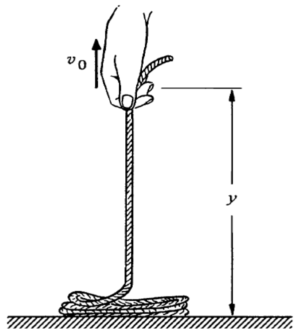
PH 101 **Tutorial 05** **Due:Sep 04,2012** **DKG**

Note Title 30-Aug-12

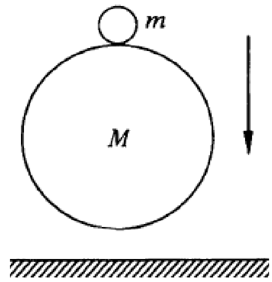
4.21 A uniform rope of mass λ per unit length is coiled on a smooth horizontal table. One end is pulled straight up with constant speed v_0.

a. Find the force exerted on the end of the rope as a function of height y.

b. Compare the power delivered to the rope with the rate of change of the rope's total mechanical energy.

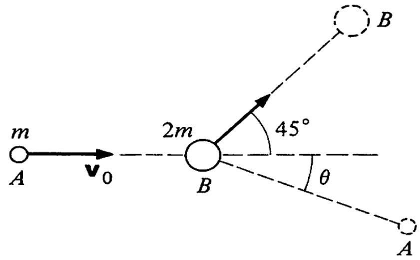


4.23 A small ball of mass m is placed on top of a "superball" of mass M, and the two balls are dropped to the floor from height h. How high does the small ball rise after the collision? Assume that collisions with the superball are elastic, and that $m \ll M$. To help visualize the problem, assume that the balls are slightly separated when the superball hits the floor.



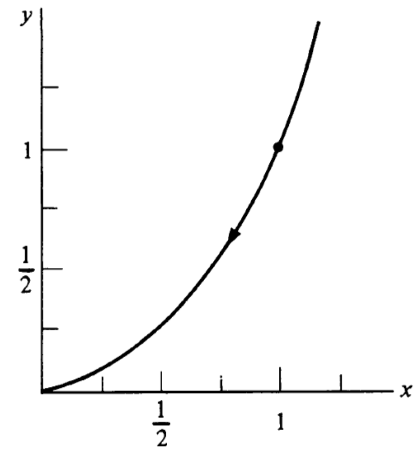
4.25 A proton makes a head-on collision with an unknown particle at rest. The proton rebounds straight back with $\frac{4}{9}$ of its initial kinetic energy.

Find the ratio of the mass of the unknown particle to the mass of the proton, assuming that the collision is elastic.



4.27 Particle A of mass m has initial velocity v_0. After colliding with particle B of mass $2m$ initially at rest, the particles follow the paths shown in the sketch at right. Find θ.

5.2 A particle of mass m moves in a horizontal plane along the parabola $y = x^2$. At $t = 0$ it is at the point (1,1) moving in the direction shown with speed v_0. Aside from the force of constraint holding it to the path, it is acted upon by the following external forces:



A radial force $\mathbf{F}_a = -Ar^3\hat{\mathbf{r}}$

A force given by $\mathbf{F}_b = B(y^2\hat{\mathbf{i}} - x^2\hat{\mathbf{j}})$

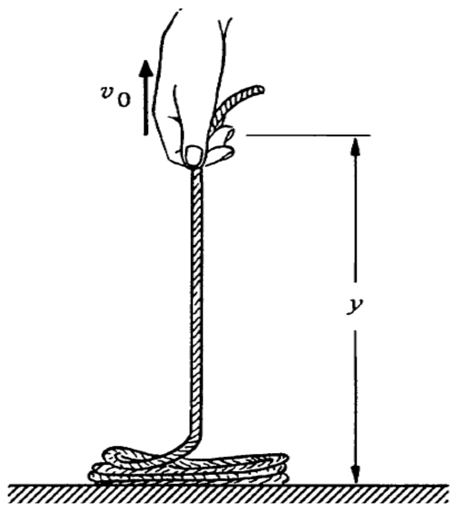
where A and B are constants.

a. Are the forces conservative?

b. What is the speed v_f of the particle when it arrives at the origin?

