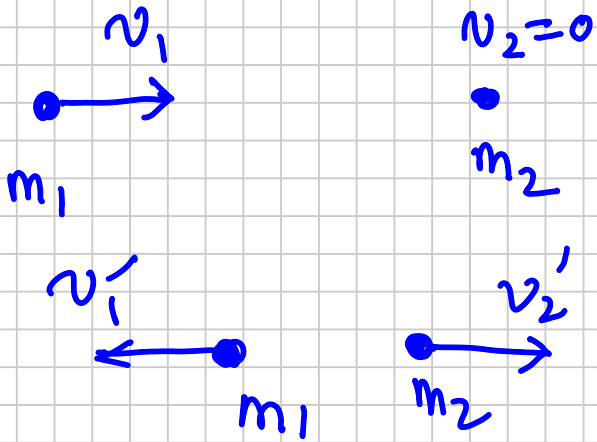
PH 101 Lecture 09 DKG

Note Title 28-Aug-12

Collision :

1D



Before collision

After collision

No external force !

momentum conservation :

$$m_1 v_1 = m_1 v_1' + m_2 v_2' \quad \text{--- (1)}$$

Energy Considerations

$$KE + Q = KE' \quad \text{--- (2)}$$

$Q > 0 \rightarrow$ Superelastic collision ; $Q = 0 \rightarrow$ Eelastic collision

$Q < 0 \rightarrow$ inelastic collision

Elastic collision:

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \quad \text{--- (3)}$$

Solving!

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \Rightarrow \text{! Can be +ve or -ve}$$

$$v_2' = \left(\frac{2 m_1}{m_1 + m_2}\right) v_1 \Rightarrow \text{always +ve !}$$

Case I

$m_1 \gg m_2$ ie, $m_2 \to 0$

$v_1' = v_1 \longrightarrow$ Expected

$v_2' = 2v_1 \longrightarrow$ not obvious atall

Case II

$m_1 \ll m_2$ ie $m_1 \to 0$

$v_1' = -v_1$ & $v_2' = 0$

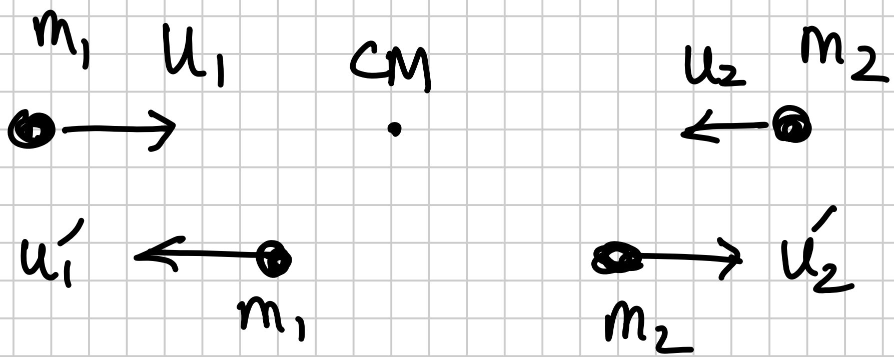
Case III

$$m_1 = m_2$$

$$v_1' = 0 \quad , \quad v_2' = v_1$$

CENTER OF MASS !

Total Momentum at CM is always zero !



BEFORE Collision

AFTER Collision

u_1 & u_2 in CM frame and $Q = 0$ [Elastic]

