Lecture - 13

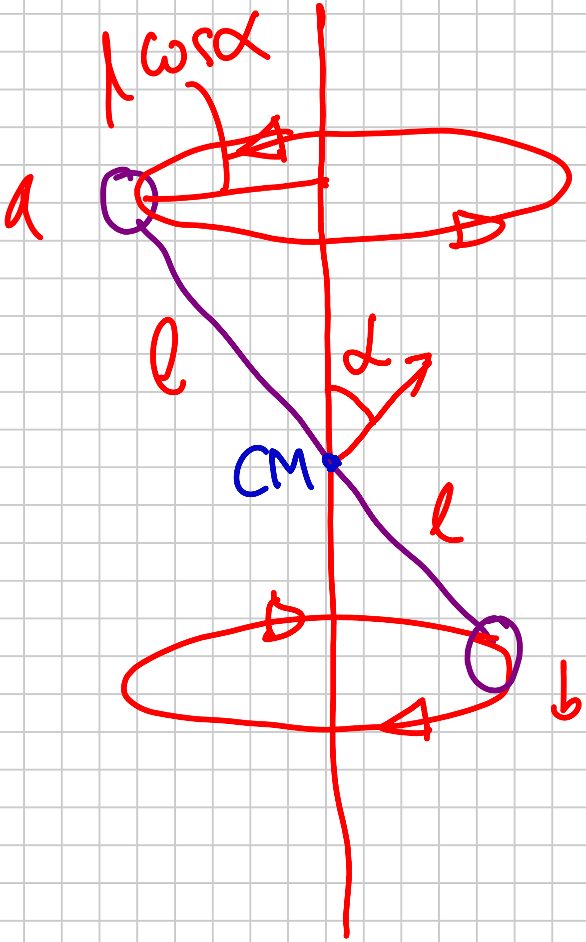
Note Title 27-Sep-12

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

$$\vec{\omega} = \omega \hat{k}$$

$$\vec{v} = \omega r \hat{\theta}$$

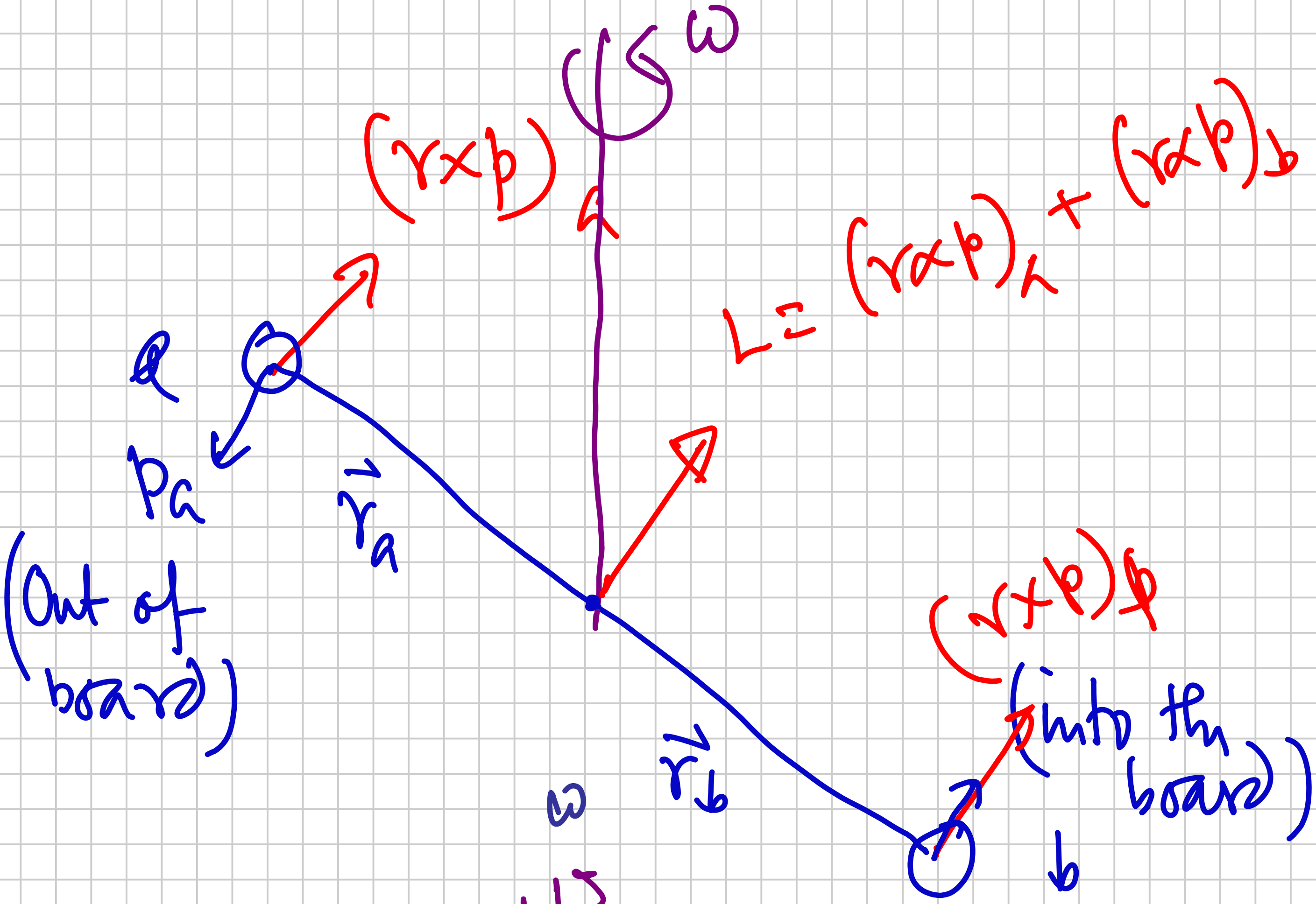


$$\vec{L} = \sum \vec{r}_j \times \vec{p}_j$$

