Indian Institute of Technology Guwahati

PH101: Physics –I

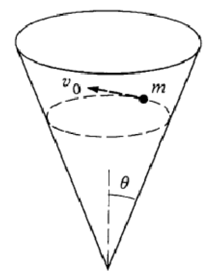
Tutorial 02

Due: Aug 7, 2012

KK 2.9 A particle of mass m slides without friction on the inside of a cone. The axis of the cone is vertical, and gravity is directed downward. The apex half-angle of the cone is θ, as shown.

The path of the particle happens to be a circle in a horizontal plane. The speed of the particle is v_0.

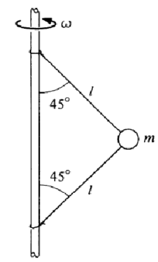
Draw a force diagram and find the radius of the circular path in terms of v_0, g, and θ.



KK 2.11 A mass m is connected to a vertical revolving axle by two strings of length l, each making an angle of 45° with the axle, as shown. Both the axle and mass are revolving with angular velocity ω. Gravity is directed downward.

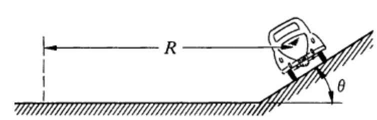
a. Draw a clear force diagram for m.

b. Find the tension in the upper string, T_{up}, and lower string, T_{low}.



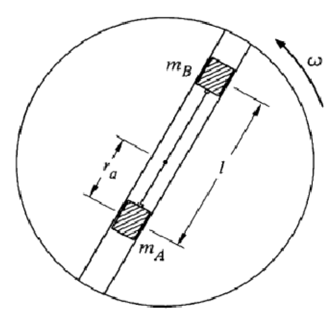
KK 2.23 A piece of string of length l and mass M is fastened into a circular loop and set spinning about the center of a circle with uniform angular velocity ω. Find the tension in the string. Suggestion: Draw a force diagram for a small piece of the loop subtending a small angle, $\Delta\theta$.

KK 2.28 An automobile enters a turn whose radius is R. The road is banked at angle θ, and the coefficient of friction between wheels and road is μ. Find the maximum and minimum speeds for the car to stay on the road without skidding sideways.



KK 2.30 A disk rotates with constant angular velocity ω, as shown. Two masses, m_A and m_B, slide without friction in a groove passing through the center of the disk. They are connected by a light string of length l, and are initially held in position by a catch, with mass m_A at distance r_A from the center. Neglect gravity. At $t = 0$ the catch is removed and the masses are free to slide.

Find $\ddot{r}_A$ immediately after the catch is removed in terms of m_A, m_B, l, r_A, and ω.

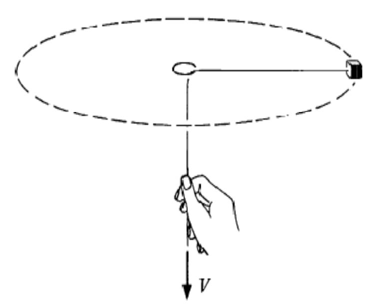


KK 2.34. A mass m whirls around on a string which passes through a ring, as shown. Neglect gravity. Initially the mass is distance r_0 from the center and is revolving at angular velocity ω_0. The string is pulled with constant velocity V starting at $t = 0$ so that the radial distance to the mass decreases. Draw a force diagram and obtain a differential equation for ω. This equation is quite simple and can be solved either by inspection or by formal integration. Find

a. $\omega(t)$.

Ans. clue. For $Vt = r_0/2$, $\omega = 4\omega_0$

b. The force needed to pull the string.



