De Broglie wavelength $\lambda = \frac{h}{p}$.

h – plank constant $(6.6 \times 10^{-34}\,\mathrm{Js})$

p – momentum

Ex. Calculate de-Broglie wavelength of an electron with a kinetic energy of 1 eV.

$$\frac{p^2}{2m} = E_{kinetic} \Rightarrow p = \sqrt{2mE_k}$$

$$= \sqrt{(2) \times (9.1 \times 10^{-34}\,\mathrm{kg}) \times (1\,\mathrm{eV}) \times (1.6 \times 10^{-19}\,\mathrm{J/eV})}$$

$$= 5.4 \times 10^{-25} \ kg.m/s$$

then $$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34} \ J.s}{5.4 \times 10^{-25} \ kg \ m/s}$$

$$= 1.2 \times 10^{-9} m = 1.2 nm$$

$$p = \sqrt{2m E_k} = \sqrt{\frac{2(mc^2)}{c^2} E_k}$$

$$= \frac{1}{c} \sqrt{2(mc^2) E_k}$$

$$cp = \sqrt{2 \times (5.1 \times 10^5 eV)(1 eV)}$$

$$cp = 1.0 \times 10^3 \text{ eV}$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1240 \text{ eV}\cdot\text{nm}}{1\times10^3 \text{ eV}}$$

$$= 1.2 \text{ nm}$$

Heisenberg Uncertainty principle

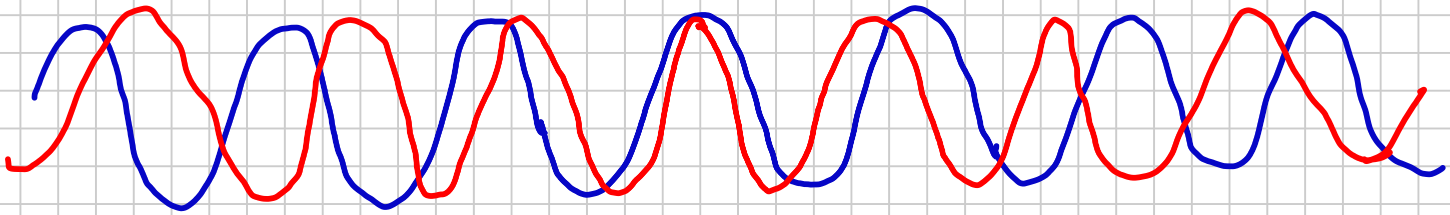
Classical particle

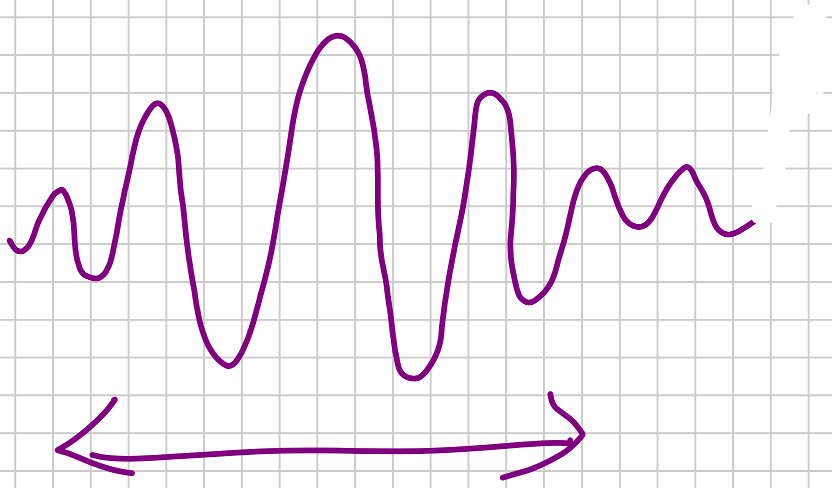
Wave

$$y' = y_1 \sin k_1 x \quad \text{--- (1)}$$

$$y'' = y_2 \sin k_2 x \quad \text{--- (2)}$$

$$y = y' + y'' = y_1 \sin k_1 x + y_2 \sin k_2 x$$





δx

$\delta k = \underline{k_2 - k}$!

take large number of waves!

