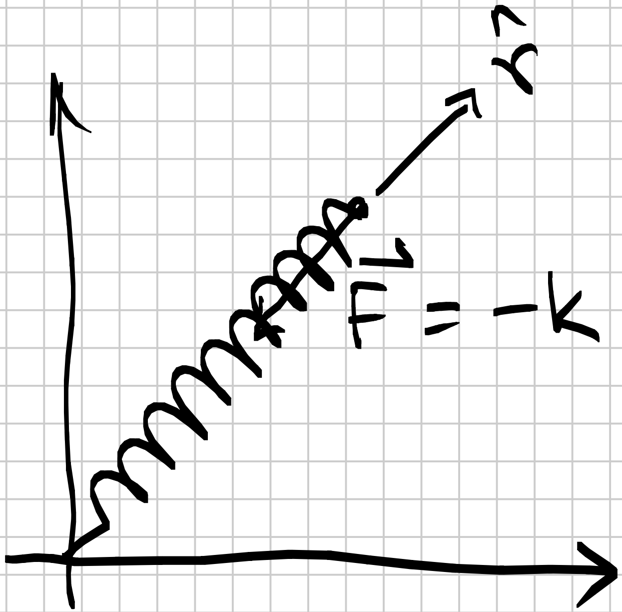
Spring Force!

(Conservative)



$$\vec{F} = -k\,(r - r_0)\,\hat{r}$$

r_0: equilibrium length

(restoring force)

$$\vec{F} = -k\,(r - r_0)\,\hat{r}$$

$$U(r) - U(a) = -\int \vec{F} \cdot d\vec{r}$$

$$U(r) - U(a) = -\int_a^r (-k)\,(r - r_0)\,dr$$

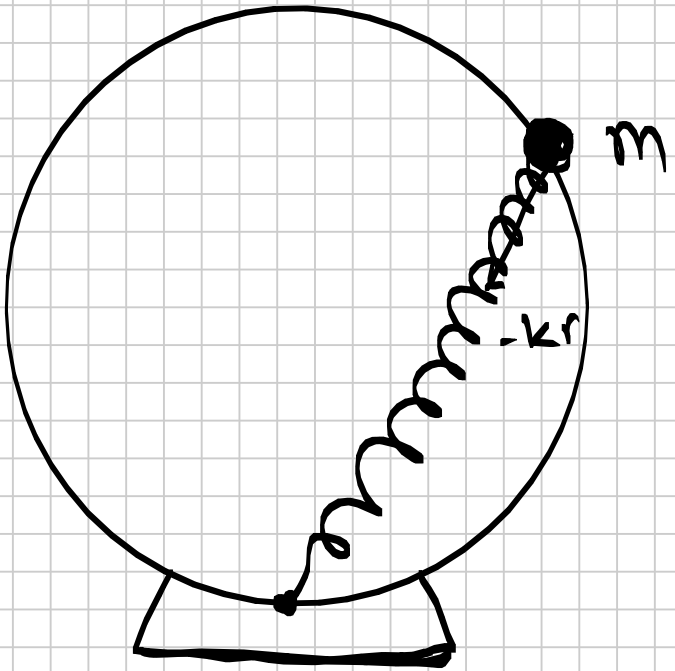
$$= \frac{1}{2} k\,(r - r_0)^2 \Big|_a^r$$

$$U(r) = \frac{1}{2} k (r - r_0)^2 + C \quad (C\text{- Constant})$$

Conventionally $\quad U(r_0) = 0$

$$C = 0$$

$$U(r) = \frac{1}{2} k (r - r_0)^2$$



Consider intial length of Spring 0 (for simplicity)

$\vec{F} = -k\vec{r}$

Release at top!

Potential Energy

→ Gravitational $= mg(2R)$

→ Spring $= \frac{1}{2}k(2R)^2$

$U_i = 2mgR + 2kR^2$

$U_f = 0$ at the bottom!

Conservative force → Mechanical Energy conserved!

$K_i + U_i = K_f + U_f$

$K_f = U_i - U_f$

$K_i = 0$

$$\frac{1}{2} m v_f^2 = 2mgR + 2kR^2$$

$$v_f = 2\sqrt{gR + \frac{kR^2}{m}}$$

→ If force [conservative] is given

Potential Energy can be calculated

$$U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{r}$$

What potential energy tells us about force;