KK : An Introduction to Mechanics, Kleppner & Kolenkow

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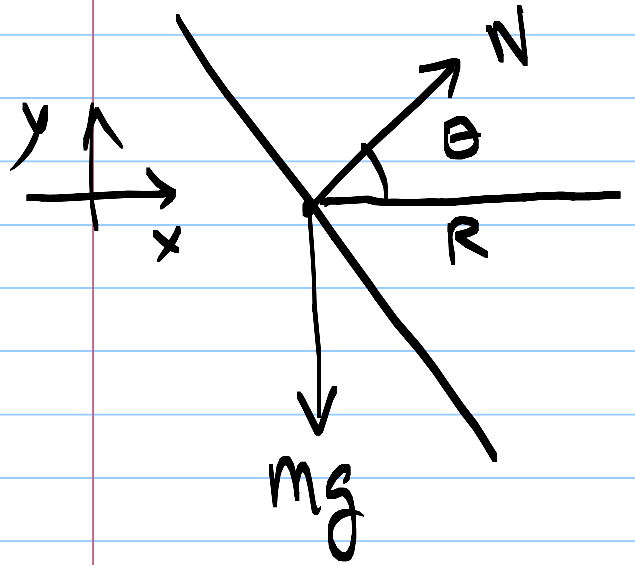
Solutions Tut 02 Due Aug 7, 2012

Note Title 8/2/2012

KK 2.9



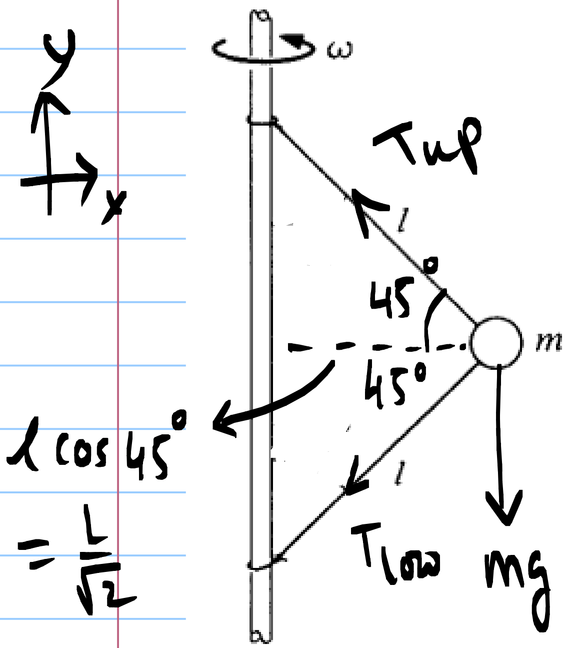
$\boxed{X}$ $N\cos\theta = mv_0^2/R$

$\boxed{Y}$ $N\sin\theta = mg$

$$\tan\theta = \frac{gR}{v_0^2}$$

$$R = \frac{v_0^2}{g}\tan\theta$$

KK 2.11



$\boxed{Y}$ $T_{up}\frac{1}{\sqrt{2}} = mg + T_{low}\frac{1}{\sqrt{2}}$

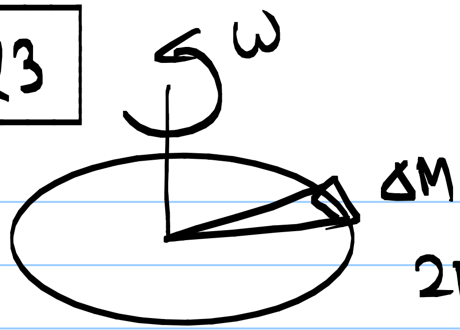
$\boxed{X}$ $(T_{up} + T_{low})\frac{1}{\sqrt{2}} = m\frac{l}{\sqrt{2}}\omega^2$

