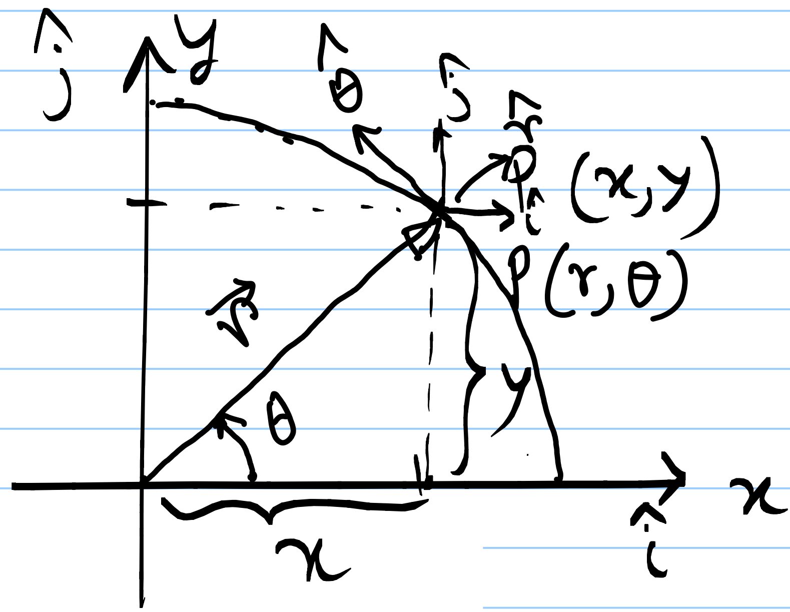
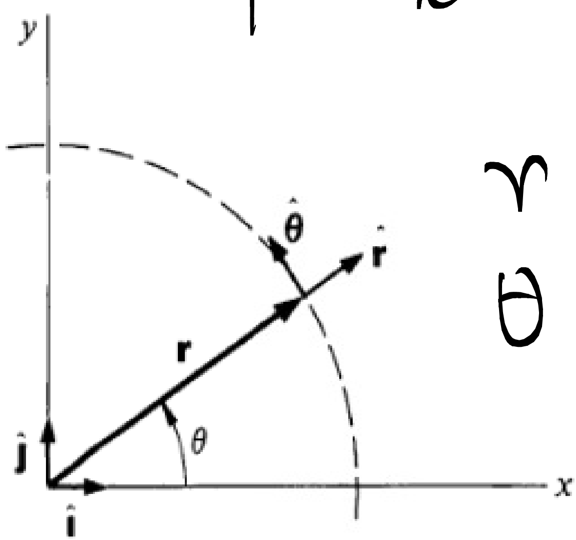
Plane Polar Coordinate system:



$$x = r \cos\theta$$
$$y = r \sin\theta$$

$$\boxed{\begin{array}{l} (x, y) \text{ — Cartesian} \\ (r, \theta) \text{ — Polar} \end{array}}$$