Ans. $v_f = (v_0{}^2 + 2A/\,m + 3B/5m)^{\frac{1}{2}}$

KK 4·21



Change of momentum along y-direction

$$\Delta p_y = (\Delta m) v_0 = F \, \Delta t$$

$$F = \frac{dm}{dt} v_0 \quad — ①$$

Now $\quad \dfrac{dm}{dt} = \dfrac{dm}{dy} \cdot \dfrac{dy}{dt} = \lambda v_0$

$$\Rightarrow \quad F = \lambda v_0^2$$

Total force $\quad F_{total} = \lambda v_0^2 + \underbrace{\lambda g y}_{\text{Gravitational force}}$

ⓑ Power delivered $= F_{tot} v_0$

$$= \lambda v_0^3 + \lambda g y v_0 \quad — ①$$

Total energy $\quad E = \frac{1}{2}(\lambda y) v_0^2 + P.E$

$$P.E = \int_0^y g y \, dm = \int_0^y g \lambda y \, dy = \frac{1}{2} g \lambda y^2$$

$$\Rightarrow \quad E = \frac{1}{2} \lambda y v_0^2 + \frac{1}{2} g \lambda y^2$$

$$\frac{dE}{dt} = \frac{1}{2} \lambda v_0^2 \frac{dy}{dt} + g \lambda y \frac{dy}{dt} = \frac{1}{2} \lambda v_0^3 + \lambda g y v_0 \quad — ②$$

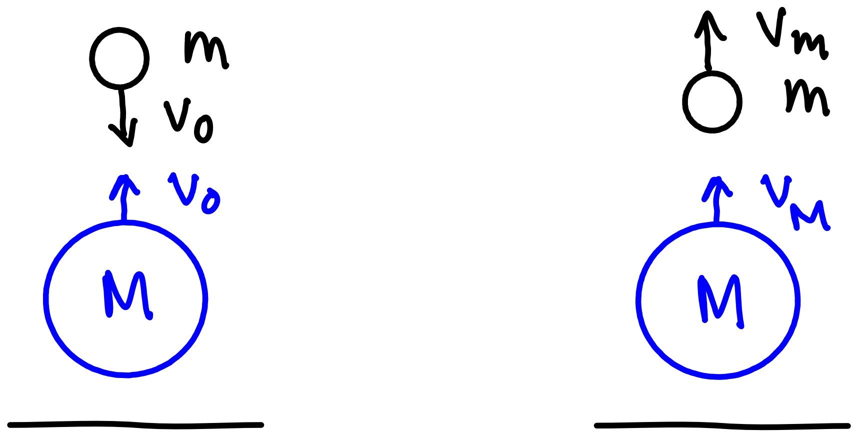
Compairing ① & ②

1st part : Mechanical energy is lost due to the friction bet^n different parts of the rope.

2nd part : No changes in the gravitational energy part as conservative force.

