Newton' law of motion:

1st law: law of inertia

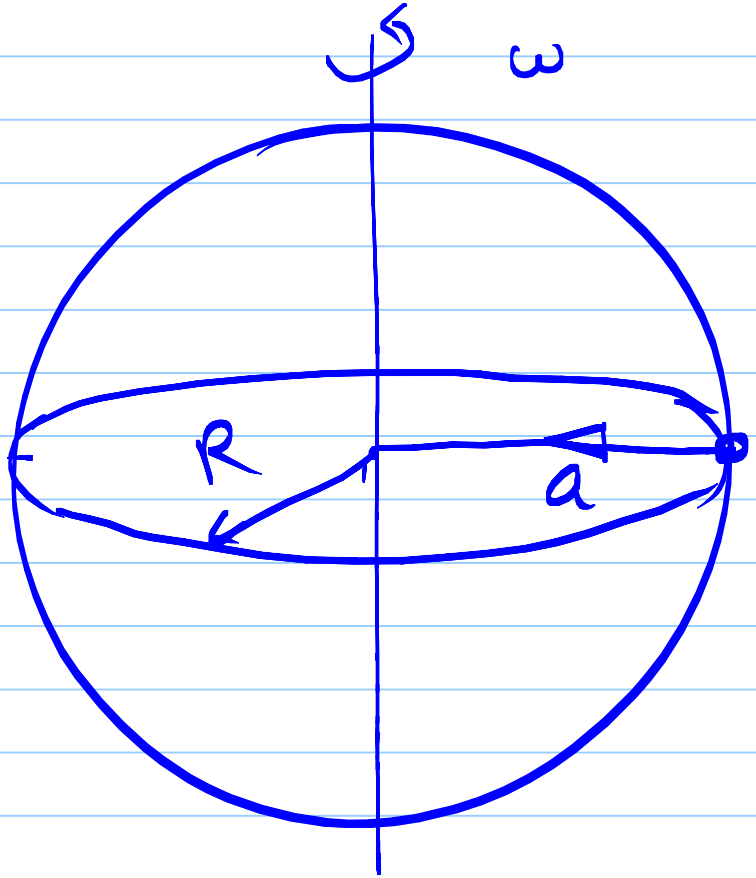
In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with constant velocity along a straight line

Does not valid in all reference frame!

Inertial frame of reference

→ No acceleration of any kind

Is EARTH inertial ref. frame??



$T = 24 \times 3600$ sec

$\omega = \frac{2\pi}{T}$ rad/sec

$R = 6400$ km

a = radial acc^n

$= \omega^2 R$

$= \underline{0.034}$ m/s^2

$g = 10$ m/sec^2

Consider other motions and re-calculate

Newton's 2nd law :

$$\vec{F} = m\,\vec{a}$$

Force　　　mass　　　accl^n

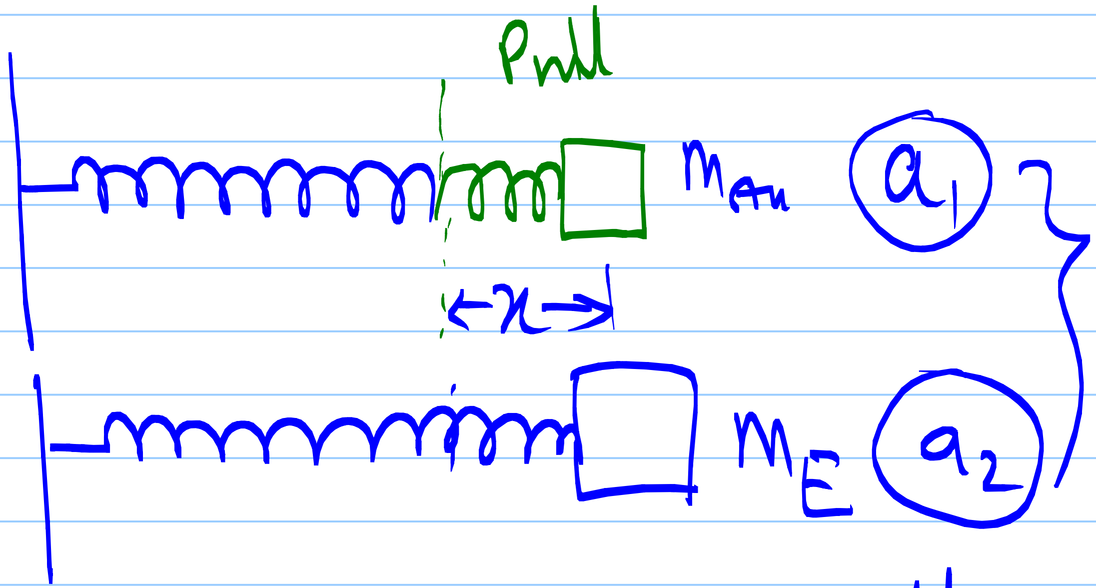
$\vec{a} \Rightarrow$ can be measured ! $= \dfrac{d\vec{v}}{dt}$

What is "m" ? [matter of convention]

Define 1 kg mass .

1 kg of mass is given — A box of Au

measure mass of Elephant!



measure acceleration!

$$\vec{a}_1 = \frac{'F'}{m_{Au}}$$

Experimentally proved

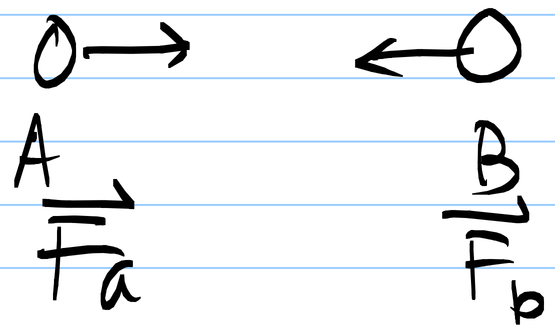
$$a_2 = \frac{'F'}{m_E}$$

'F' is not known!

$$\frac{a_{Au}}{a_E} = \frac{M_E}{M_{Au}} = 1$$

3rd law

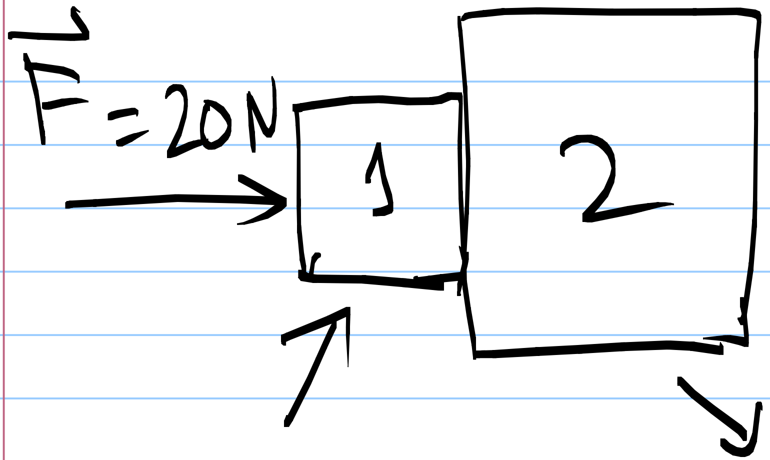
Action = Reaction



Force!

$$\vec{F}_a = -\vec{F}_b$$

Action and reaction are on different body !