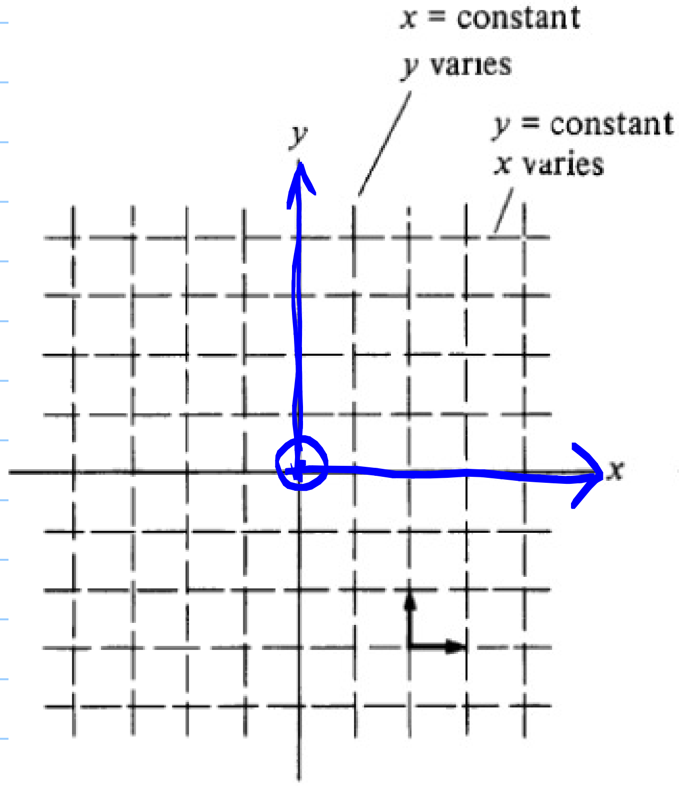


$$r = \sqrt{x^2 + y^2}$$
$$\theta = \arctan\left(y/x\right)$$

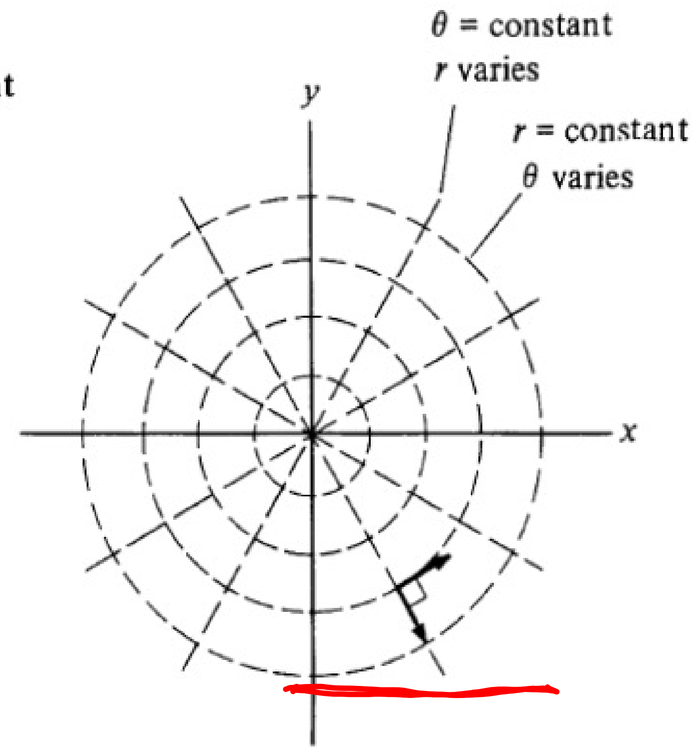
unit vector

$\hat{i}$ & $\hat{j}$

$\hat{r}$ & $\hat{\theta}$

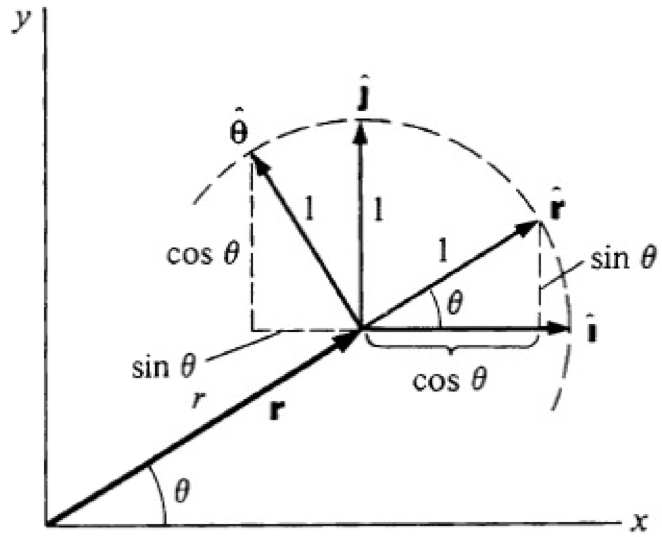


Cartesian



Plane polar

Relation betⁿ $\hat{i}, \hat{j}$ and $\hat{r}, \hat{\theta}$



$$\hat{r} = \hat{i}\cos\theta + \hat{j}\sin\theta$$
$$\hat{\theta} = -\hat{i}\sin\theta + \hat{j}\cos\theta.$$