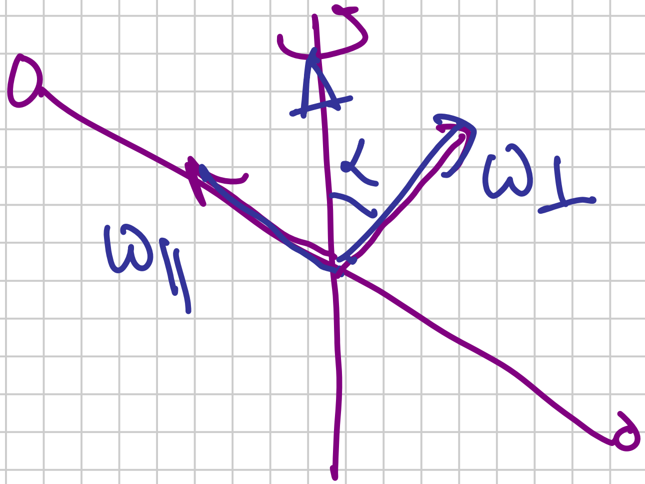
$$|\vec{r}| = l$$

$$|\vec{p}| = m \omega l \cos\alpha$$

$$|\vec{L}| = 2 m \omega l^2 \cos\alpha$$



$$L = (r \times p)_a + (r \times p)_b$$

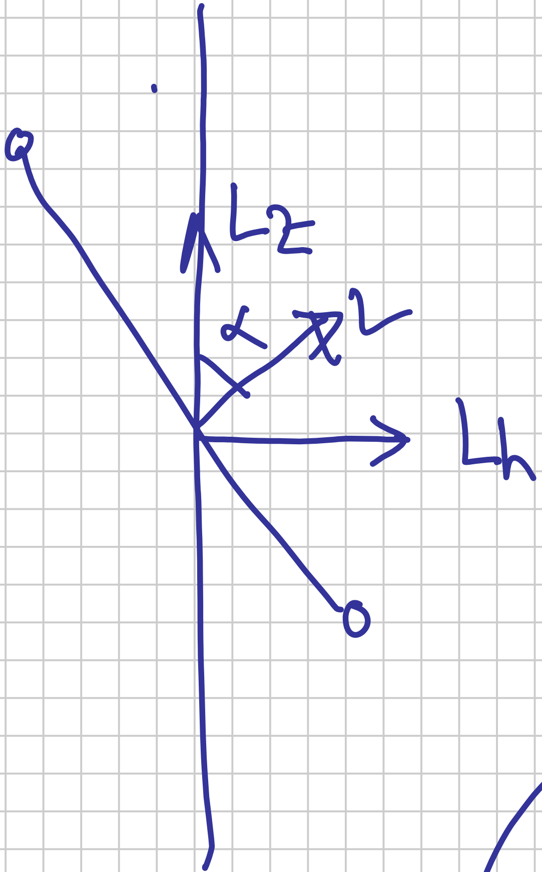


$$L = I\,\omega_\perp$$
$$= 2ml^2\omega_\perp$$
$$= 2ml^2\omega\cos\alpha$$

Torque on the rotating rod.