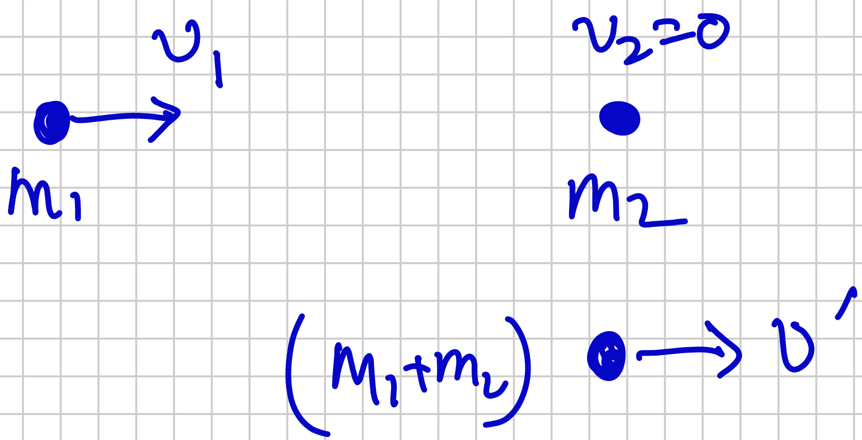
$$m_1 u_1' + m_2 u_2' = 0 \quad \text{— (1)}$$

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 u_1'^2 + \frac{1}{2} m_2 u_2'^2 \quad \text{— (2)}$$

Solving

$$u_1' = -u_1$$
$$u_2' = -u_2$$

Completely enelastic collision! Total KE = 0



LAB FRAME

look at the CM.!

$$M_{tot}\,\vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2$$

$$v_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{(m_1+m_2)}$$

and $\vec{u}_1 = \vec{v}_1 - \vec{v}_{CM}$

$\vec{u}_2 = v_2 - \vec{v}_{CM}$

CM frame
total mom = 0

If no external force $\Rightarrow$ momentum conserved!

$$m_1 v_1 = (m_1 + m_2) v'$$

$$\Rightarrow \quad v' = \frac{m_1 v_1}{(m_1 + m_2)} = v_{CM}$$

$$Q = KE' - KE = \frac{1}{2} m_1 v_1^2 - \frac{1}{2}(m_1 + m_2) \cdot \frac{m_1^2 v_1^2}{(m_1 + m_2)^2}$$

$$= -\frac{1}{2} m_1 v_1^2 + \frac{1}{2} \frac{m_1^2}{(m_1 + m_2)} v_1^2$$

$$= -\frac{1}{2} \left[\frac{m_1^2 - m_1 m_2 + m_1^2}{m_1 + m_2} \right] v_1^2$$

$$= -\frac{1}{2}\left[\frac{m_1 m_2}{(m_1+m_2)}\right] v_1^2$$

We loose KE !

CM frame !

$$u_1 = v_1 - v_{cm} = \left(\frac{m_2}{m_1+m_2}\right) v_1$$

$$u_2 = \cancel{v_2}^{\,0} - v_{cm} = -\frac{m_1}{(m_1+m_2)} v_1$$

$$K.E. = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

$$= \frac{1}{2}\left(\frac{m_1 m_2}{m_1+m_2}\right) v_1^2 \qquad \text{Internal K.E.}$$

Let $m_2 \to \infty$

$$K.E. = \frac{1}{2} m_1 v_1^2$$

Gradient Operators:

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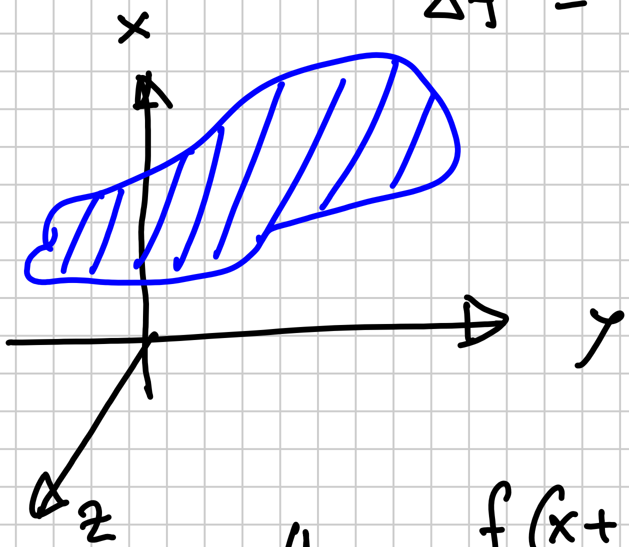
$f \rightarrow$ function $= f(x)$

$$\Delta f = \frac{df}{dx} \Delta x + \cdots$$



$f(x,y)$ — function

calculate Δf ?

