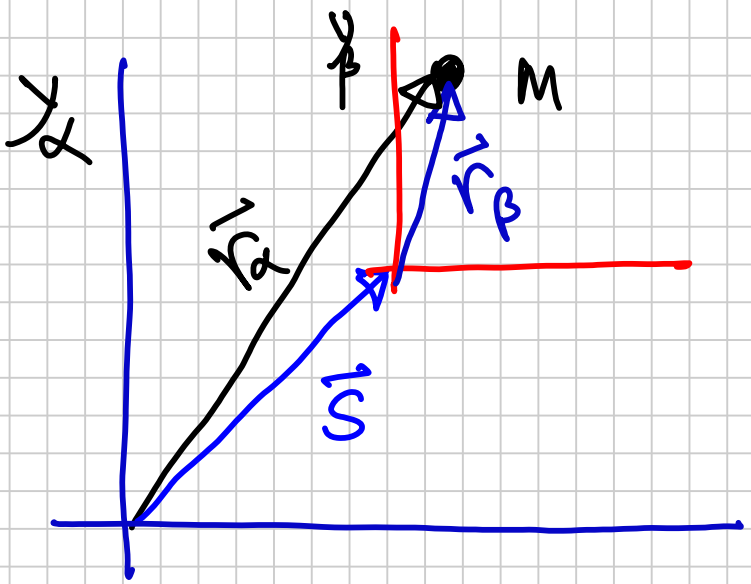
Non inertial systems and ficticious forces

$\vec{F} = m\vec{a}$ — Holds in inertial coordinate system!

Inertial frame!

'A coordinate system moving uniformly with respect to an inertial frame is inertial'



α, β — relative motion!

No rotation!

$$\vec{r}_\beta = \vec{r}_\alpha - \vec{s}$$

α:

$$\vec{a}_\alpha = \ddot{\vec{r}}_\alpha \rightarrow \vec{F}_\alpha = m\vec{a}_\alpha$$

β:

$$\vec{F}_\beta = m\vec{a}_\beta$$

What is the relation between $\vec{F}_\alpha$ & $\vec{F}_\beta$

$$\vec{r}_\beta = \vec{r}_\alpha - \vec{s} \quad \Rightarrow \quad \vec{v}_\beta = \vec{v}_\alpha - \vec{v}$$

$$\vec{a}_\beta = \vec{a}_\alpha - \vec{A}$$

$$\vec{v} = \dot{\vec{s}} \quad \text{—} \quad \text{Constant}$$

$$\Rightarrow \quad \vec{A} = 0 \quad \Rightarrow \quad \vec{a}_\beta = \vec{a}_\alpha$$

$$\text{ie,} \quad \vec{F}_\beta = m\vec{a}_\beta = m\vec{a}_\alpha = \vec{F}_\alpha$$

Force in both system is same!

Uniformly accelerating system:

Observer in a system accelerating at $\vec{A}$ w.r.t. an inertial ref. frame!

$$\vec{a}' = \vec{a} - \vec{A}$$

$$\vec{F}' = m\vec{a}' = m\vec{a} - m\vec{A}$$

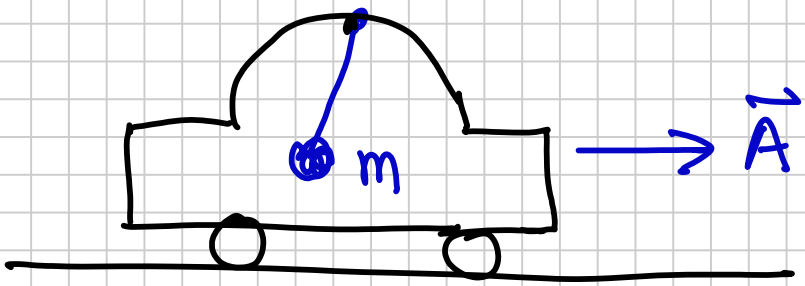
$m\vec{a} = \vec{F}$ true force!

$$\vec{F}' = \vec{F} - m\vec{A} \rightarrow \vec{F}_{fic} = -m\vec{A}$$

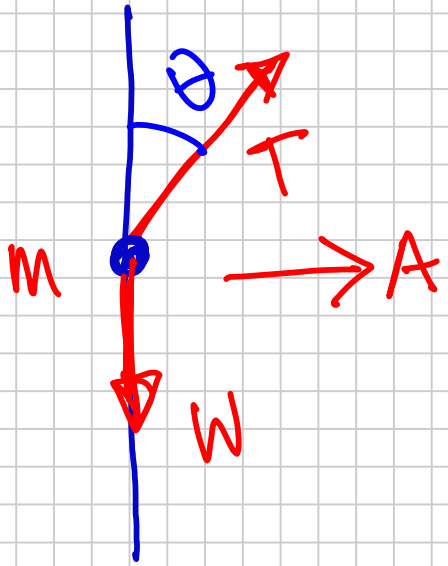
$$= \vec{F} - \vec{F}_{fic}$$

Fictitious force!