$$\bar{\tau} = \frac{d\bar{L}}{dt}.$$

$L \rightarrow$ changing

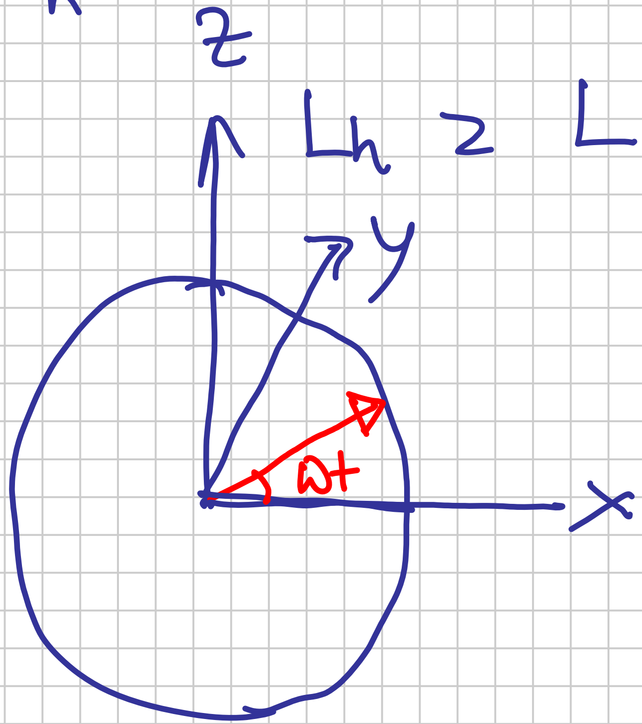


$$L_z = L\cos\alpha$$

$= \text{constant}$

$= \text{No torque}$

$$L_h = L\sin\alpha$$



$$L_x = L_h \cos\omega t$$

$$= L\sin\alpha\cos\omega t$$

$$L_y = L_h \sin\omega t$$

$$= L\sin\alpha\sin\omega t$$

$$\vec{L} = L \sin\alpha \left[\hat{i} \cos\omega t + \hat{j} \sin\omega t \right] + L \cos\alpha \, \hat{k}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} = L\omega \sin\alpha \left[-\hat{i} \sin\omega t + \hat{j} \cos\omega t \right]$$