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x} = \left.\frac{\partial f}{\partial x}\right|_y$$

$$\lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y} = \left.\frac{\partial f}{\partial y}\right|_x$$

Partial derivative

$$\delta f = \left.\frac{\partial f}{\partial x}\right|_y \Delta x + \left.\frac{\partial f}{\partial y}\right|_x \Delta y$$

$\Delta x \to 0$
$\Delta y \to 0$

$$df = \frac{\partial f(x,y)}{\partial x}\,dx + \frac{\partial f(x,y)}{\partial y}\,dy$$

Consider

$$U_b - U_a = -\int_{r_a}^{r_b} \vec{F}\cdot d\vec{r}$$

$$\Delta U = -\vec{F}\cdot\Delta\vec{r} \leftarrow \text{in 3D derived!}$$

$$= -\left(F_x \Delta x + F_y \Delta y + F_z \Delta z\right)$$

$$\delta U = \frac{\partial U}{\partial x}\Delta x + \frac{\partial U}{\partial y}\Delta y + \frac{\partial U}{\partial z}\Delta z = -F_x\Delta x - F_y \Delta y - F_z \Delta z$$

$$\frac{\partial U}{\partial x} = -F_x \qquad \frac{\partial U}{\partial y} = -F_y \qquad \frac{\partial U}{\partial z} = -F_z$$

$$\vec{F} = \hat{i}\,F_x + \hat{j}\,F_y + \hat{k}\,F_z$$

$$= -i\frac{\partial U}{\partial x} - \hat{j}\frac{\partial U}{\partial y} - \hat{k}\frac{\partial U}{\partial z}$$

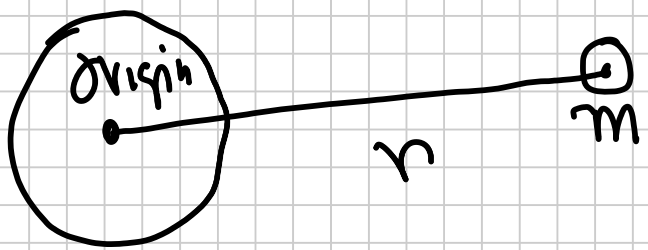
$$= -\vec{\nabla} U$$

where $\vec{\nabla} U = \frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$ [we Defined]

$\vec{\nabla} \rightarrow$ gradient operator / not a vector! vector operator

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Example:



mass M

Potential energy of mass m a distance r from origin

$$U(x, y, z) = -\frac{GMm}{r} \quad \text{[attractive]}$$

Then $\quad \vec{F} = -\vec{\nabla} U = +GMm\,\vec{\nabla}\left(\frac{1}{r}\right)$

$$r = \sqrt{x^2+y^2+z^2}$$

$$\frac{\partial}{\partial x}\left[\frac{1}{(x^2+y^2+z^2)^{1/2}}\right] = \frac{-x}{\left[(x^2+y^2+z^2)\right]^{3/2}} = -\frac{x}{r^3}$$

$$\vec{F} = GMm\left[\hat{i}\,\frac{-x}{r^3} + \hat{j}\,\frac{-y}{r^3} + \hat{k}\,\frac{-z}{r^3}\right]$$

$$= \frac{GMm}{r^3}\left[-\vec{r}\right] = -\frac{GMm}{r^2}\,\hat{r}$$

Force of gravity between two particles !