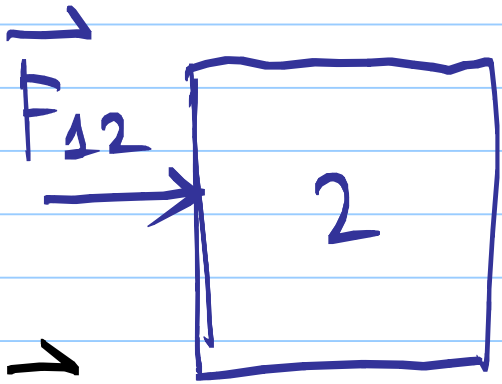
$m_1 = 5\,kg$ $\quad$ $m_2 = 15\,kg$

$$\vec{F} = m\vec{a}$$

$$= (m_1 + m_2)\vec{a}$$

$$\vec{20} = 20\,\vec{a}$$

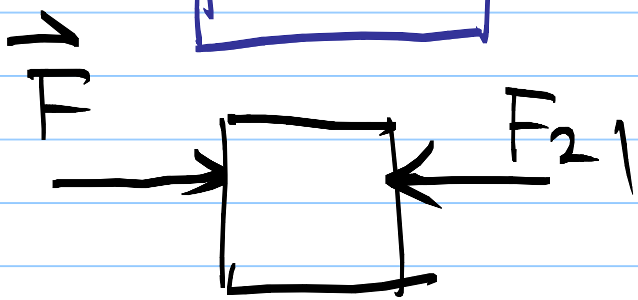
$$\Rightarrow a = 1\ m/s^2$$



$$\vec{a} = 1\ m/s^2$$

$$\vec{F}_{12} = 15 \times 1\ N$$

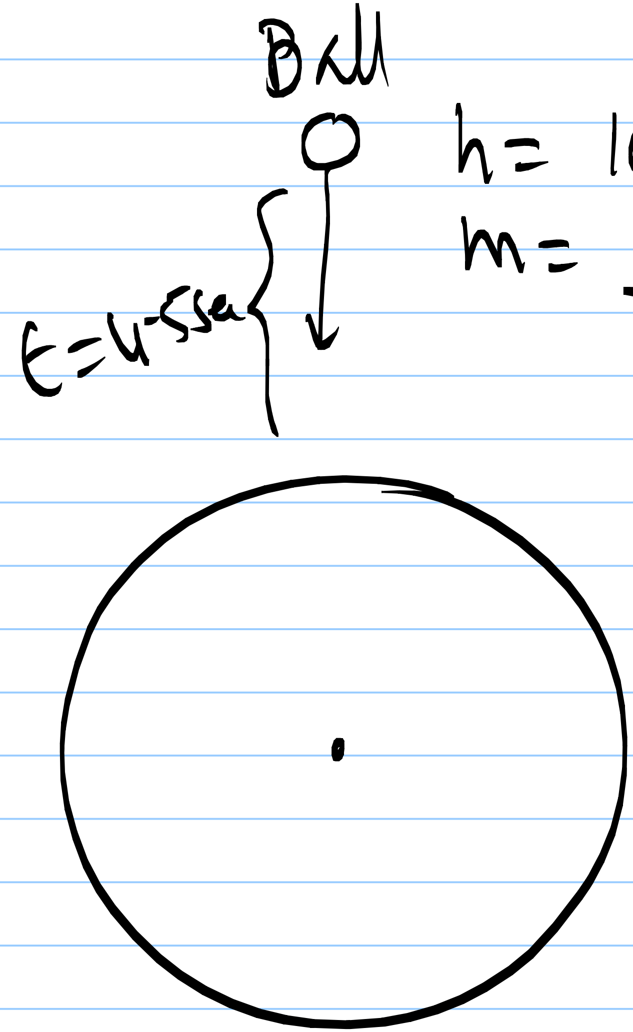
$$= 15\ N$$



$$\vec{F} + \vec{F}_{21} = m_1\vec{a}$$

$$F_{21} = 5 - 20 = -15\ N$$

Take gravitational force!



$h = 100$ m

$m = 1$ kg

$M_E = 6 \times 10^{24}$ kg

$\vec{F}_b = m\vec{g}$

$= 10$ N

Time? $\frac{1}{2} g t^2 = 100$

$t = \sqrt{20} = 4.5$ sec

Reaction force on Earth:

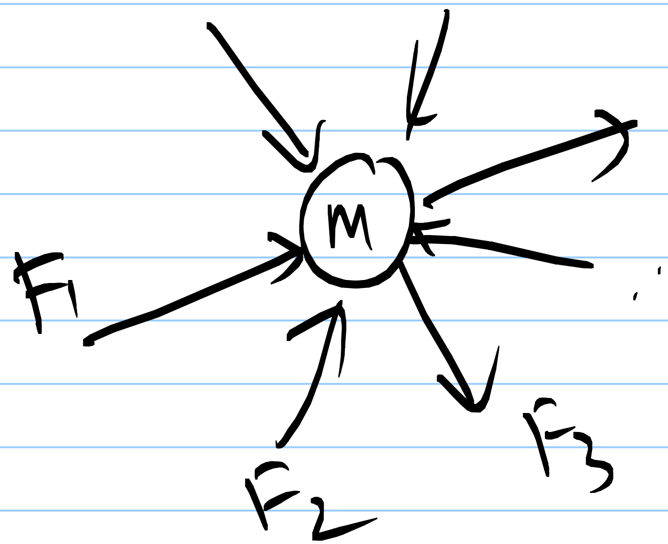
$= -\vec{F}_b = M_E \vec{a}_E$

$\vec{a}_E = \frac{F}{M} = \frac{10}{6 \times 10^{24}} = 1.7 \times 10^{-24}$ m/s^2

How much distance it goes

$= \frac{1}{2} a t^2 = \frac{1}{2} 1.7 \times 10^{-24} \times 20 = 17 \times 10^{-24}$ m

Principle of Superposition of Forces!



Calculate net force

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

[tip to tail in vectors]

$$\vec{F} = \vec{F}_{Net} = \sum_{i=1}^{N} \vec{F}_i$$

$$= \sum_{i=1}^{N} m\,\vec{a}_i$$

$$\vec{F} = m\,\vec{a}$$

Solving Newton's law problems

* Look at the bigger picture
* Devided into small systems
  - each point particle
* Draw the force diagramm for each mass in the vector form.
* When two bodies interacts then force betn them must be equal & opposite
* Use Newton's 2nd law.
* Resolve into components.

Identyfy all forces:

Fundamental forces:
- Gravitational force
- Electromagnetic forces
- Weak Nuclear forces
- Stron Nuclear forces

Gravitational force:

Newton's Law of gravitation

$$\vec{F} = G \frac{m_1 m_2}{r^2} \hat{r}$$

→ attractive

— Explain planatary motion.