How to find force from Potential Energy

1D

Force $F(x)$

$$U_a - U_b = -\int_{x_a}^{x_b} F(x)\, dx$$

Particle moves from x to $x + \Delta x$

$$U(x+\Delta x) - U(x) = \Delta U = -\int_{x}^{x+\Delta x} F(x)\, dx$$

$\Delta x \to$ small, say F is constant withi Δx

$$\Delta U \approx -F(x)\left(x + \Delta x - x\right) = -F(x)\,\Delta x$$

or $F(x) = -\dfrac{\Delta U}{\Delta x}$

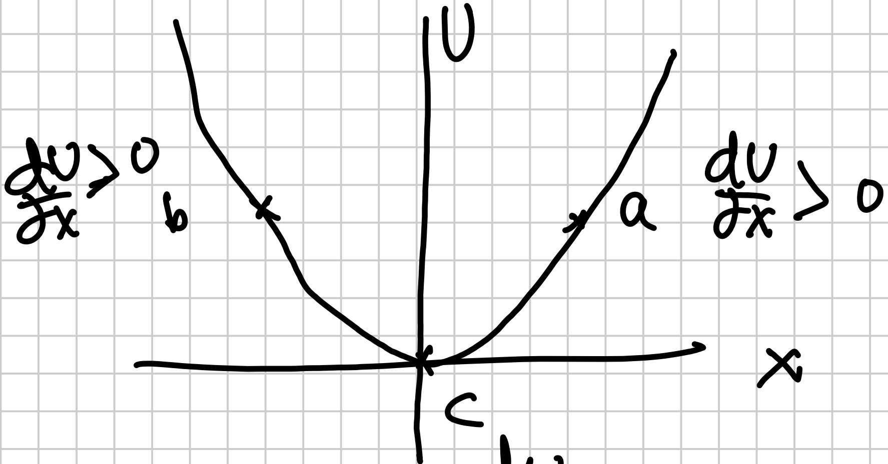
$\Delta x \to 0$ $\quad F(x) = -\dfrac{dU}{dx}$

Force is -ve derivative of potential !

$$F = -\frac{dU}{dx}$$

For harmonic oscillator

$$U = k\frac{x^2}{2} \quad \text{Parabola}$$



$\frac{dU}{dx} = 0$

(most stable)

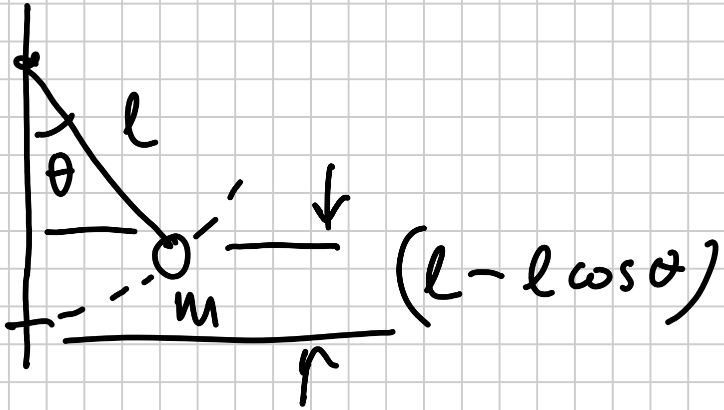
at 'a' $\quad F = -ve$

at 'b' $\quad F = +ve$

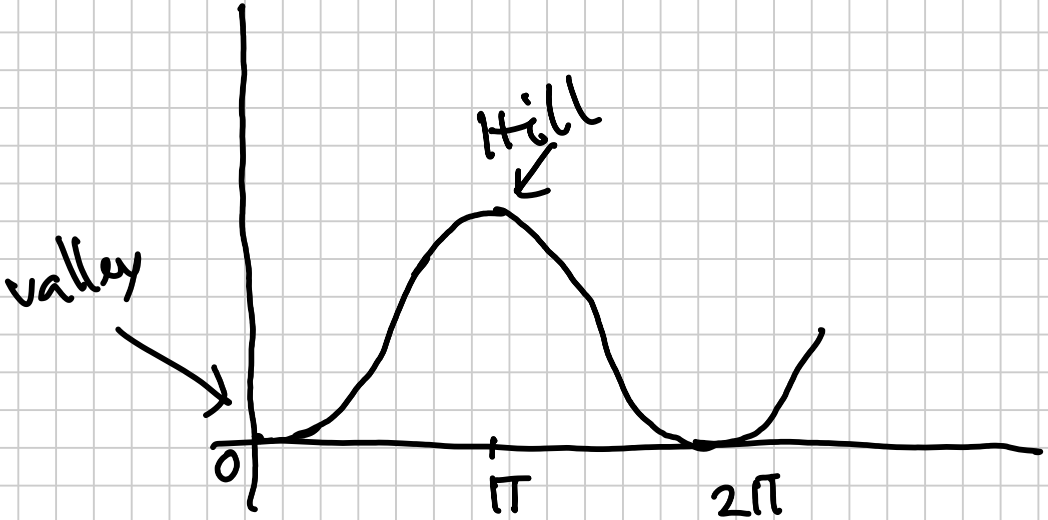
at 'c' $\quad F = 0$

most stable case!

Not always true!