Uniform gravitational field

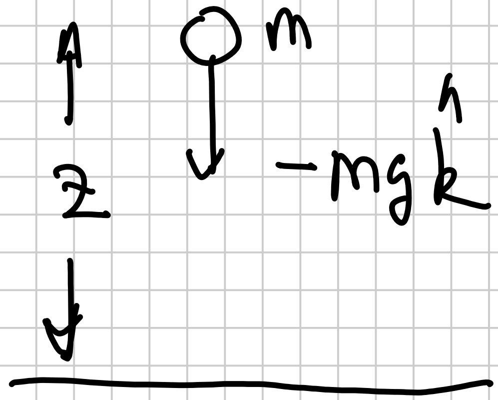
$$U(x,y,z) = mgz$$

$$\vec{F} = -\nabla U$$

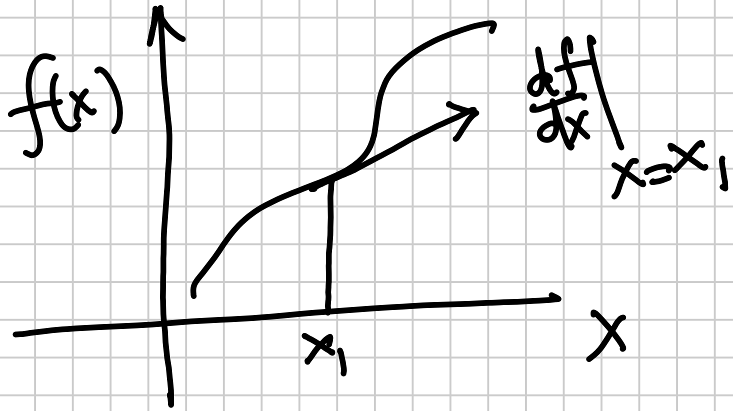
$$= -mg\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$$

$$= -mg\hat{k}$$

Force under gravity!



What is gradient?



Potential energy $U(x, y, z)$ | Conservative force!

Change in a function induced by the change in the variable

$\circledast$ (x, y, z)

$U(x, y, z)$

$\circledast$ $(x+\Delta x, y+\Delta y, z+\Delta z)$

$U(\cdot \; \cdots)$

$\Delta x \to 0$, $\Delta y \to 0$, $\Delta z \to 0$

$$dU = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy + \frac{\partial U}{\partial z} dz$$

$$dU = \vec{\nabla} U \cdot d\vec{r}$$

$$d\vec{r} = dx\, \hat{i} + dy\, \hat{j} + dz\, \hat{k}$$

$$\vec{\nabla} U = \frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}$$

Constant Energy Surface!