Solving

$$T_{up} = \frac{1}{2}ml\omega^2 + \frac{1}{\sqrt{2}}mg$$

$$T_{low} = \frac{1}{2}ml\omega^2 - \frac{1}{\sqrt{2}}mg$$

KK 2.23

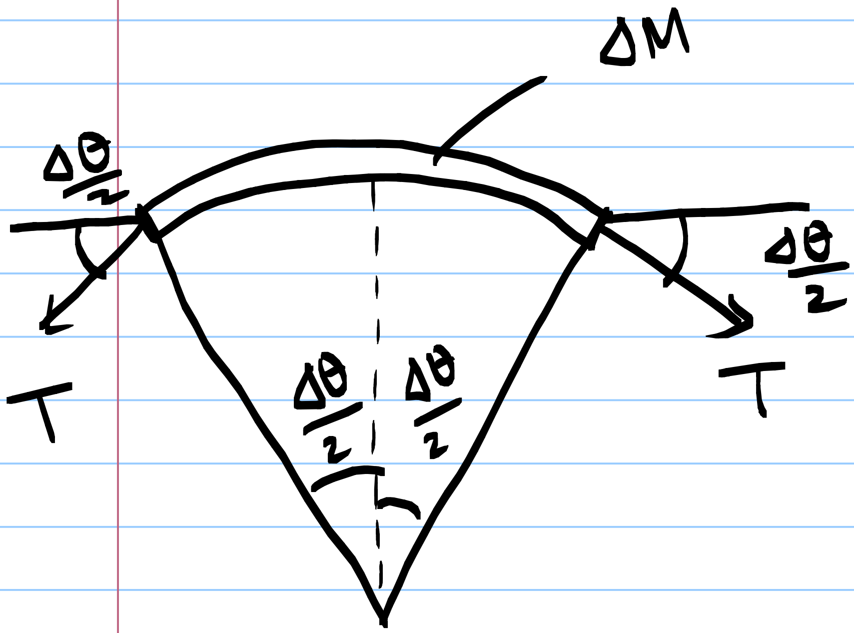


$2\pi r = l$

$$\Delta M = \frac{M}{2\pi}\Delta\theta$$

From force diagram

$$2T \sin\frac{\Delta\theta}{2} = \Delta M r \omega^2$$

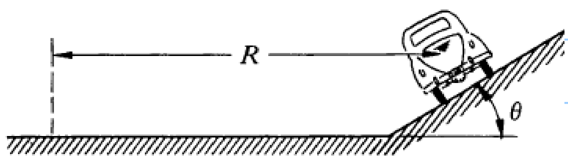


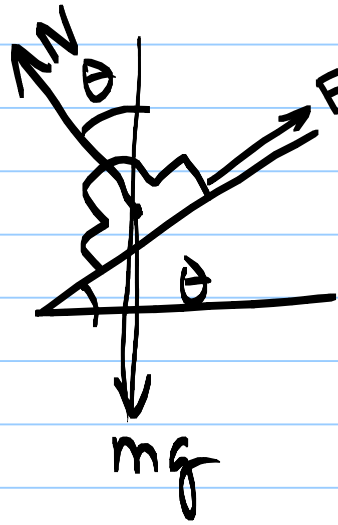
$$\Rightarrow \quad 2T\frac{\Delta\theta}{2} = \Delta M r \omega^2$$

$$T\Delta\theta = \frac{M}{2\pi}\Delta\theta \frac{l}{2\pi}\omega^2$$

$$T = \frac{M}{(2\pi)^2}\omega^2 l$$

KK 2.28





Minimum speed!

$$N\cos\theta + F\sin\theta = Mg$$

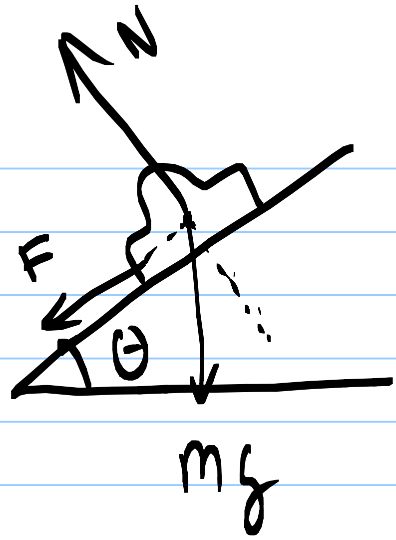
$$N\sin\theta - F\cos\theta = \frac{Mv^2}{R}$$

$$F_{max} = \mu N$$

Solving

$$V_{min} = \left[Rg\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)^2\right]^{\frac{1}{2}}$$

Maximum speed!