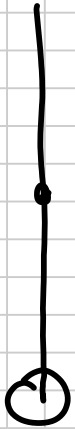
$$U(\theta) = mgz$$
$$= mgl(1 - \cos\theta)$$



$$\frac{dU}{d\theta} = 0 \quad \text{at}$$

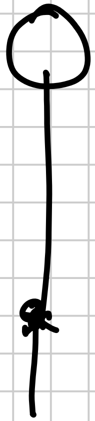
$\theta = 0, \ \pi \to$ Pendulum is equilibrium!

$\theta = 0$



Stable!

$\theta = \pi$



Not stable!

Potential: $U = 0$

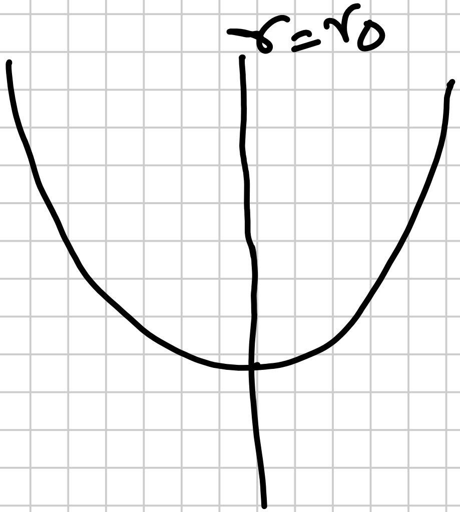
$U = 2mgl$ (maximum)

Mathematically: $\frac{d^2U}{dx^2} > 0$ stable, $\frac{d^2U}{dx^2} < 0$ not stable

Revise Taylor's series !

$f(x)$ can be expanded around $x = x_0$

$$f(x) = f(x_0) + (x - x_0) \left.\frac{df(x)}{dx}\right|_{x=x_0} + \frac{1}{2}(x - x_0)^2 \left.\frac{d^2 f(x)}{dx^2}\right|_{x=x_0} + \cdots$$



$$U(r) = U(r_0) + (r - r_0) \left.\frac{dU}{dr}\right|_{r=r_0} + \frac{1}{2}(r - r_0)^2 \frac{d^2U}{dr^2} + \cdots$$

at $r = r_0$ $\quad \frac{dU}{dr} = 0$ & $\frac{d^3U}{dr^3}$ .. neglect

$$U(r) = U(r_0) + \frac{1}{2}(r - r_0)^2 \left.\frac{d^2U}{dr^2}\right|_{r=r_0}$$

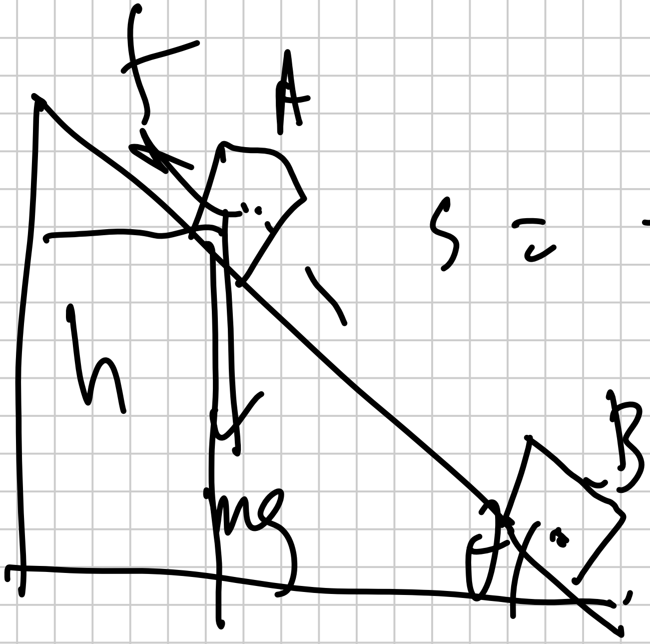
$$U(x) = \text{Constant} + \frac{1}{2} k x^2$$ Harmonic oscillator

Non Conservative force:

$$\vec{F} = \vec{F}^c + \vec{F}^{nc}$$

$a \longrightarrow b$

$$W_{ab}^{total} = \int \vec{F} \cdot d\vec{r} = \underbrace{\int \vec{F}^c \cdot d\vec{r}}_{U_a - U_b} + \underbrace{\int \vec{F}^{nc} \cdot dr}_{W_{ab}^{nc}}$$



$$s = \frac{h}{\sin\theta}$$

$U = 0$

	A	B
	$U_a = mgh$	$U_B = 0$
	$K_a = 0$	$K_b = \frac{1}{2}mv^2$
	$E_a = mgh$	$E_B = \frac{1}{2}mv^2$

$$W_{ab}^{nc} = \int_a^b f \cdot dr = -f \cdot s$$

$$f = \mu mg \cos\theta$$

$$= -\mu \cot\theta \, mgh.$$

$$E_a - E_b = W_{ab}^{nc} \Rightarrow \frac{1}{2}mv^2 - mgh = -\mu \cot\theta \, mgh$$

$$v = [2(1-\mu\cot\theta)gh]^{1/2}$$