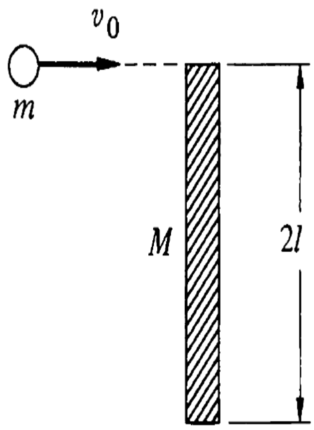
PH101 TUTORIAL 08 DUE: OCT 4, 2012

Note Title 28-Sep-12

6.31 A cylinder of radius R spins with angular velocity ω_0. When the cylinder is gently laid on a plane, it skids for a short time and eventually rolls without slipping. What is the final angular velocity, ω_f?

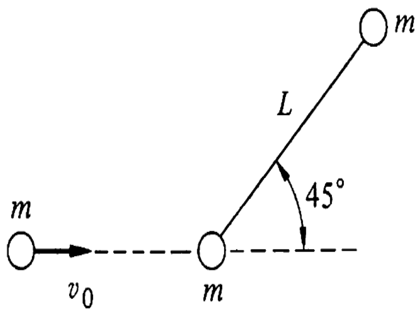
Ans. clue. If $\omega_0 = 3$ rad/s, $\omega_f = 1$ rad/s



6.37 a. A plank of length $2l$ and mass M lies on a frictionless plane. A ball of mass m and speed v_0 strikes its end as shown. Find the final velocity of the ball, v_f, assuming that mechanical energy is conserved and that v_f is along the original line of motion.

b. Find v_f assuming that the stick is pivoted at the lower end.

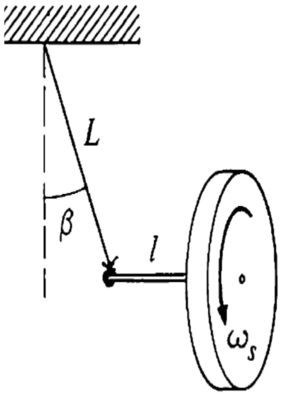
Ans. clue. For $m = M$, (a) $v_f = 3v_0/5$; (b) $v_f = v_0/2$



6.38 A rigid massless rod of length L joins two particles each of mass m. The rod lies on a frictionless table, and is struck by a particle of mass m and velocity v_0, moving as shown. After the collision, the projectile moves straight back.

Find the angular velocity of the rod about its center of mass after the collision, assuming that mechanical energy is conserved.

Ans. $\omega = (4\sqrt{2}/7)(v_0/L)$



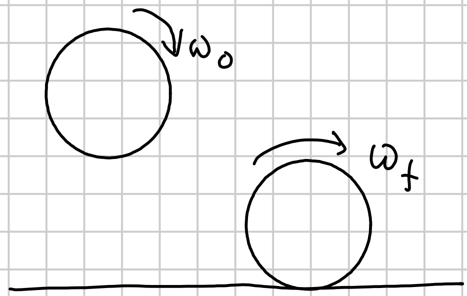
7.3 A gyroscope wheel is at one end of an axle of length l. The other end of the axle is suspended from a string of length L. The wheel is set into motion so that it executes uniform precession in the horizontal plane. The wheel has mass M and moment of inertia about its center of mass I_0. Its spin angular velocity is ω_s. Neglect the mass of the shaft and of the string.

Find the angle β that the string makes with the vertical. Assume that β is so small that approximations like $\sin\beta \approx \beta$ are justified.

TUTORIAL: 08 [SOLUTIONS] DUE: OCT 4, 2012

Note Title 27-Sep-12

$\boxed{6.31}$ Angular momentum is conserved about point of impact.



$$I_0\omega_0 = I_0\omega_f + MvR$$

For pure rolling

$$\omega_f = \frac{v}{R}$$

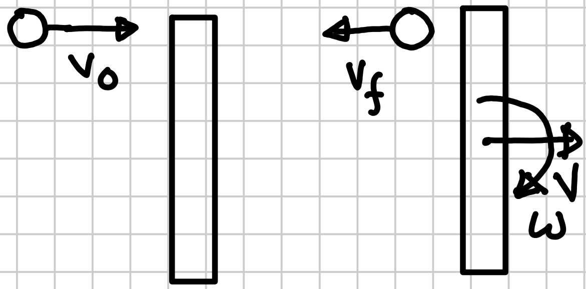
$$I_0\omega_0 = I_0\omega_f + MR^2\omega_f$$

$$I_0 = \frac{1}{2}MR^2$$

$$\Rightarrow \quad \frac{1}{2}MR^2\omega_0 = \frac{1}{2}MR^2\omega_f + MR^2\omega_f$$

$$\Rightarrow \quad \omega_f = \frac{\omega_0}{3}$$

$\boxed{6.37}$



(a) Considering momentum and energy conservation:

$$m v_0 = MV - m v_f \quad \text{—①}$$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} MV^2 + \frac{1}{2} I_0 \omega^2 \quad \text{—②}$$

About CM $\quad m v_0 l = -m v_f l + I_0 \omega \quad$ —③

$$\Rightarrow \quad V = \frac{m}{M}(v_0 + v_f)$$

& $\quad \omega = m(v_0 + v_f)\, l / I_0$

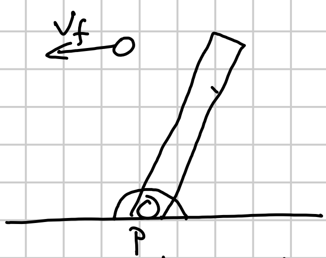
Here $\quad I_0 = \frac{1}{12} M (2l)^2 = \frac{1}{3} M l^2$