Single wave $\rightarrow$ $\Delta k = 0$

Δx = infinite!

$\Delta k \rightarrow$ increases

$\Delta x \rightarrow$ decreases

Inverse relation!

$$\Rightarrow \quad \boxed{\Delta x \, \Delta k \sim 1}$$

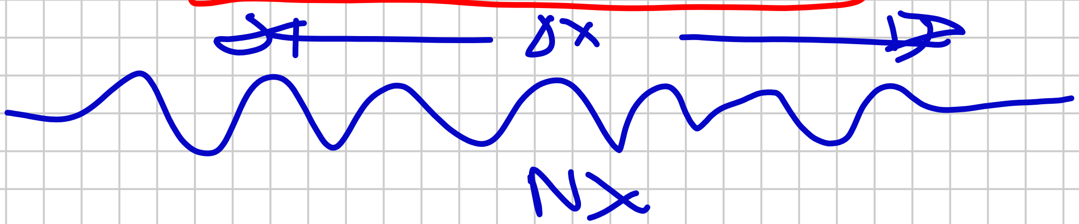
$\Delta x \rightarrow \lambda$

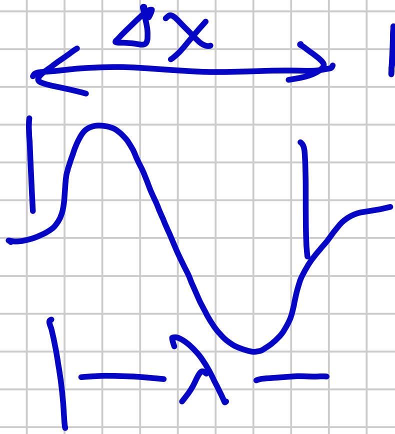
max^m value one can measure!

$$\Delta x \sim \lambda$$

$$\Delta \lambda \sim \lambda$$

$$\boxed{\delta x \, \Delta \lambda \sim \lambda^2}$$





$T \to$ time period

Time interval! $\Delta t \sim NT$ for N cycle!

$\Delta T \sim T/N$

$\Delta T \, \Delta t \sim T^2$

$$\boxed{\Delta \omega \, \Delta t \sim 1}$$

$\omega = \frac{2\pi}{T}$

$\Delta \omega = -\frac{\Delta T \, 2\pi}{T^2}$

$\Delta T \approx -\frac{T^2}{2\pi} \Delta \omega$

$\Delta x \, \Delta p_x \sim \hbar$

$p = \hbar k$

$\Delta k = \Delta p / \hbar$

$E = \hbar \omega \Rightarrow \Delta E = \hbar \Delta \omega$

$$\Delta\omega\,\Delta t \sim 1$$

$$\Rightarrow \quad \Delta E\,\Delta t \sim \hbar$$

$$\boxed{\Delta x\,\Delta p_x \geq \hbar/2}$$

Ex.

$e \rightarrow$ x-direction

$v_e = 3.6 \times 10^{6}$ m/s $\rightarrow$ Error 1%

a) precision of position measurement

b) Motion in y direction

$$p_x = m v_x = (9.11 \times 10^{-31}\ \text{kg})(3.6 \times 10^{6}\ \text{m/s})$$

$$= 3.3 \times 10^{-24}\ \text{kg.m/s}$$

$$\Delta p_x = 1\% = 3.3 \times 10^{-26}\ \text{kg.m/s}$$

$$\Delta x \sim \frac{\hbar}{\Delta p_x} = \frac{1.05 \times 10^{-34}\ \text{J.s}}{3.3 \times 10^{-26}\ \text{kg.m/s}} = 3.2\ \text{nm}$$