$$U(x,y,z) = \text{constant} = c$$

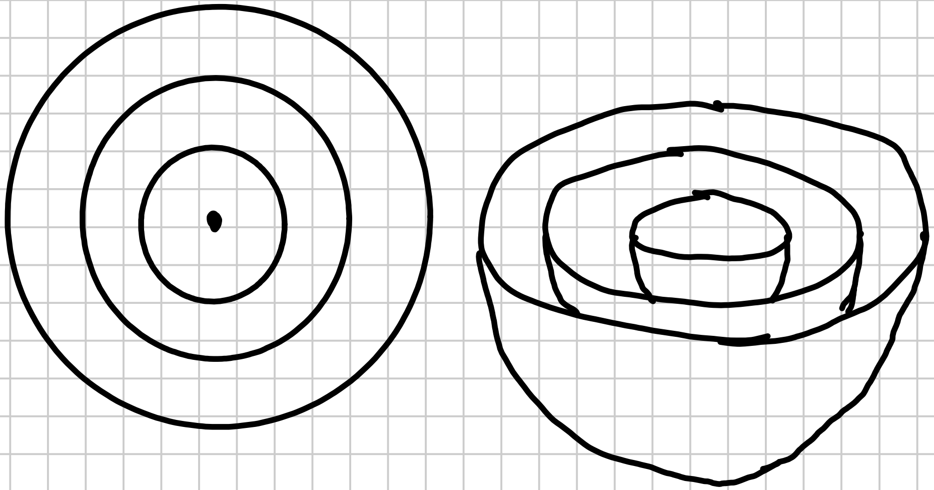
! Constant Energy Surface

Gravitational P.E. $U = -\frac{GMm}{r}$

$$-\frac{GMm}{r} = c \Rightarrow r = -\frac{GMm}{c}$$

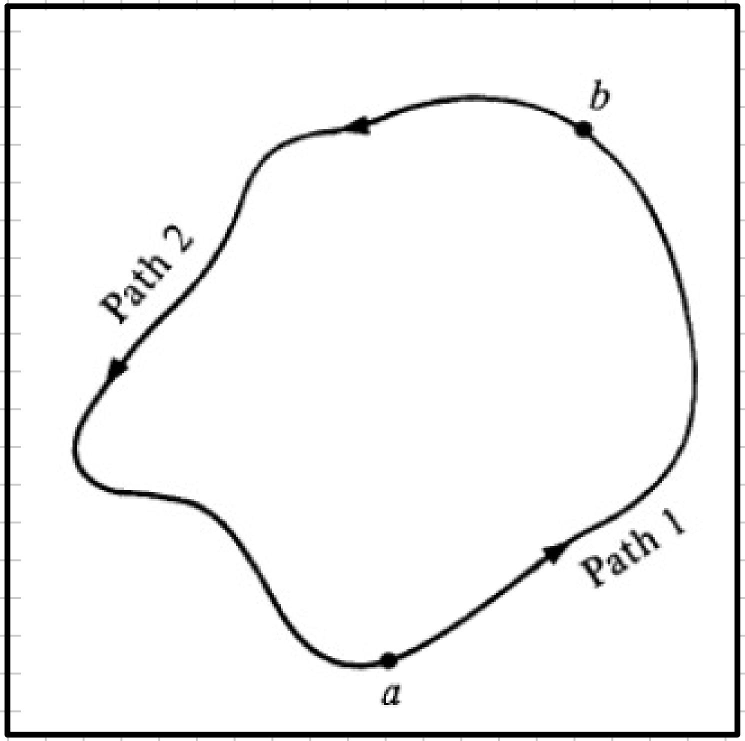
Energy Sphere!

Constant radius



$\vec{F}(r)$ — is conservative??

Consider conservative!



Work done from $a \to b$ and $b \to a$

$$\underset{\text{Path 1}}{\oint_a^b} \mathbf{F} \cdot d\mathbf{r} + \underset{\text{Path 2}}{\oint_b^a} \mathbf{F} \cdot d\mathbf{r}$$

$$= (-U_b + U_a) + (-U_a + U_b) = 0.$$

$$\boxed{\oint \mathbf{F} \cdot d\mathbf{r} = 0}$$

Conservative force!

Another way to check if a function is conservation

$\vec{\nabla} \times \vec{F} = 0$ ie

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix} = 0$$

example!

Gravitational force!

