Indian Institute of Technology Guwahati

Mid Semester Examination

PH101: Physics I

Date: Sept 21, 2012; Time: 2:00 – 4:00 PM

Total Marks: 5×6=30

General Instructions: Answer all five questions. Please be sure to keep all parts of a question together and circle your final answer.

1. A block of mass m is constrained to slide inside a ring of radius r as shown in Fig.1. The ring is fixed on a frictionless horizontal table. At $t=0$, the block is moving along the inside of the ring with tangential velocity v_0. The coefficient of friction between the block and the ring is μ. Neglect gravity.
   (a) Find the speed of the block at any later time t.
   (b) Find the angular position of the block at any later time t.

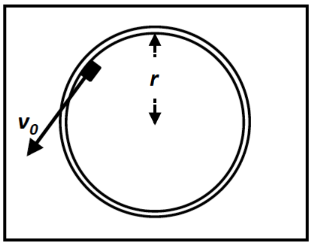

Fig. 1

[3+3]

2. Derive the moment of inertia of a thin circular disk of radius R and mass M using plane polar coordinate system about an axis
   (a) passing through its center and parallel to the disk
   (b) passing through its center and perpendicular to the disk

[3+3]

3. A uniformly filled crate of mass M, length L and height H is kept across the floor. A force F is applied at the top back edge of the crate as shown in Fig.2. Calculate the minimum frictional coefficient between the crate and the floor for which the crate will topple.

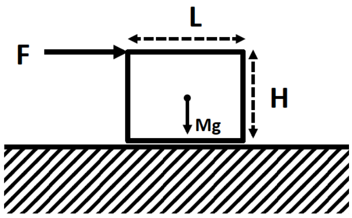

Fig.2 [6]

4. A particle of mass m is inserted into a central force of the form

$$\vec{F} = -\frac{k}{r^n}\hat{r},$$

where, r is the radial distance from the origin, k & n are constants. Magnitude of the angular momentum about origin is L.
   (a) Derive the effective potential for the system.
   (b) Derive the general expression for r to calculate equilibrium points of the effective potential.
   (c) Determine the values of n for which the stable orbits are possible.

[2+2+2]

5. A cone of height h and base radius R is free to rotate about a fixed vertical axis as shown in Fig.3. It has a thin groove cut in the surface. The cone is set rotating freely with angular speed ω_0 and a small block of mass m is released in the top of the frictionless groove and allowed to slide under gravity. Assume that the block stays in the groove. Take the moment of inertia of the cone about the vertical axis to be I_0.

   (a) What is the angular velocity of the cone when the block reaches the bottom?
   (b) Find the speed of the block in inertial space when it reaches the bottom.

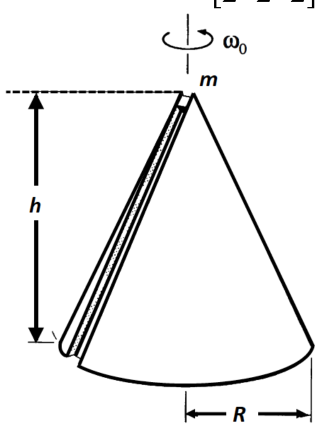

Fig.3

[3+3]

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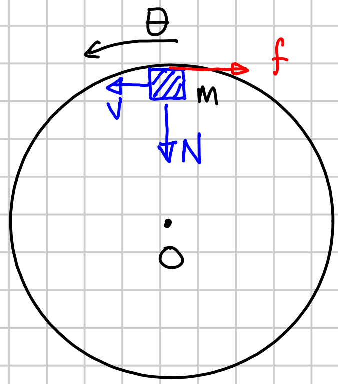
1. (a) Consider at $t=0$, position and speed of mass is θ_0 and v_0

Accelaration of mass m in plane polar coordinate system can be written as



$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

Here $\dot{r} = 0$ and $\ddot{r} = 0$

$\dot{\theta} = \omega$ and $\ddot{\theta} = \dot{\omega}$

$$\Rightarrow \vec{a} = -r\dot{\theta}^2\hat{r} + r\ddot{\theta}\hat{\theta}$$

$$= -r\omega^2\hat{r} + \frac{dv}{dt}\hat{\theta}$$

since,

$v = \omega r$

$\frac{dv}{dt} = r\frac{d\omega}{dt} = r\dot{\omega}$

$= r\ddot{\theta}$

$\therefore$ Force on mass m

$$\vec{F} = -mr\omega^2\hat{r} + m\frac{dv}{dt}\hat{\theta}$$

considering the components of the force as per force diagram.