(b) $\Delta p_y = 0$ — moving along x-directi

$\delta y \, \Delta p_y \sim \hbar$

$\delta y \sim$ infinite (Nothing can be said about motion along y-direction)

$$y(x,t) = A\cos(k_1 x - \omega_1 t) + A\cos(k_2 x - \omega_2 t)$$

phase velocity

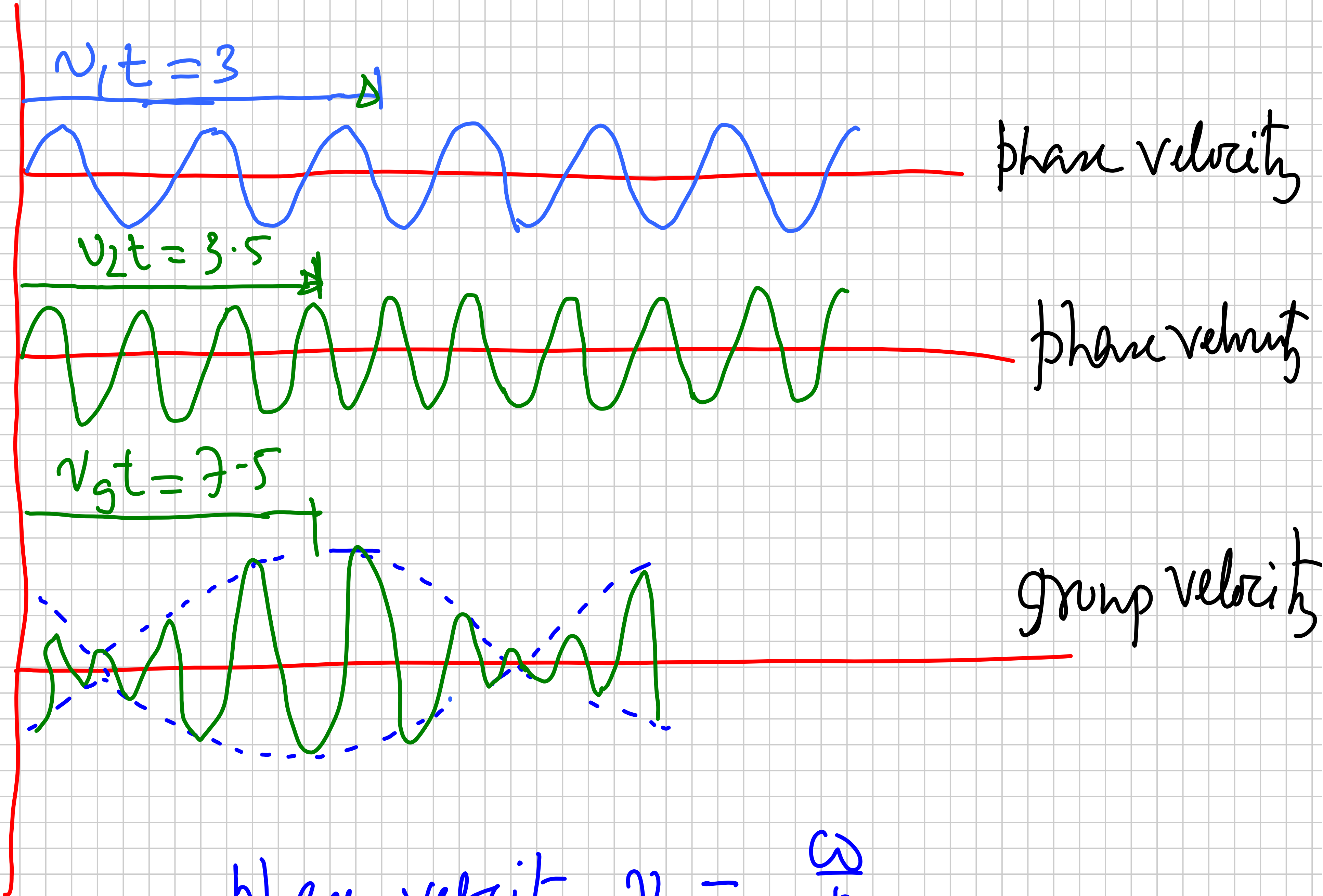
$$\boxed{v_1 = \omega_1 / k_1}$$

$$\boxed{v_2 = \frac{\omega_2}{k_2}}$$

$$y(x,t) = 2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right) \times \cos\left(\frac{k_1+k_2}{2}x - \frac{\omega_1+\omega_2}{2}t\right)$$

$$\Delta\omega = \omega_2 - \omega_1$$

$$\Delta k = k_2 - k_1$$



phase velocity $v_p = \frac{\omega}{k}$

Group velocity $v_g = \frac{d\omega}{dk}$

Particle localized in certain region of space

$\rightarrow$ de Broglie wavelength.