Electromagnetic forces:

Coulomb force → $\vec{F} = \frac{q q'}{r^2} \hat{r}$ → attractive or repulsive

Magnetic force → $\vec{E}$ and $\vec{B}$

Lorentz force → $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Nuclear forces:

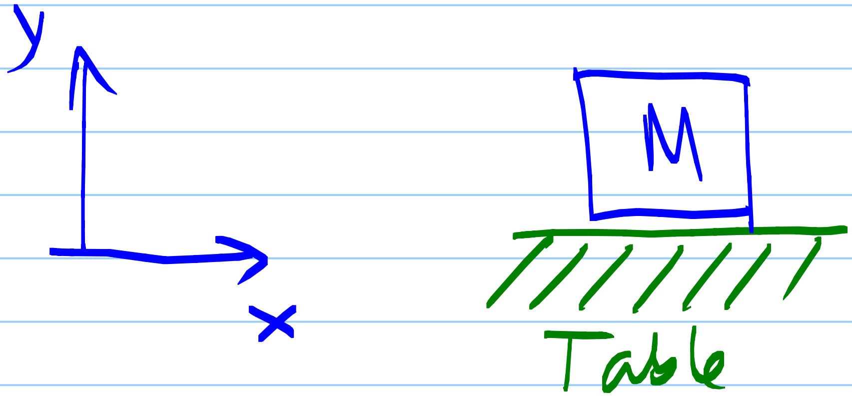
— Weak & Strong

— range 10^{-15} m

Everyday forces:

* Tension
* Normal force
* Viscous force
* Spring
* Atomic forces (10^{-10} m)

Example 1 — Block at rest!



$\vec{N} = N\hat{j}$ ($\perp$ to surface)

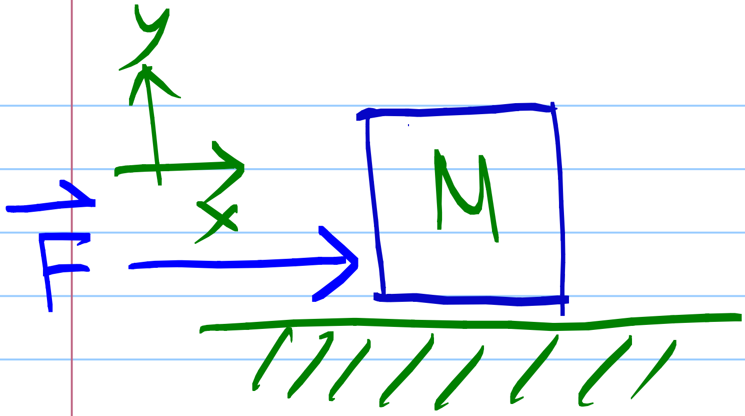
M

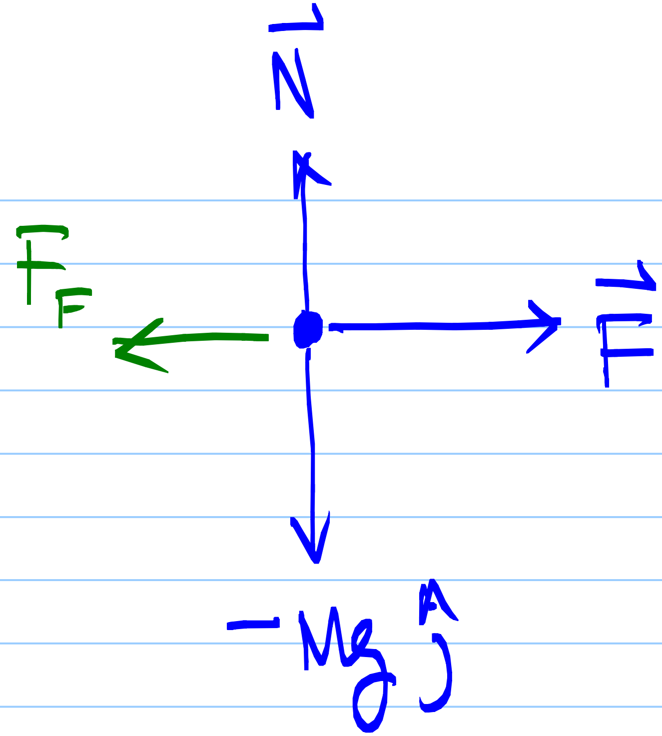
$-mg\hat{j}$

But $a = 0$

$$M\vec{a} = M \cdot 0 = N\hat{j} - mg\hat{j}$$

$$\Rightarrow \boxed{N = mg} \quad \text{Normal force!}$$





resists force || surface

$\vec{F}_F$ || surface & opposes other forces.

$$|\vec{F}_F| \leq \mu_s |\vec{N}| \qquad \mu_s \to \text{No dimension}$$

(static case)

Vertical $\quad N - Mg = 0 \implies N = Mg$

Block does not move $|F_F| \leq \mu_s Mg$

Applied force exceeds $\mu_s Mg$

$\Rightarrow$ Block starts moving

$\Rightarrow$ Force of friction goes down!

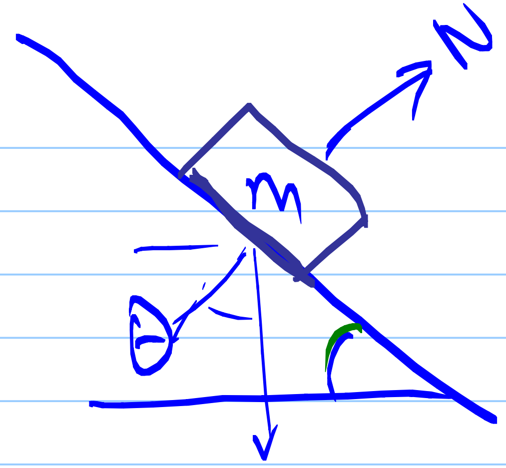
in case of sliding

$$|\vec{F}_F| = \mu_k |\vec{N}|$$

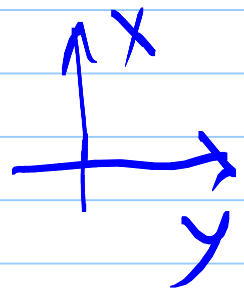
$\mu_s \rightarrow$ Coefficient of static friction