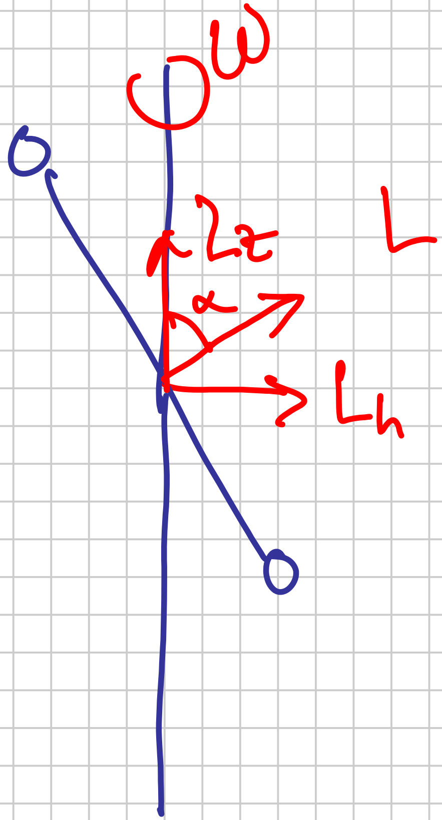
$$L = 2ml^2 \omega \cos\alpha$$

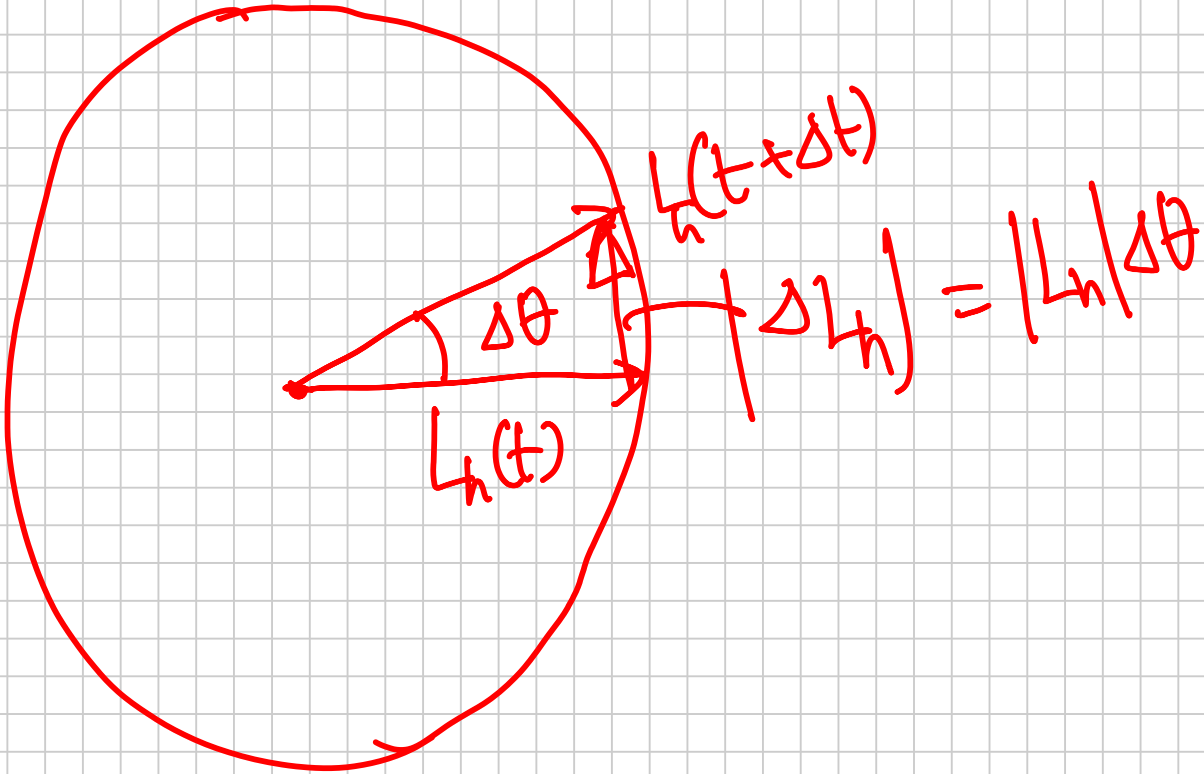
$$\tau_x = -2ml^2\omega^2 \sin\alpha \cos\alpha \sin\omega t$$

$$\tau_y = 2ml^2\omega^2 \sin\alpha \cos\alpha \cos\omega t$$

$$\tau = \sqrt{\tau_x^2 + \tau_y^2} = \omega L \sin\alpha .$$

when $\alpha = 0, \pi/2$ [?]

