$ $k \in \mathbb{N}$ $T \in \mathbb{R}_{\geq 0}$

Types of Execution

(6)

Execution $\alpha = \tau_0 a_1 \tau_1 a_2 \tau_2 \ldots \tau_n$

α is finite if it is a finite sequence

infinite .. .. .. not finite

Closed if it is finite and τ_n is Closed

Open if not closed.

SPACEEx

Flow *

Closed α $\alpha.lstate = \tau_n.lstate$

$\alpha.ltime = \sum_i \tau_i.ltime$

if α is finite it can be open iff τ_n is open $\tau_0 a_1 \tau_1 \ldots \tau_n$

$\boxed{[0,T)}$

$[0,\infty)$

α is admissible if $\alpha.ltime = \infty$

(i) α infinite $\alpha.ltime = \infty = \sum \tau_i.ltime$

(2) α finite with $\tau_n.ltime = \infty$

α is zeno if neither closed nor abmissible

Zeno type 1

$\dot{x}_1 = x_2$

$\dot{x}_2 = -g$

Pre $x_1 \leq 0 \wedge x_2 \leq 0$

$x_2 = -c x_2$