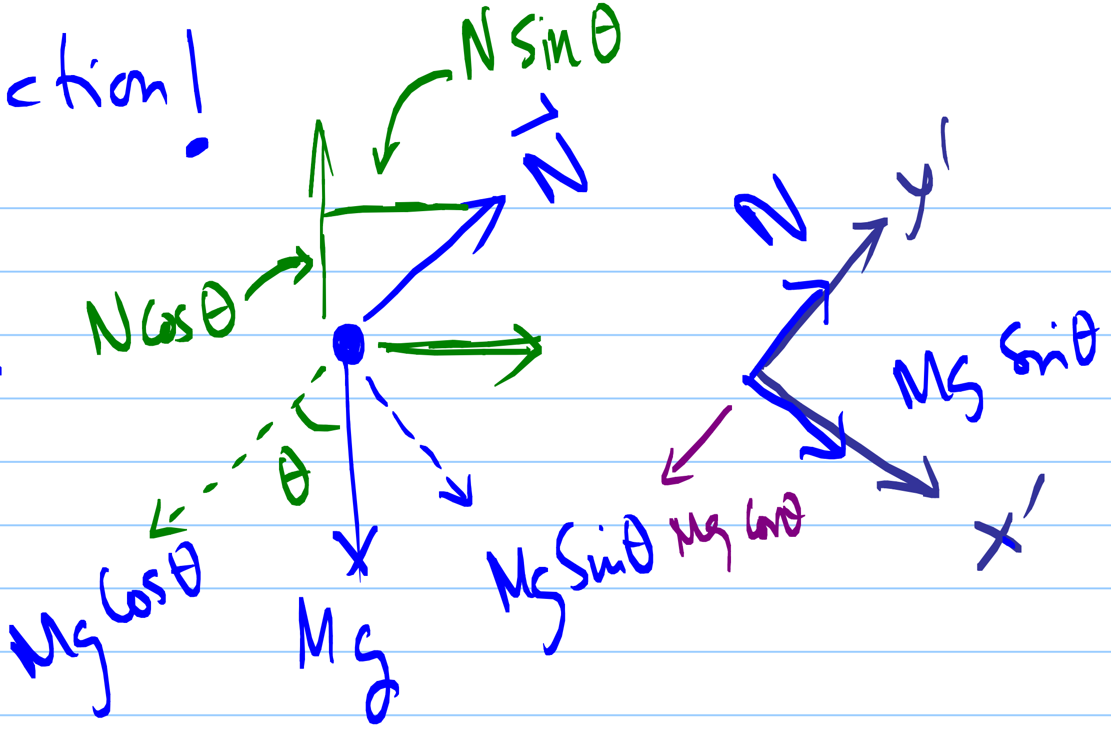
$\mu_k \rightarrow$ " " kinetic "

usually $\mu_k < \mu_s$



No friction!





x : $N \sin\theta = M\ddot{x}$

y : $N\cos\theta - Mg = M\ddot{y}$

$N = Mg\cos\theta$

$M\ddot{x} = Mg\cos\theta\sin\theta$

$M\ddot{y} = Mg(\cos^2\theta - 1)$

$= -Mg\sin^2\theta$

$M\ddot{x}' = Mg\sin\theta$

$M\ddot{y}' = 0$

$= N - Mg\cos\theta$

$M\ddot{x} = M\ddot{x}'\cos\theta$

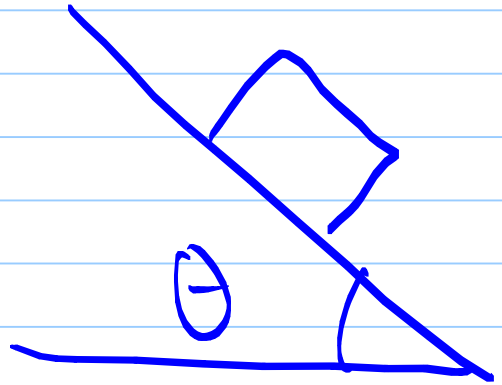
$= Mg\sin\theta\cos\theta$

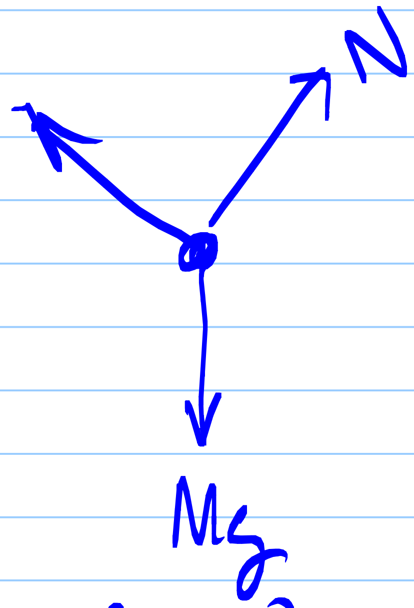
$M\ddot{y} = -M\ddot{x}'\sin\theta$

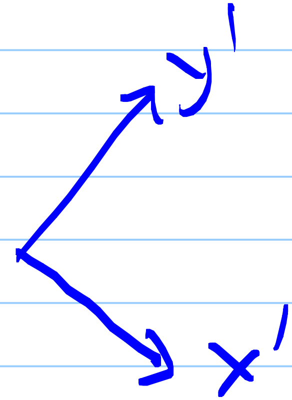
$= -Mg\sin^2\theta$

Now add friction!

How large can θ get before block slides?







y':
$$N - Mg\cos\theta = 0$$
$$N = Mg\cos\theta$$
$$|\vec{F}_f| \le \mu_s N = \mu_s Mg\cos\theta$$

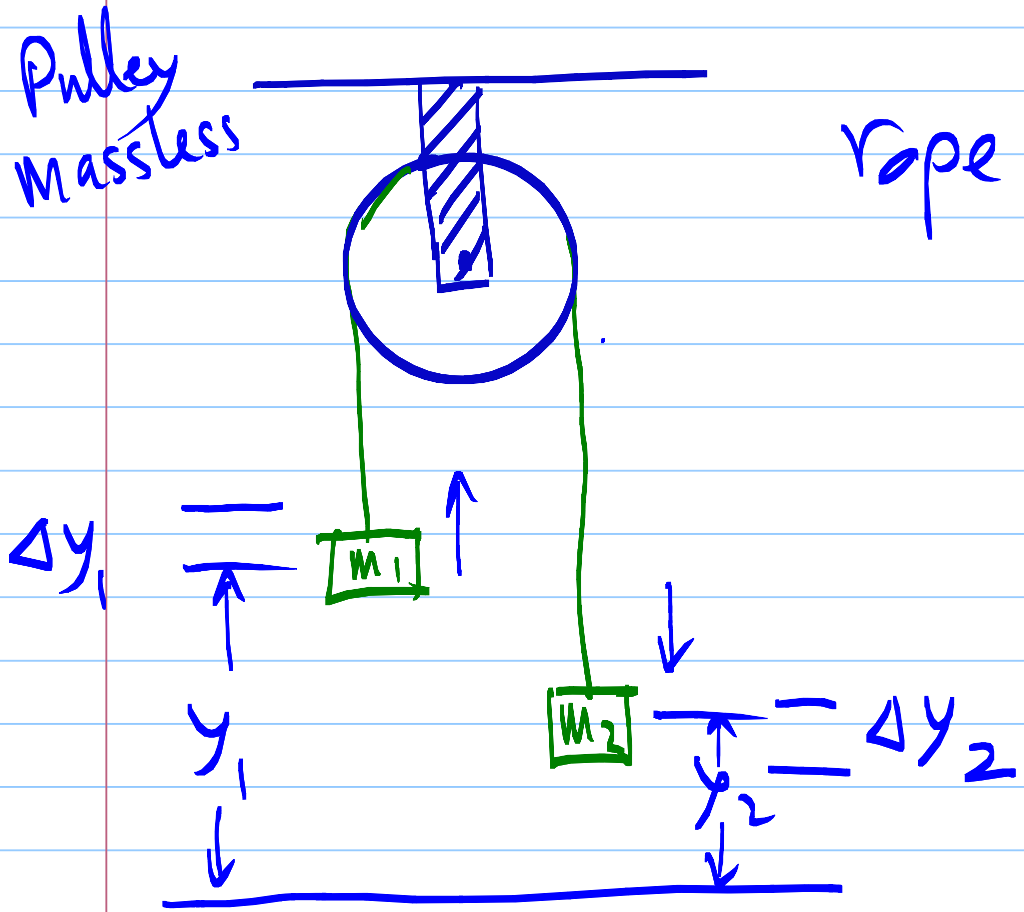
$\ddot{x}$ $\quad Mg\sin\theta - F_F = M\ddot{x}$