We find,

$$\left(1 + \frac{4m}{M}\right) v_f^2 + \left(\frac{8m}{M} v_0\right) v_f - \left(1 - \frac{4m}{M}\right) v_0^2 = 0$$

Solving $\quad v_f = \left[\dfrac{1 - \frac{4m}{M}}{1 + \frac{4m}{M}}\right] v_0$

When $m = M \Rightarrow v_f = -\frac{3}{5} v_0$



(b) Conserving mechanical energy

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} I_p \omega^2$$

Taking angular momentum about the pivot

$$I_p = \frac{1}{3} M (2l)^2 = \frac{4}{3} M l^2$$

$$m v_0 (2l) = -m v_f (2l) + I_p \omega$$

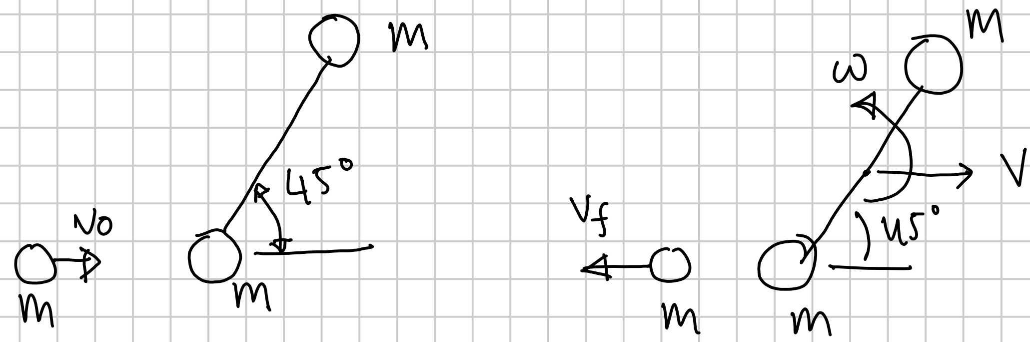
$$\Rightarrow \quad \omega = \frac{2ml(v_0 + v_f)}{I_p}$$

and $\left(1+\frac{3m}{M}\right)v_f^2 + \left(6\frac{m}{M}v_0\right)v_f - \left(1-\frac{3m}{M}\right)v_0^2 = 0$

$$\Rightarrow \quad v_f = \left[\frac{1-3m/M}{1+3m/M}\right]v_0$$

If $m = M \Rightarrow v_f = -\frac{v_0}{2}$

$\boxed{6.38}$



$I_0 = 2m\left(\frac{L}{2}\right)^2$
$= \frac{1}{2}mL^2$

$mV_0 = 2mV - mV_f$

& $\frac{1}{2}mV_0^2 = \frac{1}{2}mV_f^2 + \frac{1}{2}(2m)V^2 + \frac{1}{2}I_0\omega^2$

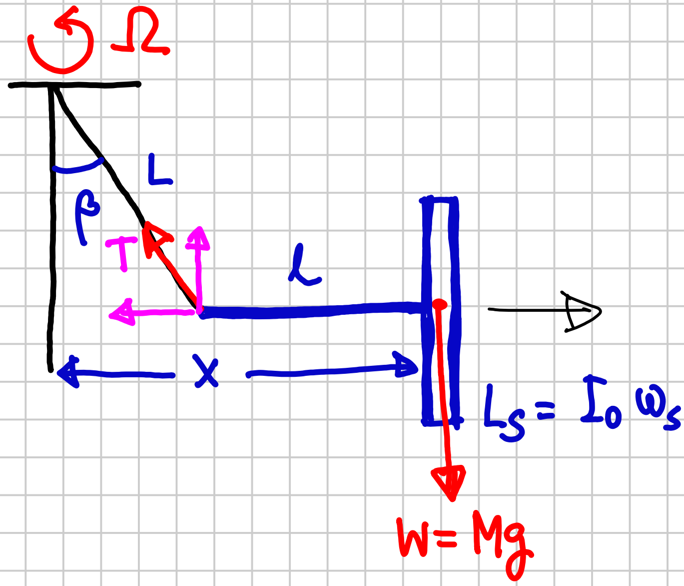
Taking angular momentum about CM

$$mv_0\left(\frac{L}{2}\right)\sin 45^\circ = -mv_f\left(\frac{L}{2}\right)\sin 45^\circ + I_0\omega$$

$$\Rightarrow \quad V_0 = 2V - V_f$$
$$V_0^2 = V_f^2 + 2V^2 + \frac{L^2}{2}\omega^2$$
$$V_0 = -V_f + \sqrt{2}L\omega$$

$$\Rightarrow \quad \omega = \frac{4\sqrt{2}}{7}\frac{V_0}{L}$$

$\boxed{7.3}$



The linear equations of motion are

$$T \cos\beta - Mg = 0$$
$$T \sin\beta = M\Omega^2 x$$

$$\text{Torque} = \ell M g = \frac{dL_s}{dt} = \Omega L_s = \Omega I_0 \omega_s$$

with $\cos\beta \approx 1$, $\sin\beta = \beta$

$$\beta \sim \frac{M\Omega^2 x}{T} = \frac{\Omega^2 x}{g} = \frac{1}{g}\left(\frac{Mg\ell}{I_0\omega_s}\right)^2 x$$

But $x = \ell + L\sin\beta \sim \ell + L\beta$

$$\beta = \frac{1}{g}\left(\frac{Mg\ell}{I_0\omega_s}\right)^2 [\ell + L\beta]$$

$$\Rightarrow \beta = \beta_0 + \frac{L}{\ell}\beta_0\beta$$

$$\Rightarrow \beta = \frac{\beta_0}{1 - \frac{L}{\ell}\beta_0}$$