In cartesian coordinates

$$\vec{r} = x\hat{i} + y\hat{j}$$

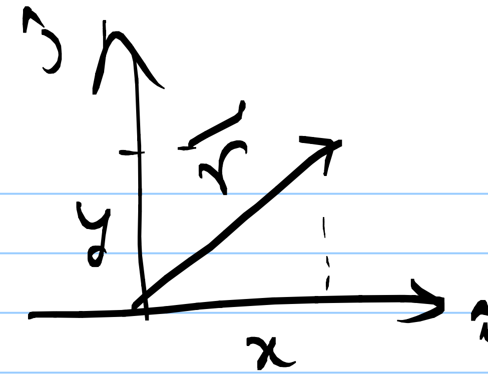
In polar coordinates

$$\vec{r} = r\hat{r}$$

$\hat{r} \cdot \hat{\theta} = 0$ (Orthogonal)

$\hat{r} \times \hat{\theta} = ?$ [Home work]

Velocity in Polar coordinates



$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(x\hat{i} + y\hat{j})$$

$$= \dot{x}\hat{i} + \dot{y}\hat{j}$$

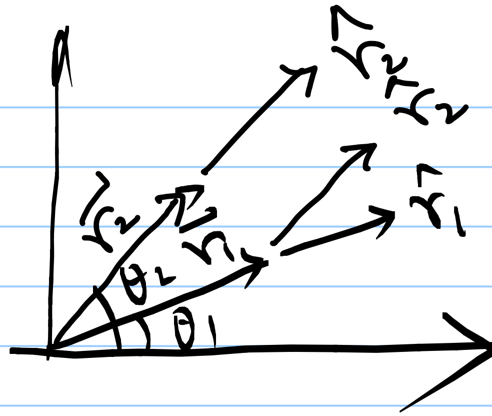
$$\left[\begin{array}{l} \dot{x} = \frac{dx}{dt} \\ \dot{y} = \frac{dy}{dt} \end{array}\right]$$

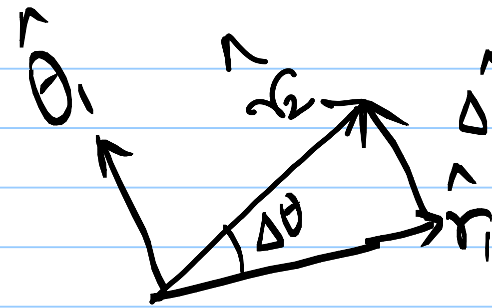
$$\vec{v} = \frac{d}{dt}(r\hat{r})$$

$$= \underbrace{\dot{r}\hat{r}} + r\frac{d\hat{r}}{dt}$$

Component of velocity directed radially outward

$$\frac{d\hat{r}}{dt} = ?$$





$$\Delta\hat{r} = |\hat{r}|\Delta\theta = \Delta\theta$$

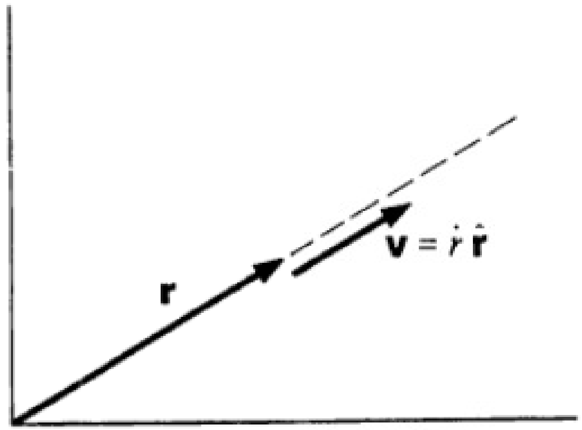
$$\frac{|\Delta\hat{r}|}{\Delta t} \approx \frac{\Delta\theta}{\Delta t} \Rightarrow \left|\frac{d\hat{r}}{dt}\right| = \frac{d\theta}{dt}$$