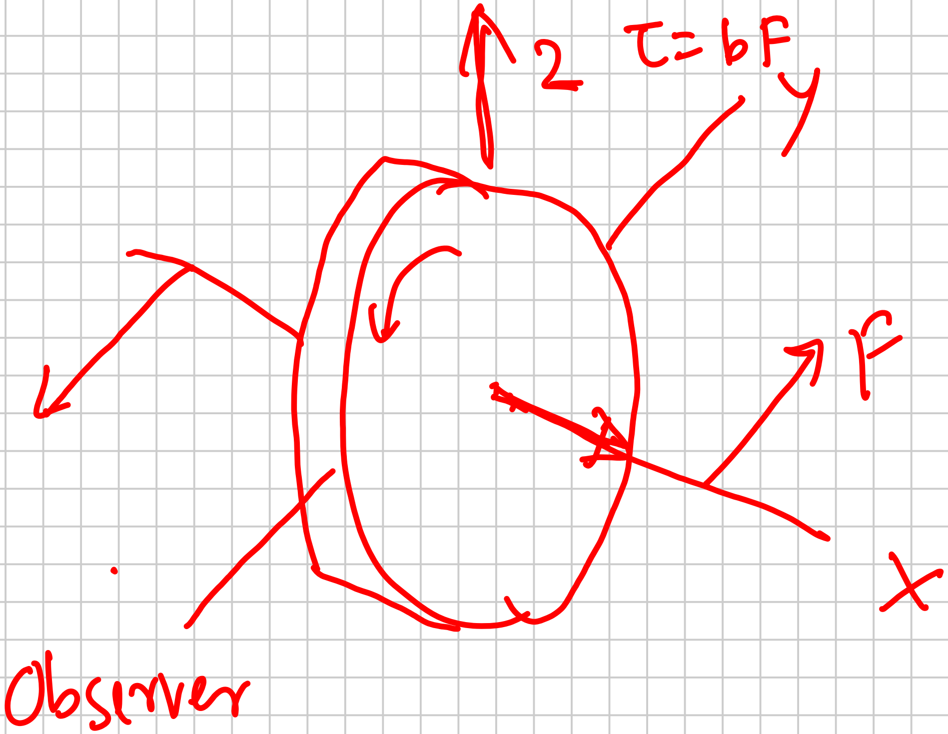


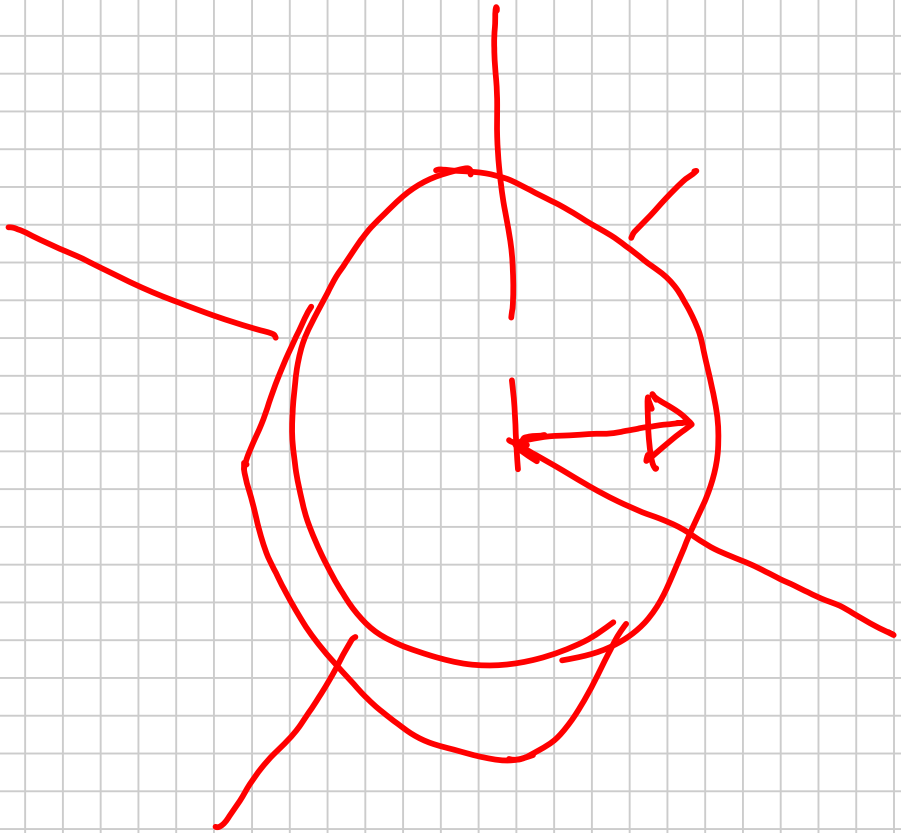


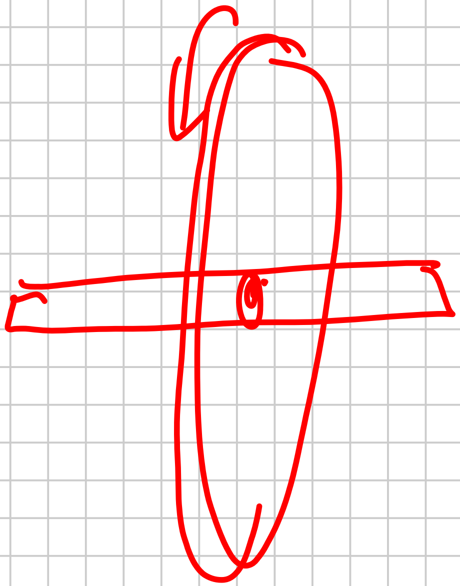
$$|\Delta \vec{L_h}| = |L_h|\,\Delta\theta$$