Example



Inertial system



$$T\cos\theta - W = 0$$

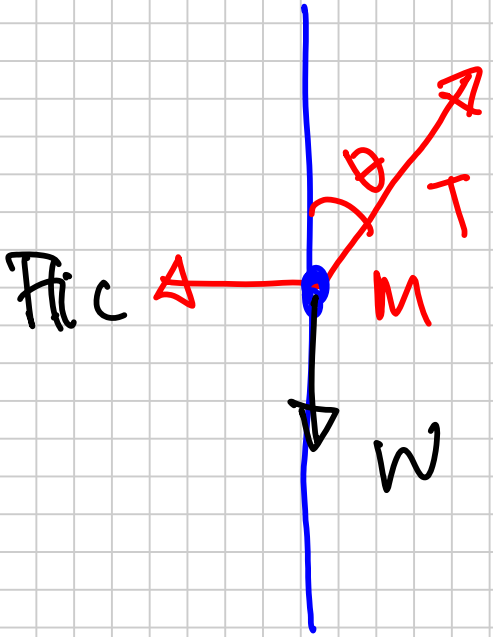
$$T\sin\theta = MA$$

$$\tan\theta = \frac{MA}{W} = \frac{A}{g}$$

$$T = M\left(g^2 + A^2\right)^{\frac{1}{2}}$$

System accelerating with a

[observer inside the car]



$$T\cos\theta = W$$

$$T\sin\theta - F_{fic} = 0$$

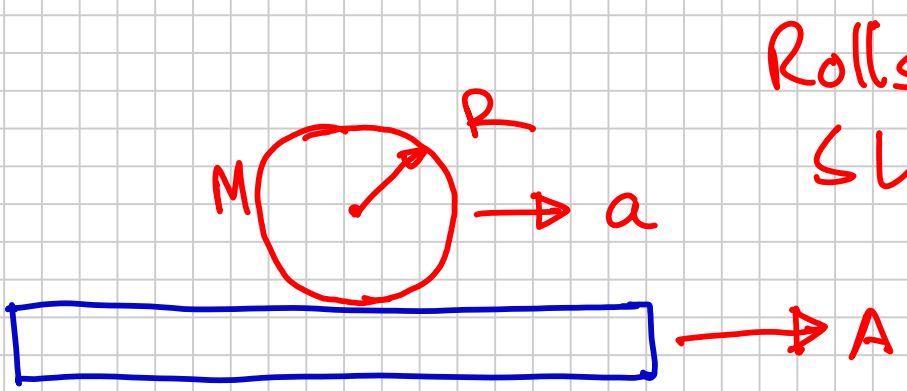
$$\vec{F}_{fic} = -MA$$

$$\tan\theta = \frac{A}{g}$$

$$T = M\left(g^2 + A^2\right)^{\frac{1}{2}}$$

F_{fict} → inertial force!

Cylinder on an accelerating Plank:



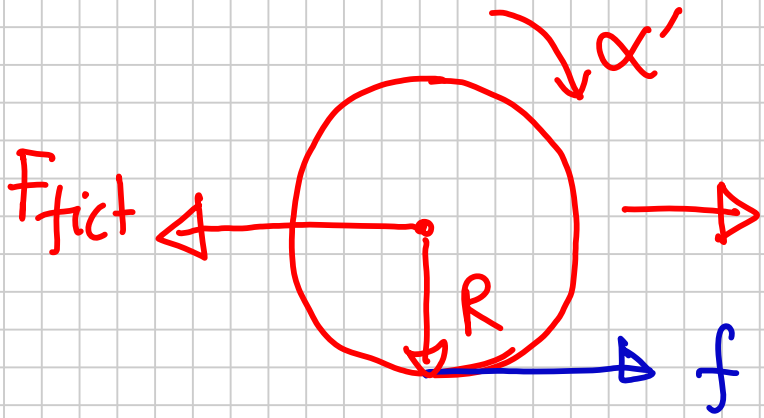
Rolls without slipping

$a = ?$

a' – accl^n of the cylinder as observed in a system fixed to plank

$f \rightarrow$ friction force

$F_{fict} = -MA$



$$f - F_{fict} = Ma'$$

$$Rf = -I_0 \alpha'$$

$$\alpha = \dot{\omega} = \frac{d\omega}{dt} = \frac{d\, V/r}{dt} = \frac{1}{r}\frac{dV}{dt}$$

Cylinder rolls without slipping!

$$\alpha' R = a'$$

$$Ma' = -\frac{I_0 \alpha'}{R} - F_{fict}$$

$$\Rightarrow a' = -\frac{F_{fict}}{M + I_0/R^2}$$

Now, $I_0 = MR^2/2$ & $F_{fict} = MA \Rightarrow a' = -\frac{2}{3}A$

$$\Rightarrow a = A + a' = \frac{1}{3}A$$

[inertial frame]