$\mu_s Mg\cos\theta > Mg\sin\theta$ — Block won't move

$\tan\theta < \mu_s$

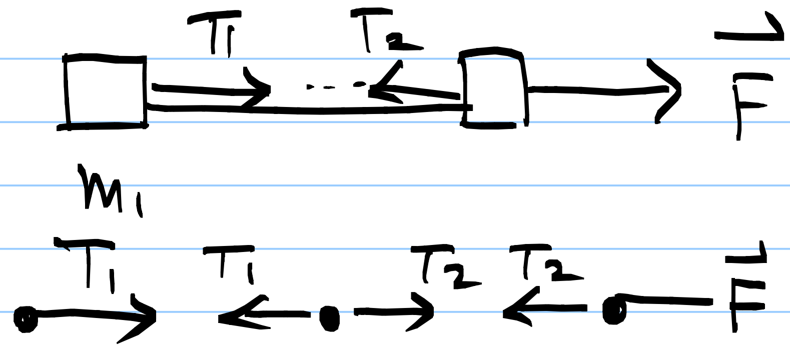
$\theta_{max} = \tan^{-1}(\mu_s)$

At wood's Machine



rope – massless and does not stretch

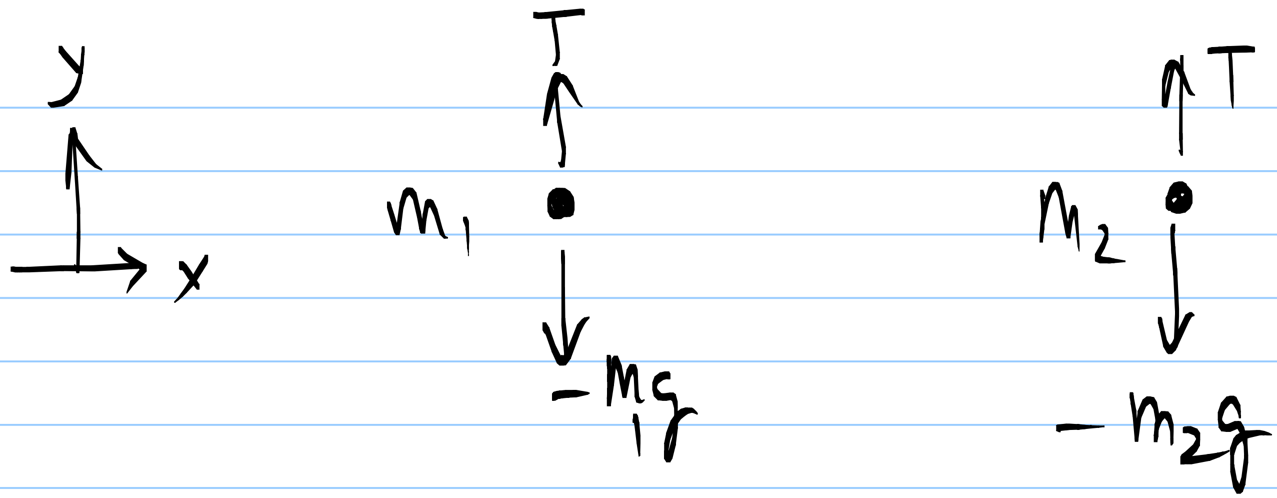
Imagine



$m_1 \ddot{y}_1 = T_1$ $\quad$ $m_s \ddot{y}_s = T_2 - T_1$ $\quad$ $m_2 \ddot{y}_2 = F - T_2$

$m_s = 0$

$T_1 = T_2$



$$m_1 \ddot{y}_1 = T - m_1 g \qquad m_2 \ddot{y}_2 = T - m_2 g$$

$$\Delta y_1 = -\Delta y_2 \quad \& \quad \ddot{y}_1 = -\ddot{y}_2$$

$$m_1\left[\frac{-T}{m_2} + g\right] = T - m_1 g \quad \Bigg| \quad -\ddot{y}_1 = \frac{T}{m_2} - g$$

$$\Rightarrow 2 m_1 g = T\left(1 + \frac{m_1}{m_2}\right) \Rightarrow T = \frac{2 m_1}{1 + \frac{m_1}{m_2}} g = 2\frac{m_1 m_2}{m_1 + m_2} g$$

$$\ddot{y}_1 = -\frac{T}{m_2} + g = -2\frac{m_1 m_2}{m_2(m_1+m_2)} g + g$$

$$\ddot{y}_1 = \left[\frac{(m_2 - m_1)}{(m_1+m_2)}\right] g$$

$m_2 > m_1$
$\ddot{y}_1 > 0$

$$\ddot{y}_2 = -\ddot{y}_1 = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] g$$

$m_2 > m_1$
$\ddot{y}_2 < 0$

$$T = 2\frac{1}{\frac{m_1+m_2}{m_1 m_2}} g = 2\left(\frac{1}{\frac{1}{m_1}+\frac{1}{m_2}}\right) g = 2\mu g$$

$\mu \Rightarrow$ reduced mass $\Rightarrow \frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$ | smaller mass dominates

$$T = 2\frac{m_1 m_2}{m_1+m_2}g$$

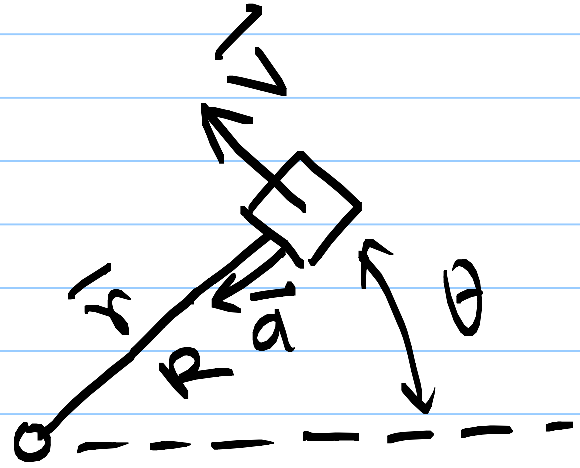
$$\frac{dT}{dm_2} = 2\left[\frac{m_1}{m_1+m_2} - \frac{m_1 m_2}{(m_1+m_2)^2}\right]g$$