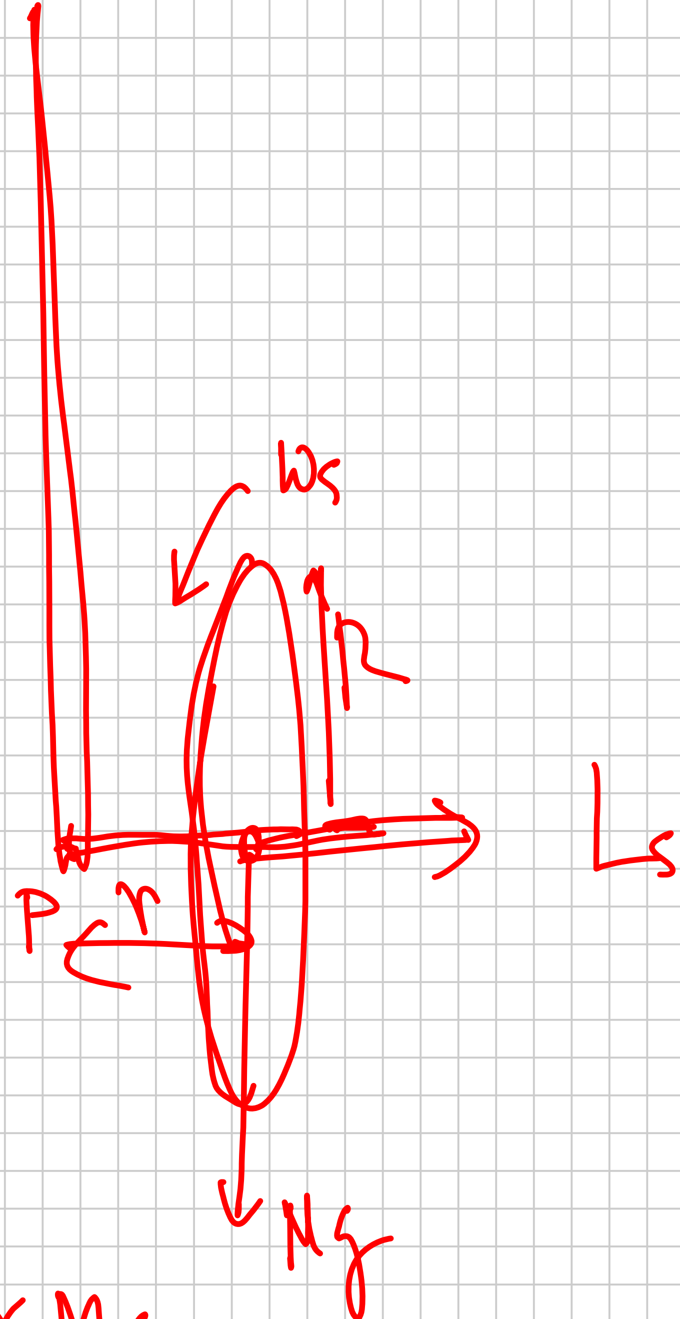
$$\tau = \frac{dL_h}{dt} = L_h \frac{d\theta}{dt} = L_h\,\omega_{pr}$$

$$\tau = L_h\,\omega_{pr} \Rightarrow \omega_{pr} = \frac{\tau}{L_h}$$


z
τ = bF
y
r
x
Observer







$$\tau_p = r M g$$

$$\omega_{pr} = \frac{\tau}{L_S} = \frac{r M g}{I \omega_s}$$

$r = 17$ cm

$R = 29$ cm

$I = MR^2$

$f_s = 5$ Hz

$\omega_{pr} =$

$\omega_s = 2\pi f_s$

$$\omega_{pr} = \frac{rMg}{MR^2 \omega_s} = \frac{rg}{R^2 \omega_s}$$

$$T = \frac{2\pi}{\omega_p} = 10 \text{ sec}$$

2 - 3 PM