$$\vec{F} = \frac{A}{r^2}\hat{r} = \frac{A\,\vec{r}}{r^3}$$

$$= A \cdot \frac{x\hat{i} + y\hat{j} + z\hat{k}}{r^3}$$

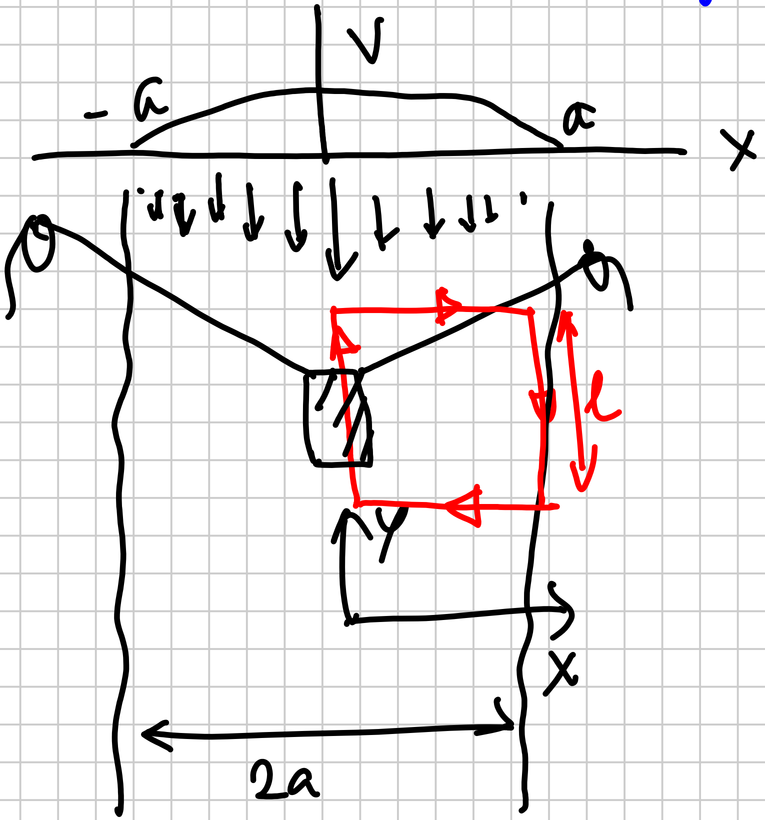
$$F_x = \frac{x}{(x^2+y^2+z^2)^{3/2}} \qquad F_y = \frac{y}{(x^2+y^2+z^2)^{3/2}}$$

$$\vec{F}_z = \frac{z}{(x^2+y^2+z^2)^{3/2}}$$

calculate $(\vec{\nabla} \times \vec{F})_x \Rightarrow ? = 0$

$$\vec{\nabla} \times \vec{F} = 0$$

Non Conservative force!



Velocity of water

$$\vec{v} = -v_0\left(1-\frac{x^2}{a^2}\right)\hat{j}$$

Barge is pulled up

Force on Barge $\vec{F}_{river} = b\,\vec{v}$

Force at winches

$$\vec{F} = -\vec{F}_{river} = -b\vec{v}$$

$$= +b\,v_0\left(1-\frac{x^2}{a^2}\right)\hat{j}$$

calculate $\left|\vec{\nabla}\times\vec{F}\right| = -\dfrac{2bv_0}{a^2}\,x$ Not vanishes!

Work done

$$W = F(x=0)\,\ell - F(x=a)\,\ell$$

$$= v_0 b\,\ell - v_0 b\,\ell\left(1-\frac{a^2}{a^2}\right) = v_0 b\,\ell$$

Potential Energy Function:

Force $\vec{F} = A\,(x^2\,\hat{i} + y\,\hat{j})$ — ①

U exists if $\nabla \times F = 0$ [Check]

$$-\frac{\partial U}{\partial x} = F_x = Ax^2 \quad \text{— ②}$$

$$-\frac{\partial U}{\partial y} = F_y = Ay \quad \text{— ③}$$

$$U(x,y) = -\frac{A}{3}x^3 + f(y) \quad \text{— ④}$$

$$-\frac{\partial}{\partial y}\left(-\frac{A}{3}x^3 + f(y)\right) = Ay$$

$$-\frac{\partial f}{\partial y} = Ay = -\frac{df}{dy} \Rightarrow f(y) = -\frac{A}{2}y^2 + C$$

Potential energy $U = -\frac{A}{3}x^3 - \frac{A}{2}y^2 + C$

Apply for non conservative force!

$$\vec{F} = A\left(xy\,\hat{i} + y^2\,\hat{j}\right)$$

$$\vec{\nabla} \times \vec{F} \neq 0$$

$$-\frac{\partial U}{\partial x} = F_x = A\,xy$$

$$U = -\frac{A}{2}x^2 y + f(y)$$

$$-\frac{\partial U}{\partial y} = F_y = Ay^2$$

$$-\frac{A}{2}x^2 - \frac{\partial f(y)}{\partial y} = Ay^2$$

$$f(y) = -\frac{A}{2}x^2 - Ay^2$$

$f(y)$ can not depends on x.