$$-mr\omega^2 = -N \quad \text{--- (1)}$$

$$m\frac{dv}{dt} = -f \quad \text{--- (2)}$$

From Eq (1) $N = mr\omega^2 = \frac{mv^2}{r}$ — (3)

So $m\frac{dv}{dt} = -f = -\mu N = -\mu\frac{mv^2}{r}$

$$\Rightarrow \frac{dv}{dt} = -\frac{\mu v^2}{r}$$

$$\int_{v_0}^{v}\frac{dv}{v^2} = -\int_0^t\frac{\mu}{r}dt \Rightarrow \frac{1}{v_0} - \frac{1}{v} = -\frac{\mu}{r}t$$

$$\boxed{v = \frac{v_0}{\left(1 + \frac{\mu v_0 t}{r}\right)}}$$

1 (b) Angular position of mass m is θ

Then, $\frac{d\theta}{dt} = \frac{v}{r} = \frac{v_0/r}{\left(1+\frac{\mu v_0 t}{r}\right)}$

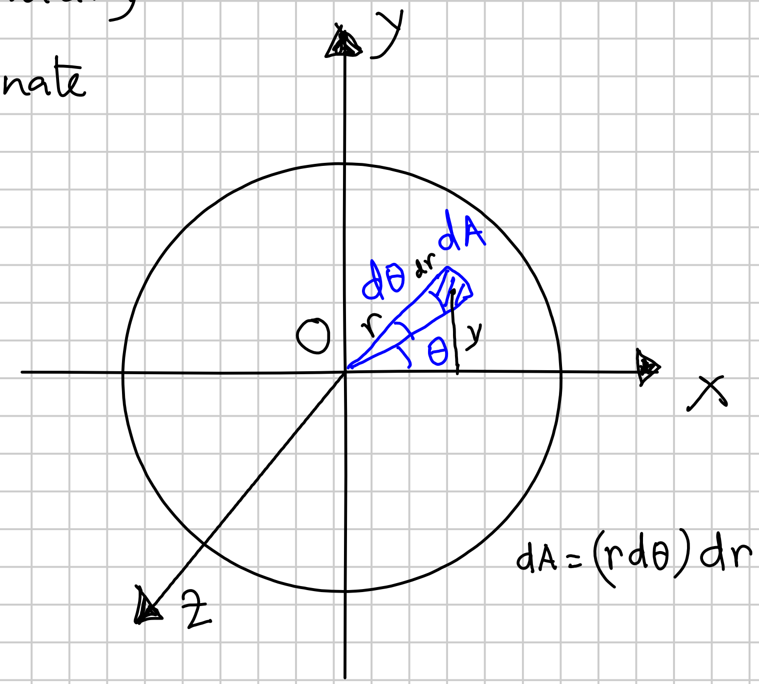
$$\int_{\theta_0}^{\theta} d\theta = \int_0^t \left(\frac{v_0/r}{1+\frac{\mu v_0 t}{r}}\right) dt$$

$$\Rightarrow \theta - \theta_0 = \frac{v_0}{r} \frac{1}{\frac{\mu v_0}{r}} \ln\left[1 + \frac{\mu v_0 t}{r}\right]$$

$$\Rightarrow \boxed{\theta = \theta_0 + \frac{1}{\mu} \ln\left[1 + \frac{\mu v_0 t}{r}\right]}$$

2 (a) Considering an elementary area in plane polar coordinate system

$$dA = (r\,d\theta)\,dr \quad \text{— ①}$$



Mass of this area

$$dm = M\frac{dA}{A} = M\frac{r\,dr\,d\theta}{\pi R^2}$$

$$\Rightarrow dm = \frac{M}{\pi R^2} r\,dr\,d\theta \quad \text{— ②}$$

The moment of inertia of the elemental mass about an axis passing through the center and parallel to the disk is x-axis.

Therefore

$$dI_x = y^2\,dm$$

$$= (r\sin\theta)^2 \frac{M}{\pi R^2} r\,dr\,d\theta$$

$$= \frac{M}{\pi R^2}\left(r^3 \sin^2\theta\,dr\,d\theta\right)$$

Total moment of inertia

$$I_x = \frac{M}{\pi R^2}\int_0^{2\pi}\int_0^R r^3\,dr\,\sin^2\theta\,d\theta$$

$$= \frac{M}{\pi R^2}\cdot\frac{R^4}{4}\;\frac{2\pi}{2} = \frac{1}{4}MR^2$$

$$\boxed{I_x = \frac{1}{4}MR^2}$$

(b) The moment of inertia of elemental mass dm about an axis passing through the center and perpendicular to the disk (z-axis as shown in Figure) is dI_z