KK 4.23

Velocity of the balls at the ground $v_0 = \sqrt{2gh}$



Before collision   After collision

$$(M-m)\,v_0 = M v_M + m v_m \quad -①$$

$$\frac{1}{2}(M+m)v_0^2 = \frac{1}{2} M v_M^2 + \frac{1}{2} m v_m^2 \quad -②$$

Solving Eq ① & Eq ② with $m \ll M$

$$v_m^2 - 2v_0 v_m - 3v_0^2 = 0$$

$$v_m = \frac{2v_0 + \sqrt{4v_0^2 + 12v_0^2}}{2}$$

$$= v_0 + 2v_0 = 3v_0 = 3\sqrt{2gh}$$

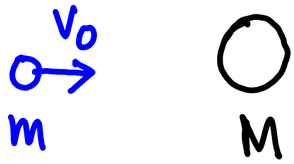
If marble rise to a height H then

$$mgH = \frac{1}{2} m v_m^2 = 9\,mgh$$

$$H \cong 9h$$

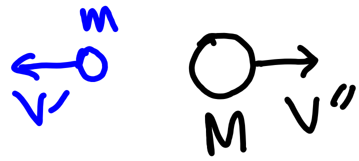
KK 4.25

considering momentum & energy conservation



$$mV_0 = Mv'' - mv'$$

$$\frac{1}{2}mV_0^2 = \frac{1}{2}Mv''^2 + \frac{1}{2}mv'^2$$



$$\frac{1}{2}mv'^2 = \frac{4}{9}\left(\frac{1}{2}mV_0^2\right)$$

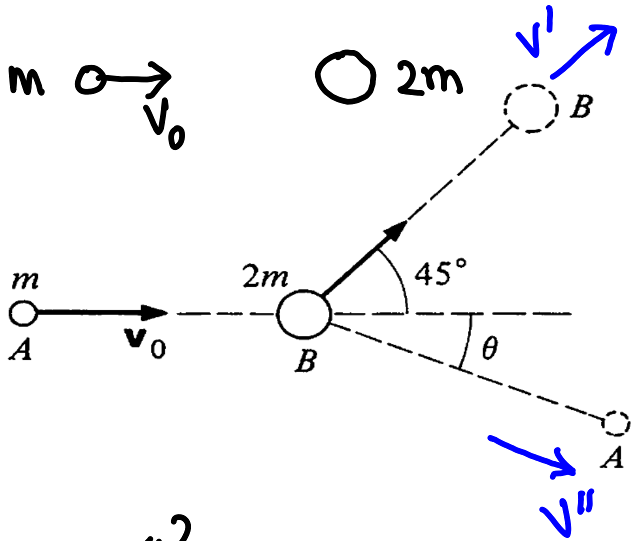
$$v' = \frac{2}{3}V_0$$

$$V_0 = \frac{M}{m}v'' - \frac{2}{3}V_0$$

$$v_0^2 = \frac{M}{m}v''^2 + \frac{4}{9}v_0^2$$

Solving $\quad \frac{M}{m} = 5$

KK 4.27



$$mV_0 = \frac{2mv'}{\sqrt{2}} + mv''\cos\theta$$

$$\frac{2mv'}{\sqrt{2}} = mv''\sin\theta$$

Assuming elastic collision!

$$\frac{1}{2}mv_0^2 = \frac{1}{2}(2m)v'^2 + \frac{1}{2}mv''^2$$

Solving

$$V_0 = V''(\sin\theta + \cos\theta)$$

$$v_0^2 = V''^2(\sin^2\theta + 1)$$

Hence, $\sin^2\theta + 1 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$$= 1 + 2\sin\theta\cos\theta$$

$$\Rightarrow \quad \sin\theta = 2\cos\theta$$

$$\tan\theta = 2 \quad \Rightarrow \quad \theta = \tan^{-1}(2)$$

KK 5.2

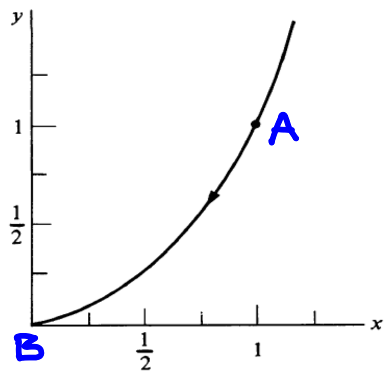
(a) $\vec{F}_a = -Ar^3\hat{r}$ $\rightarrow$ central force
so conservative force

or directly $\vec{\nabla}\times\vec{F} = 0$ can be shown

$$\vec{\nabla}\times\vec{F}_b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ By^2 & -Bx & 0 \end{vmatrix} = (-2Bx - 2By) \neq 0$$

Not Conservative

(b) $U_a = \dfrac{Ar^4}{4}$



$$K_B + U_B - (K_A + U_A) = \int_A^B \vec{F}_b \cdot d\vec{r}$$

$$\frac{1}{2}mv_f^2 + U_a(0,0) - \frac{1}{2}mv_0^2 - U_a(1,1) = \int_{1,1}^{0,0} \vec{F}_b \cdot d\vec{r}$$

$$= B\int_{1,1}^{0,0} (y^2\,dx - x^2\,dy)$$

But $U_a(0,0) = 0$

$$U_a(1,1) = \frac{A}{4}\left[\sqrt{2}\right]^4 = A$$

$y = x^2$ So that

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_0^2 + A + B\int_1^0 x^4\,dx - B\int_1^0 y\,dy$$

$$= \frac{1}{2}mv_0^2 + A - B/5 + B/2$$

$$\Rightarrow v_f = \left(v_0^2 + \frac{2A}{m} + \frac{3B}{5m}\right)^{\frac{1}{2}}$$