$$= 2\frac{m_1^2}{(m_1+m_2)^2}g$$

$$\left.\frac{dT}{dm_2}\right|_{m_2=0} = 2g \qquad \left.\frac{dT}{dm_2}\right|_{m_2=\infty} = 0$$

$$m_1 = m_2 \Rightarrow T = m_1 g$$

Block on a string: No Gravity

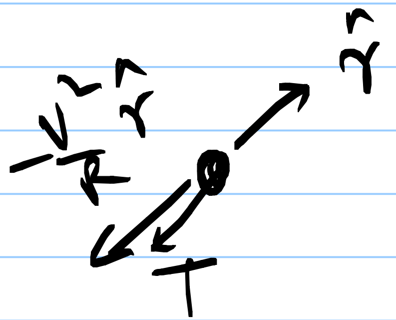


velocity $\vec{v}$ = constant

$$r = R \qquad \theta = \omega t$$
$$\dot{r} = 0 \qquad \;\; = \frac{v}{R} t$$
$$\ddot{r} = 0 \qquad \dot{\theta} = \frac{v}{R} = \omega$$
$$\ddot{\theta} = 0$$

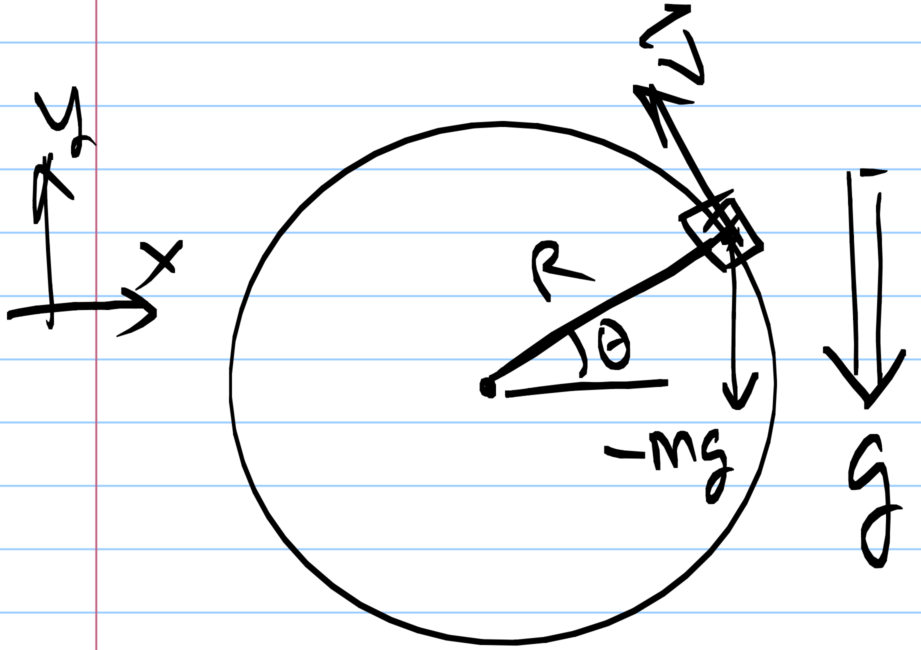
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\theta}$$

$$\vec{a} = -R\frac{v^2}{R^2}\hat{r} = -\frac{v^2}{R}\hat{r}$$



$$\vec{F}_{Net} = m\vec{a}$$
$$-T = -m\frac{v^2}{R} \Rightarrow T = \frac{mv^2}{R}$$

Add Gravity! — Not radial



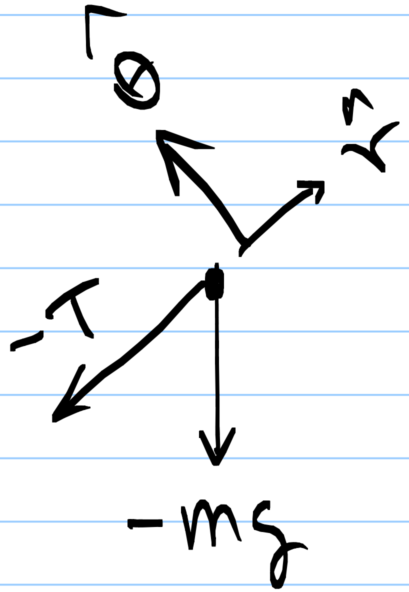
$$|\vec{r}| = R \Rightarrow \dot{r} = \ddot{r} = 0$$

Circular motion!

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\theta}$$

$$\vec{a} = -r\dot{\theta}^2\,\hat{r} + r\ddot{\theta}\,\hat{\theta}$$

$$= -R\dot{\theta}^2\,\hat{r} + R\ddot{\theta}\,\hat{\theta}$$



$\boxed{\hat{r}}$ $$-T - mg\sin\theta = -mR\dot{\theta}^2$$

$\boxed{\hat{\theta}}$ $$-mg\cos\theta = mR\ddot{\theta}$$

String can pull but not push!

