Indian Institute of Technology Guwahati

PH101: Physics –I

Tutorial 01

Due: Jul 31, 2012

KK 1.5 Prove that the diagonals of an equilateral parallelogram are perpendicular.

KK 1.8 Find a unit vector perpendicular to

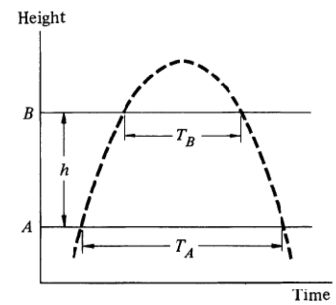
$$\mathbf{A} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \quad \text{and} \quad \mathbf{B} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}).$$

KK 1.11 Let $\mathbf{A}$ be an arbitrary vector and let $\hat{\mathbf{n}}$ be a unit vector in some fixed direction. Show that $\mathbf{A} = (\mathbf{A} \cdot \hat{\mathbf{n}})\hat{\mathbf{n}} + (\hat{\mathbf{n}} \times \mathbf{A}) \times \hat{\mathbf{n}}$.

KK 1.12 The acceleration of gravity can be measured by projecting a body upward and measuring the time that it takes to pass two given points in both directions.

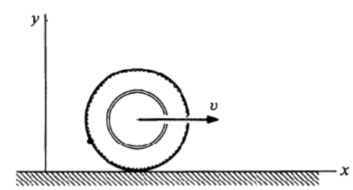
Show that if the time the body takes to pass a horizontal line A in both directions is T_A, and the time to go by a second line B in both directions is T_B, then, assuming that the acceleration is constant, its magnitude is

$$g = \frac{8h}{T_A{}^2 - T_B{}^2},$$



KK 1.17 A particle moves in a plane with constant radial velocity $\dot{r} = 4$ m/s. The angular velocity is constant and has magnitude $\dot{\theta} = 2$ rad/s. When the particle is 3 m from the origin, find the magnitude of (*a*) the velocity and (*b*) the acceleration.

KK 1.19 A tire rolls in a straight line without slipping. Its center moves with constant speed V. A small pebble lodged in the tread of the tire touches the road at $t = 0$. Find the pebble's position, velocity, and acceleration as functions of time.



KK : An Introduction to Mechanics, Kleppner & Kolenkow

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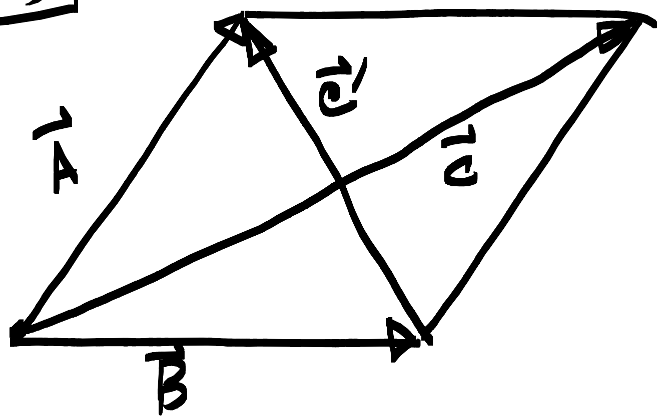
Tutorial 01 Solutions Due: 31:07:12

Note Title 7/25/2012

KK 1.5



$$|\vec{A}| = |\vec{B}|$$

$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{C}' = \vec{A} - \vec{B}$$

$$\vec{C} \cdot \vec{C}' = (\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B})$$

$$= A^2 - B^2 = 0$$

KK 1.8

$$\vec{A} = (\hat{i} + \hat{j} - \hat{k})$$

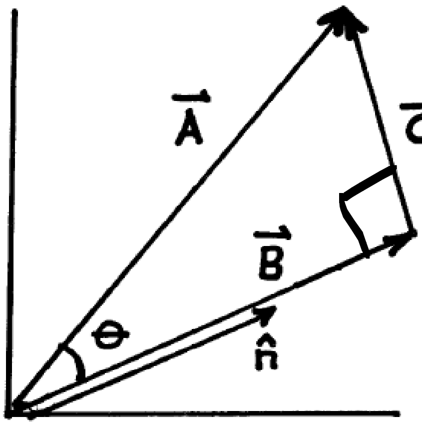
$$\vec{B} = (2\hat{i} - \hat{j} + 3\hat{k})$$

Unit vector $\perp$ to $\vec{A}$ & $\vec{B}$ is $\hat{n}$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} \quad \text{or} \quad \frac{\vec{B} \times \vec{A}}{|\vec{B} \times \vec{A}|}$$