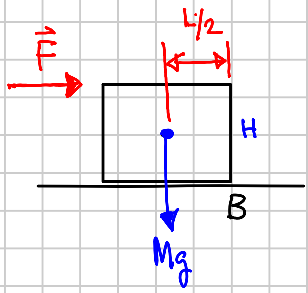
$$dI_z = r^2 \, dm$$

$$= r^2 \frac{M}{\pi R^2} r \, dr \, d\theta$$

$$\Rightarrow \quad I_z = \frac{M}{\pi R^2} \int_0^{2\pi} \int_0^{R} r^3 \, dr \, d\theta$$

$$= \frac{M}{\pi R^2} \cdot \frac{R^4}{4} \cdot 2\pi = \frac{1}{2} M R^2$$

$$\boxed{I_z = \frac{1}{2} M R^2}$$



3. If crate is not sliding

$$F < \mu M g \quad — (1)$$

If the crate topples, then it will be about lower right corner B.

Considering torque about point B.

For tipping, clockwise torque due to force F must exceeds counter clockwise torque due to gravity.

$$\Rightarrow \quad F H > \frac{MgL}{2} \quad — (2)$$

Combining Eq (1) and Eq (2) [toppling before sliding]

$$\frac{MgL}{2H} < F < \mu M g$$

$$\Rightarrow \quad \mu > \frac{L}{2H}$$

Therefore minimum frictional coefficient between crate and floor for which the crate will topple is $\boxed{L/2H}$

4 (a) $F(r) = -\frac{k}{r^n}\hat{r}$

The effective potential

$$U_{eff} = U(r) + \frac{L^2}{2mr^2}$$

where $U(r) = -\int \vec{F}(r)\cdot d\vec{r} = \int \frac{k}{r_n}(\hat{r}\cdot\hat{r})\,dr$

$$= -\frac{1}{n-1}\frac{k}{r^{n-1}} + C$$

Now, when $r\to\infty$ $U(r)\to 0$

$n>1$ $C=0$

$$\boxed{U_{eff} = -\frac{1}{n-1}\frac{k}{r^{n-1}} + \frac{L^2}{2mr^2}}$$

(b) The equilibrium point satisfies

$$\left.\frac{dU_{eff}}{dr}\right|_{r=r_{eq}} = 0$$

$\frac{d}{r} r^n = n r^{n-1}$

$$\Rightarrow -\frac{k}{n-1}(-)(n-1)r_{eq}^{-(n-1)-1} + \frac{L^2}{2m}\cdot(-2)\,r_{eq}^{-3} = 0$$

$$\Rightarrow \frac{k}{r_{eq}^n} - \frac{L^2}{mr_{eq}^3} = 0$$

$$r_{eq}^{n-3} = \frac{mk}{L^2} \Rightarrow \boxed{r_{eq} = \left(\frac{mk}{L^2}\right)^{\frac{1}{n-3}}}$$

(c) For a stable orbit

$$\left.\frac{d^2U_{eff}}{dr^2}\right|_{r_{eq}} > 0 \Rightarrow -\frac{nk}{r_{eq}^{n+1}} + \frac{3L^2}{2mr_{eq}^4} > 0$$

$$\Rightarrow r_{eq}^{n-3} > \frac{nkm}{3L^2}$$

$$\frac{mk}{L^2} > \frac{nkm}{3L^2} \Rightarrow \boxed{n < 3}$$

5. (a) Angular momentum along the vertical direction is conserved

$$L_i = I_0 \omega_0 \qquad (1)$$

$$L_f = I_0 \omega_f + mR^2 \omega_f \qquad (2)$$

$$\Rightarrow \quad L_i = L_f \quad \Rightarrow \quad \boxed{\omega_f = \left[\frac{I_0}{I_0 + mR^2}\right] \omega_0}$$

(b) Consider the conservation of total energy.

$$E_i = \frac{1}{2} I_0 \omega_0^2 + mgh$$

$$E_f = \frac{1}{2} I_0 \omega_f^2 + \frac{1}{2} m v_f^2$$

Here $E_i = E_f$

$$\frac{1}{2} I_0 \omega_0^2 + mgh = \frac{1}{2} I_0 \omega_f^2 + \frac{1}{2} m v_f^2$$

$$\Rightarrow \quad v_f^2 = \frac{I_0}{m}\left(\omega_0^2 - \omega_f^2\right) + 2gh$$

$$\Rightarrow \quad \boxed{v_f = \left[\frac{I_0}{m}\left\{\omega_0^2 - \left(\frac{I_0}{I_0 + mR^2}\right)^2\right\} + 2gh\right]^{\frac{1}{2}}}$$