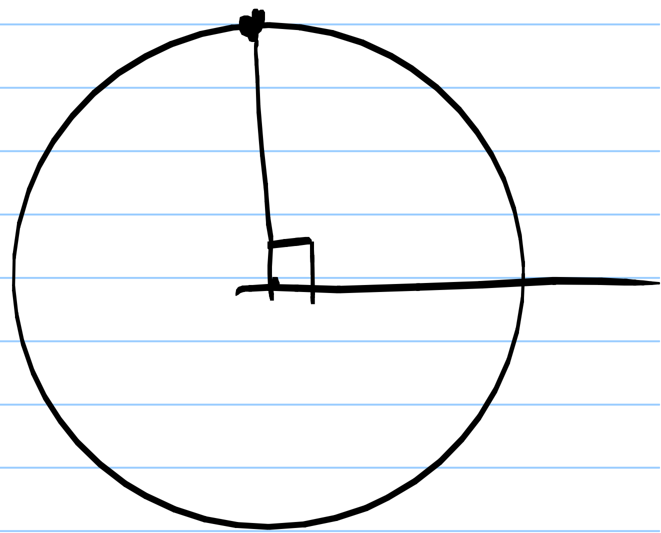
$T > 0$

$$T = mR\dot{\theta}^2 - mg\sin\theta > 0$$

$$R\dot{\theta}^2 > g\sin\theta$$

$$\frac{v^2}{R} > g\sin\theta$$

when $\theta = 90^\circ = \pi/2$



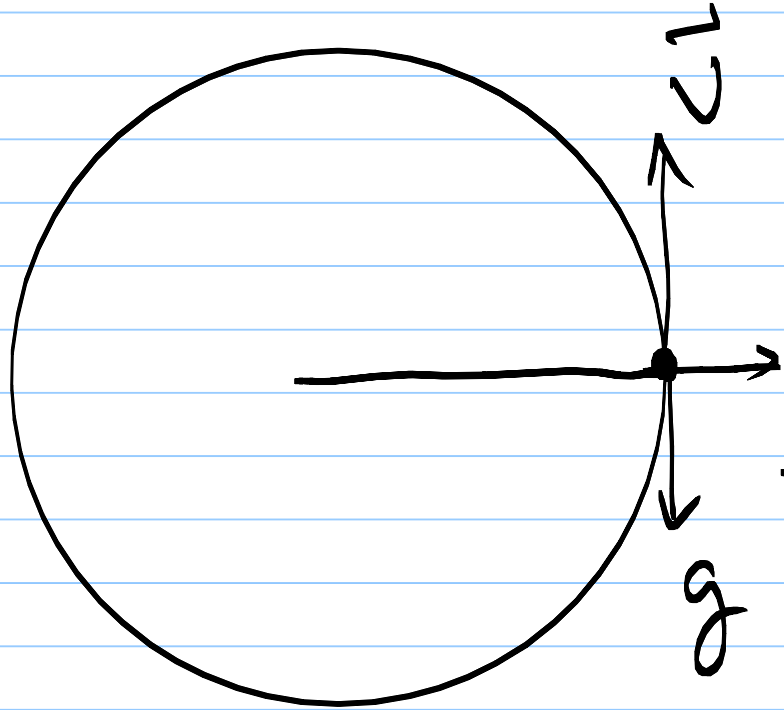
$$v^2 > gR$$

$$\Rightarrow v > \sqrt{gR}$$

$$-mg\cos\theta = mR\ddot{\theta}$$

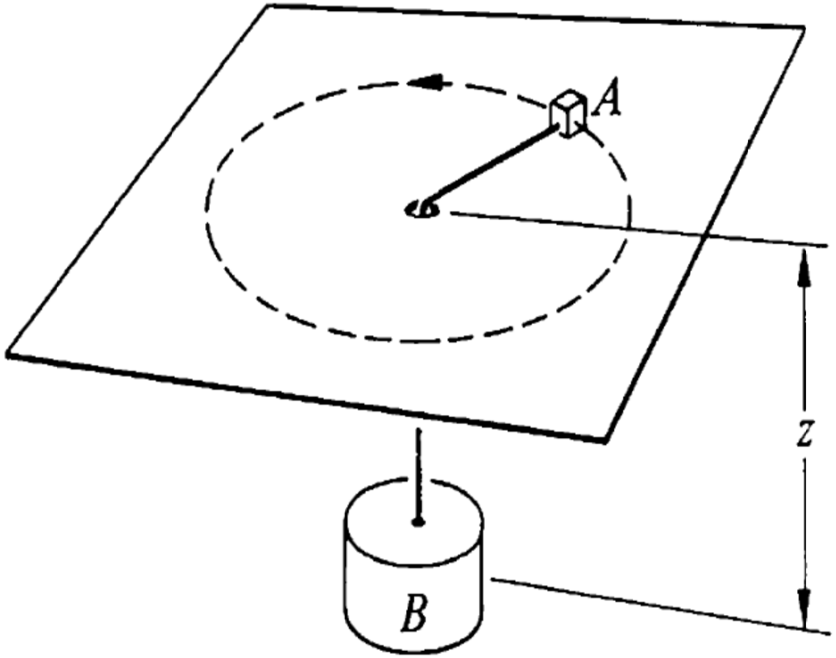
$$R\ddot{\theta} = -g\cos\theta$$ Tangential acceleration!

Down swing tangent speed increases!

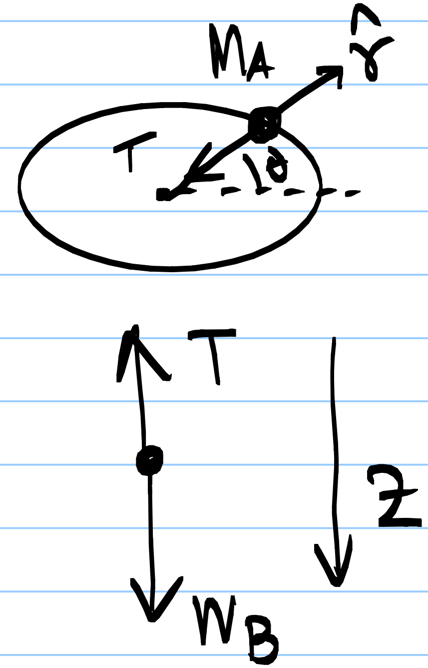


upswing tangential speed decreases!

Force diagram



B released at $t=0$.
Instantaneous accel^n?



$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\theta}$$

$$-T = M_A(\ddot{r} - r\dot{\theta}^2) \qquad \text{Radial} \quad \text{— (1) ✓}$$

$$0 = M_A(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \qquad \text{Tangential} \quad \text{— (2)}$$

$$W_B - T = M_B\ddot{z} \qquad \text{Vertical} \quad \text{— (3) ✓}$$

length of the string l

$$z + r = l$$

$$\ddot{r} = -\ddot{z} \qquad — \quad (5) \checkmark$$

$$\ddot{z} = \frac{W_B - M_A r \dot{\theta}^2}{M_A + M_B}$$

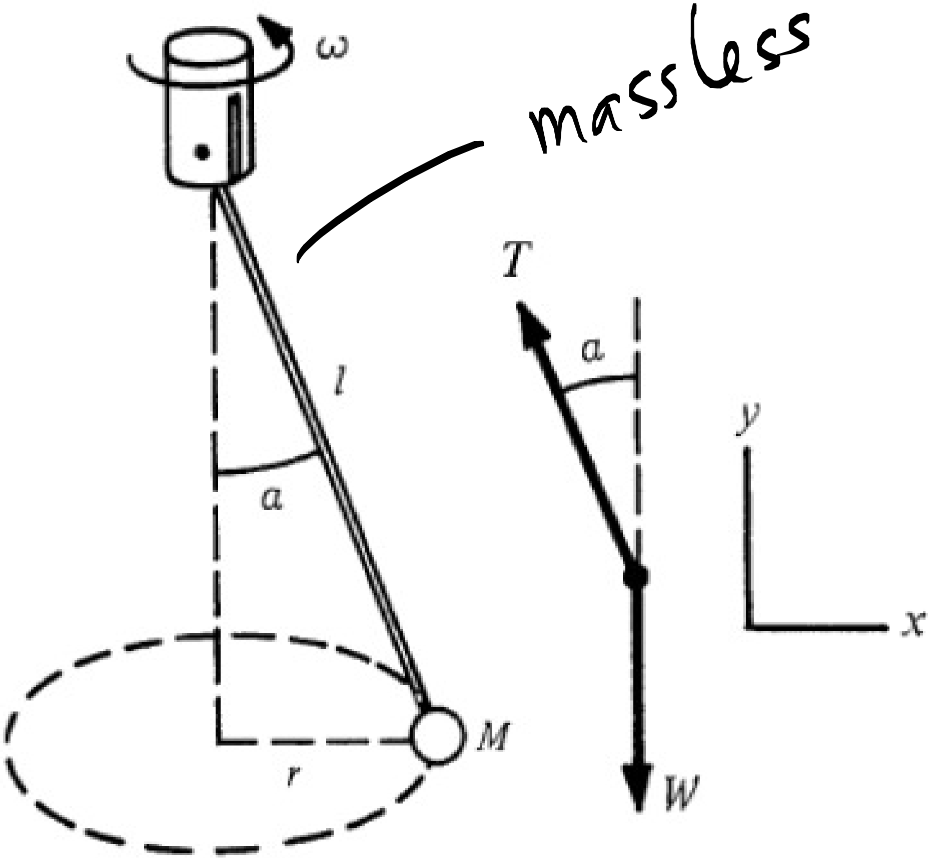
Immediately after B is released

accl^n — changes

Position & angular velocity — No change

$$\ddot{z}(0) = \frac{W_B - M_A r_0 \omega_0^2}{M_A + M_B}$$

Conical Pendulum:



mass moves with steady speed in a circular path of constant radius!

