Laboratory Assignment 3

PH 508 Numerical Methods & Programming

1. Write a program each to implement the following methods:

   (a) Bisection Method,

   (b) Newton-Raphson Method.

   Each program should output the approximate solution, the value of the function at the solution, tolerance and number of iterations required for convergence. If the solution is not obtained in 100 iterations, print a message to that effect.

2. Verify above programs by finding a root of the function $f(x) = e^{2x} - e^{x} - 2$ which lies in the interval [0,1]. Compare convergence of these methods. (The solution is $0.693146 \pm 5 \times 10^{-6}$.)

3. For each of the following equations, determine an iterating function $g(x)$ and an interval I that satisfies the conditions for convergence for the methond of fixed point iterations. ($x > 0$)

   (a) $x^3 - x - 1 = 0$

   (b) $x = 2\tanh(x)$ (Langevin Equation).

4. Write a program based on fixed point iteration algorithm to find the smallest positive roots of the examples given in problem 3.

5. The energies of the bound states of a square well potential are given by $E_n = \left[\beta_n^2 \frac{\Delta}{V_0} - 1\right] V_0$ where β_n is a solution of

$$\left(\tfrac{\Delta}{V_0}\right)^{1/2} \beta = |\cos\beta| \quad \text{if} \quad m\pi < \beta < (m + 1/2)\,\pi$$
$$\left(\tfrac{\Delta}{V_0}\right)^{1/2} \beta = |\sin\beta| \quad \text{if} \quad (m - 1/2)\pi < \beta < m\pi$$

   where m is an integer. Find and sketch the energy spectrum (in units of V_0) for $\frac{V_0}{\Delta} = 4$ and 70.