Wave packet $\rightarrow$ large number of waves

$$v_g = \frac{d\omega}{dk}$$

localized particle $\rightarrow$ velocity 'v'

λ - de Broglie wavelength

$$E = h\nu = \hbar\omega$$

$$p = \frac{h}{\lambda} = \hbar k$$

$$v_g = \frac{d\omega}{dk}$$

$$v_g = \frac{d\omega}{dk} = \left(\frac{d\omega}{dE}\right)\left(\frac{dE}{dp}\right)\left(\frac{dp}{dk}\right)$$

$$= \frac{1}{\hbar}\left(\frac{dE}{dp}\right)\hbar = \frac{dE}{dp}$$

$$E = K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\frac{dE}{dp} = \frac{p}{m} = v$$

$$v_{group} = v_{particle}$$

Wave packet and particle move together!

Newton's equation

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$\Rightarrow r(t)$ Trajectory

$\Rightarrow v(t)$

Schrödinger Eq^n (SE)

$\Rightarrow$ Wavefunction of the particle.

$$H\psi = E\psi$$

Can not be derived

$\rightarrow$ Explain Expt. results!

SE

* Energy Conserves $K + U = E$
* Consistent with de Broglie Hypothesis!

$$p = \hbar k \quad , \quad K = \frac{\hbar^2 k^2}{2m}$$

* Solution → $\psi(x)$

→ continious

→ well behaved

→ simple valued

$$\psi(x,t) = A \sin(kx - \omega t)$$

$$\frac{\partial \psi}{\partial x} = Ak \cos(kx - \omega t)$$

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = -Ak^2 \sin(kx - \omega t)$$

$$= -k^2 \psi(x,t)$$

$$E = T + U$$

$$T = E - U = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$k^2 = \frac{2m(E-U)}{\hbar^2}$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{2m(E-U)}{\hbar^2}\psi$$

$$\Rightarrow \boxed{-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + U\psi = E\psi}$$

Momentum of a wave function:

Wave fn describes the particle $\psi(x)$

$\psi(x)$ — normalized!

Probability $P(x) = |\psi(x)|^2$ for any x

$\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}$ — de Broglie wavelength!

$\psi(x) =$ sum of all wave fns. for different states

$$\psi(x) = \sum_p A_p \psi_p(x)$$

$\psi_p(x) \rightarrow$ wave function with definite p

$$= \sum_E A_E \psi_E(x)$$

$$A_p = \int \psi_p^*(x)\, \psi(x)\, dx$$

Sch Eq^n: $-\frac{\hbar^2}{2m} \frac{d^2\psi_E(x)}{dx^2} + V(x)\psi_E(x) = E\psi_E(x)$

Probability: $P(E) = |A_E|^2, \quad A_E = \int \psi_E^*(x)\psi(x)\,dx$

Example

$V(x) = 0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_E(x)}{dx^2} = E\,\psi_E(x)$$

$$\Rightarrow \psi_E''(x) + k^2\,\psi_E(x) = 0$$

$$k^2 = \frac{2mE}{\hbar^2}, \quad E = \frac{\hbar^2 k^2}{2m}$$

$$\psi(x) = A\,e^{ikx} + B\,e^{-ikx}$$

Two states with $p_1 = \hbar k$ and $p_2 = -\hbar k$.

having same energy!

called Degenerate states!

Time dependent wave function!

$$\Psi(x,t) = \int a(k)\, e^{i(kx-\omega t)}\, dk$$

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{x},t) + V(x)\,\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)}$$