So, $\hat{n} = \pm (2\hat{i} - 5\hat{j} - 3\hat{k})/\sqrt{38}$

KK 1.11



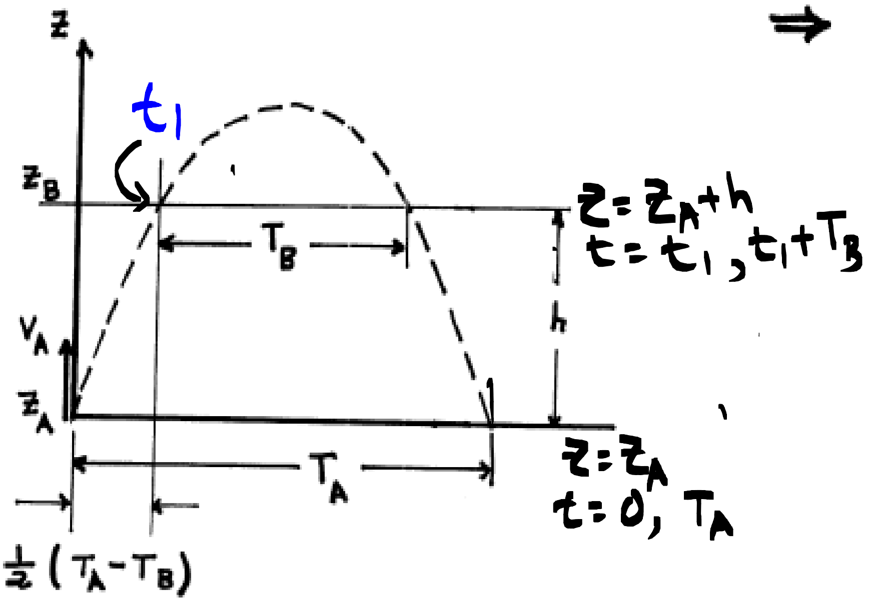
$$\vec{A} = \vec{B} + \vec{C}$$
$$B = A\cos\theta$$
$$\vec{B} = (\vec{A}\cdot\hat{n})\,\hat{n}$$
$$C = A\sin\theta$$
$$\vec{C} = (\hat{n}\times\vec{A})\times\hat{n}$$

$$\begin{aligned}\vec{A} &= \vec{B} + \vec{C}\\ &= (\vec{A}\cdot\hat{n})\,\hat{n} + (\hat{n}\times\vec{A})\times\hat{n}\end{aligned}$$

KK 1.12

Initial velocity : V_A

$$z = z_A + v_A t - \tfrac{1}{2} g t^2$$
$$z_A = z_A + v_A T_A - \tfrac{1}{2} g T_A^2$$
$$\Rightarrow \quad v_A = \tfrac{1}{2} g T_A$$



where

$$t_1 = \tfrac{1}{2}(T_A - T_B)$$

$$h = v_A \frac{(T_A - T_B)}{2} - \frac{1}{2} g \left(\frac{T_A - T_B}{2}\right)^2 = \frac{1}{4} g \left[T_A^2 - T_A T_B - \tfrac{1}{2} T_A^2 + T_A T_B - \tfrac{1}{2} T_B^2\right]$$

$$\Rightarrow \quad g = \frac{8h}{T_A^2 - T_B^2}$$

KK 1.17

(a) $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = 4\hat{r} + 2r\hat{\theta}$

$$v = \sqrt{(4)^2 + (2.3)^2} = \sqrt{52} \text{ m/s}$$

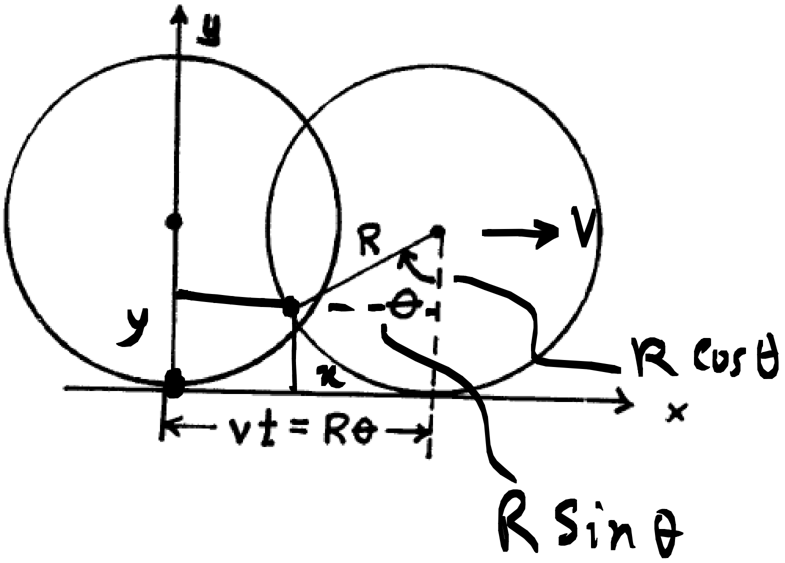
(b) $\vec{a} = (\ddot{r} - r^2\dot{\theta})\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$

$$= -2r^2\hat{r} + (2)(4)(2)\hat{\theta}$$

$$a = \sqrt{(2.9)^2 + (16)^2} = 24.1 \text{ m/s}$$

KK 1.19

$$vt = R\theta$$



$$x = R\theta - R\sin\theta$$
$$y = R(1 - \cos\theta)$$

$$\dot{x} = R\dot{\theta} - R\cos\theta\,\dot{\theta}$$

$$\dot{x} = V - V\cos\theta$$

$$\dot{y} = R\sin\theta\,\dot{\theta} = V\sin\theta$$

$$\Rightarrow \ddot{x} = V\omega\sin\theta, \quad \ddot{y} = V\omega\cos\theta$$

$$\text{with } \theta = \frac{vt}{R}$$