r swings in the $\hat{\theta}$ direction

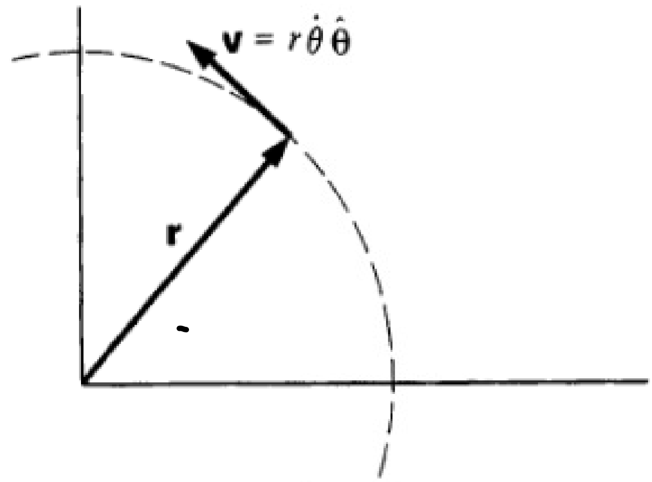
$$\frac{d\hat{r}}{dt} = \dot{\theta}\,\hat{\theta} \qquad \Bigg| \qquad \frac{d\hat{\theta}}{dt} = ? \quad \text{[Home Work]}$$

$$\vec{v} = \underbrace{\dot{r}\,\hat{r}}_{\text{radial part}} + \underbrace{r\dot{\theta}\,\hat{\theta}}_{\text{angular part.}}$$

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$



Case 1



Case 2

Case I   If θ = constant $\rightarrow$ $\vec{v} = \dot{r}\hat{r}$
one dimentional motion in a fixed radial direction.

Case II   if r = constant $\rightarrow$ $v = r\dot{\theta}\hat{\theta}$

Motion in general, both r & θ changes with time !

Acceleration in Polar coordinate System:

$$\mathbf{a} = \frac{d}{dt}\mathbf{v}$$

$$= \frac{d}{dt}(\dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}})$$

$$= \ddot{r}\hat{\mathbf{r}} + \dot{r}\frac{d}{dt}\hat{\mathbf{r}} + \dot{r}\dot{\theta}\hat{\boldsymbol{\theta}} + r\ddot{\theta}\hat{\boldsymbol{\theta}} + r\dot{\theta}\frac{d}{dt}\hat{\boldsymbol{\theta}}.$$

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\boxed{\begin{aligned}\frac{d\hat{\mathbf{r}}}{dt} &= \dot{\theta}\hat{\boldsymbol{\theta}} \\ \frac{d\hat{\boldsymbol{\theta}}}{dt} &= -\dot{\theta}\hat{\mathbf{r}}.\end{aligned}}$$

$$\mathbf{a} = \ddot{r}\hat{\mathbf{r}} + \dot{r}\dot{\theta}\hat{\boldsymbol{\theta}} + \dot{r}\dot{\theta}\hat{\boldsymbol{\theta}} + r\ddot{\theta}\hat{\boldsymbol{\theta}} - r\dot{\theta}^2\hat{\mathbf{r}}$$

$$= (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\boldsymbol{\theta}}.$$

$\left.\begin{array}{l}\ddot{r} \to \text{radial acc}l^n. \\ r\ddot{\theta} \to \text{tangential acc}l^n.\end{array}\right\}$ we discussed!

$\left.\begin{array}{l}-r\dot{\theta}^2 \to \text{Centripetal acc}l^n \\ 2\dot{r}\dot{\theta} \to \text{Coriolis acc}l^n.\end{array}\right\}$ We will discuss!