$$N\cos\theta - F\sin\theta = Mg$$

$$N\sin\theta + F\cos\theta = \frac{Mv^2}{R}$$

$$F_{max} = \mu N$$

Solving

$$V_{max} = \left[Rg \left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} \right) \right]^{\frac{1}{2}}$$

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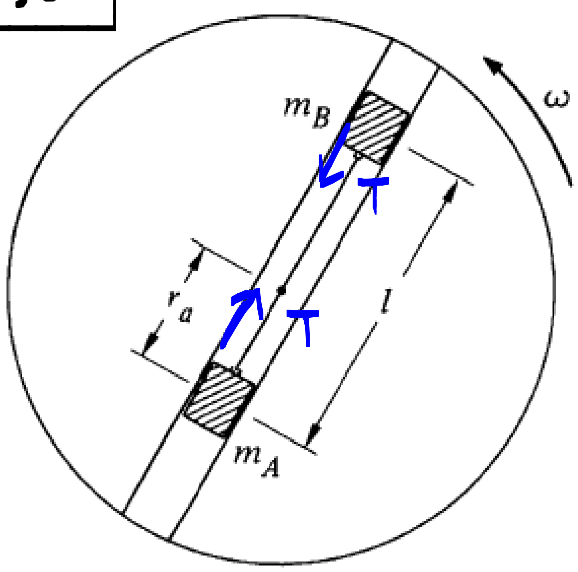
kk 2.30



$$-T = m_A \left(\ddot{r}_A - r_A \omega^2 \right)$$

$$-T = m_B \left(\ddot{r}_B - r_B \omega^2 \right)$$

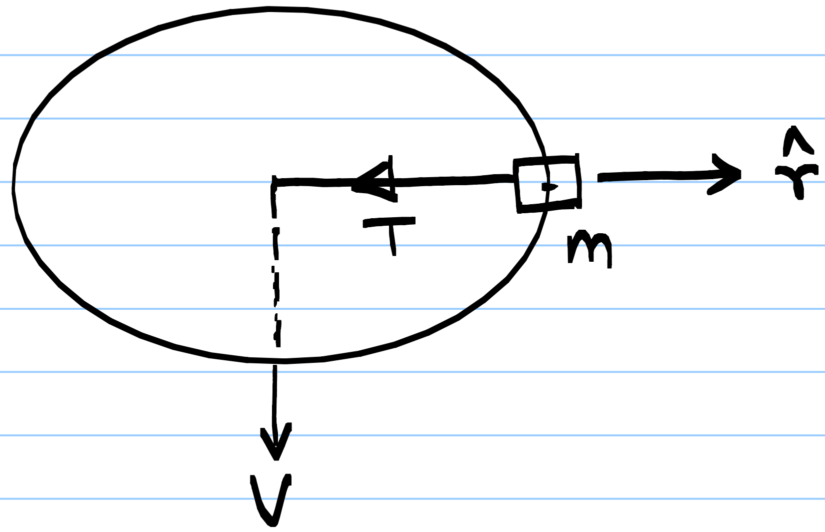
$$r_B = l - r_A$$

$$-T = m_B \left[-\ddot{r}_A - (l - r_A)\omega^2 \right]$$

Solving

$$\ddot{r}_A = r_A \omega^2 - \frac{l\omega^2}{\left(1 + \frac{m_A}{m_B}\right)}$$

KK 2.34



Equations of motion

$$-T = m(\ddot{r} - r\omega^2) \quad \text{Radial}$$

$$0 = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \quad \text{Tangential}$$

(a)

$$r\dot{\omega} + 2\dot{r}\omega = 0$$

$$\int_{\omega_0}^{\omega} \frac{d\omega}{\omega} = -2\int_{r_0}^{r} \frac{dr}{r}$$

$$\omega = \omega_0 \left(\frac{r_0}{r}\right)^2 = \omega_0 \left(\frac{r_0}{r_0 - Vt}\right)^2$$

(b)

$$T = m r \omega^2$$

$$= m\omega_0^2 \frac{r_0^4}{r^3}$$