$\alpha = ?$

Identify forces!

$T\cos\alpha - W = 0$ | $r = l\sin\alpha$

$a_r = -\omega^2 r$

$-T\sin\alpha = -M\omega^2 r = -M\omega^2 l\sin\alpha$

$T = M\omega^2 l$

$$M l \omega^2 \cos\alpha = N$$

$$\cos\alpha = \frac{Mg}{M l \omega^2} = \frac{g}{l \omega^2}$$

when $\omega \to \infty$

$\cos\alpha \to 0$

$\alpha \to \pi/2$

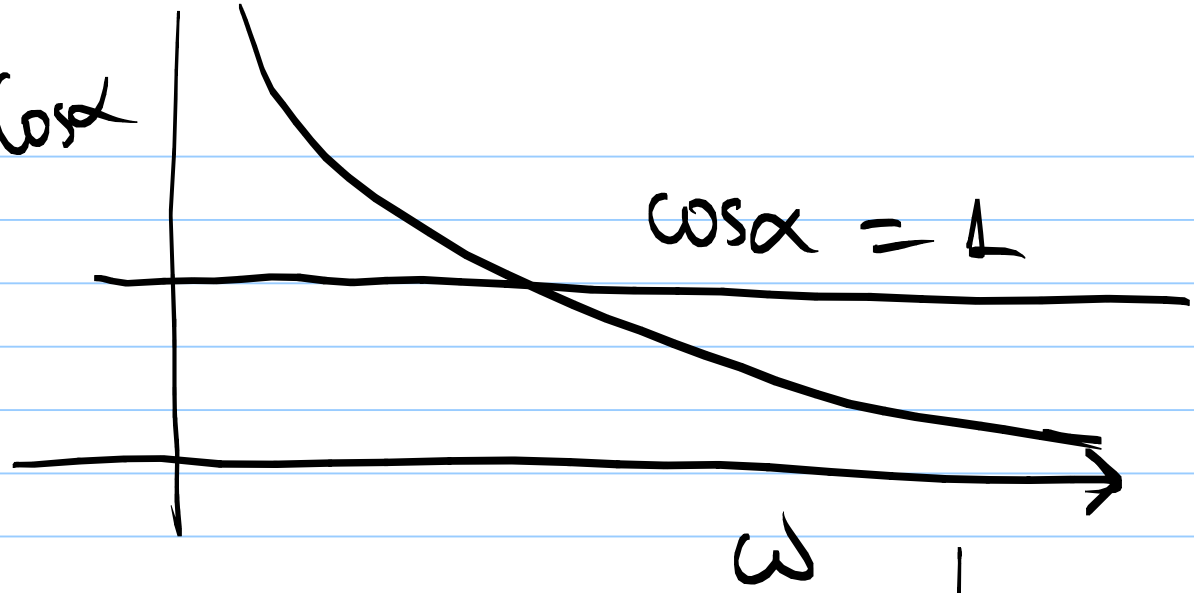
} Makes sense

if $\omega \to 0$

$\cos\alpha \to \infty$

Makes no sense!

Something is wrong!



$$\cos\alpha = \frac{g}{l\omega^2} = 1$$

$$\omega = \sqrt{g/l}$$

$$\sin\alpha = 0$$

$\cos\alpha > 1$ Condition

$$\frac{g}{l\omega^2} > 1 \Rightarrow \omega < \sqrt{\frac{g}{l}}$$

$$r = l\sin\alpha = 0$$

! Bob simply hangs vertically

$$\sin\alpha = 0 \rightarrow r = 0$$

Another solution

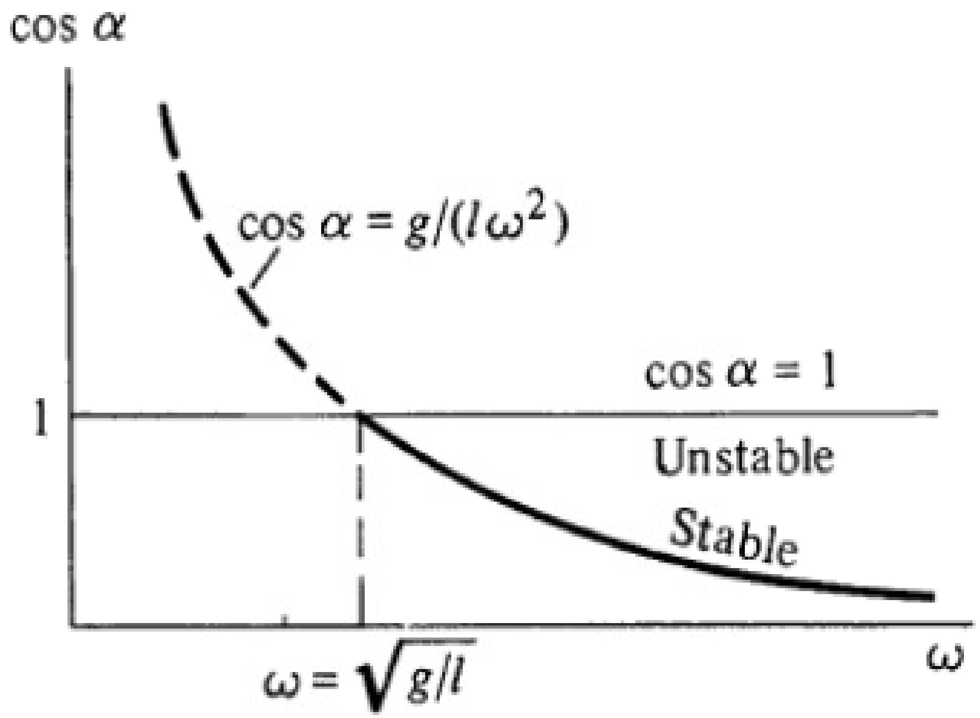
$$T = W$$

True for all ω!

$\omega \le \sqrt{g/l}$ is acceptable

if $\alpha = 0$

For $\omega > \sqrt{g/l}$



Two solutions

$\rightarrow$ $\cos\alpha = 1$ $\rightarrow$ Bob hanging vertically & rotating rapidly

$\rightarrow$ $\cos\alpha = g/l\omega^2$