

5.5 The potential energy function for a particular two dimensional force field is given by  $U = Cxe^{-y}$ , where  $C$  is a constant.

a. Sketch the constant energy lines.

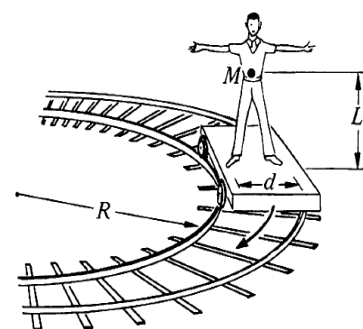
b. Show that if a point is displaced by a short distance  $dx$  along a constant energy line, then its total displacement must be  $d\mathbf{r} = dx(\hat{i} + \hat{j}/x)$ .

c. Using the result of b, show explicitly that  $\nabla U$  is perpendicular to the constant energy line.

6.1 a. Show that if the total linear momentum of a system of particles is zero, the angular momentum of the system is the same about all origins.

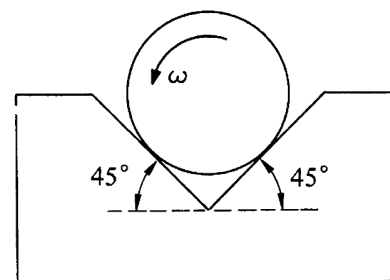
b. Show that if the total force on a system of particles is zero, the torque on the system is the same about all origins.

6.6 A man of mass  $M$  stands on a railroad car which is rounding an unbanked turn of radius  $R$  at speed  $v$ . His center of mass is height  $L$  above the car, and his feet are distance  $d$  apart. The man is facing the direction of motion. How much weight is on each of his feet?



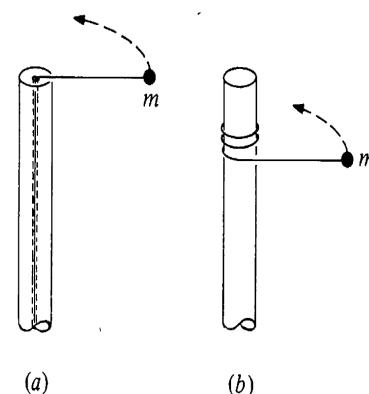
6.10 A cylinder of mass  $M$  and radius  $R$  is rotated in a uniform V groove with constant angular velocity  $\omega$ . The coefficient of friction between the cylinder and each surface is  $\mu$ . What torque must be applied to the cylinder to keep it rotating?

Ans. clue. If  $\mu = 0.5$ ,  $R = 0.1$  m,  $W = 100$  N, then  $\tau \approx 5.7$  N·m



6.13 Mass  $m$  is attached to a post of radius  $R$  by a string (see right hand sketch below). Initially it is distance  $r$  from the center of the post and is moving tangentially with speed  $v_0$ . In case (a) the string passes through a hole in the center of the post at the top. The string is gradually shortened by drawing it through the hole. In case (b) the string wraps around the outside of the post.

What quantities are conserved in each case? Find the final speed of the mass when it hits the post for each case.



## KK 5.5

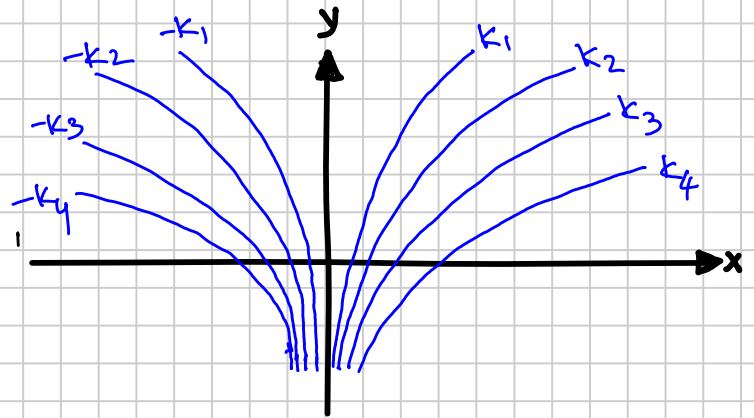
(a) For constant energy lines

$$U = Cx e^{-y} = \text{Constant}$$

Lines of constant potential are given by

$$y = \ln \left[ \frac{x}{K} \right]$$

where  $K = \text{constant}$



(b)  $d\vec{r} = dx \hat{i} + dy \hat{j}$

For constant potential line i.e.  $U = \text{Constant}$

$$dU = 0 = C e^{-y} dx - Cx e^{-y} dy$$

$$\Rightarrow \frac{dx}{x} = dy$$

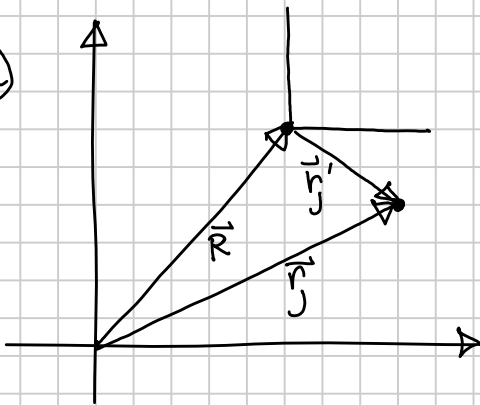
$$d\vec{r} = dx \left( \hat{i} + \hat{j} \frac{1}{x} \right) \quad \text{along a line } U = \text{Constant}$$

(c)  $\vec{\nabla} U = C \left[ e^{-y} \hat{i} - x e^{-y} \hat{j} \right]$

$$\vec{\nabla} U \cdot d\vec{r} = C dx \left[ e^{-y} - e^{-y} \right] = 0$$

## KK 6.1

(a)



$$\vec{L} = \sum_j \vec{r}_j \times \vec{p}_j$$

$$\vec{r}_j = \vec{R} + \vec{r}'_j$$

$$\vec{p}_j = m \vec{v}_j = m \vec{v}'_j = \vec{p}'_j$$

$$\vec{L} = \sum_j \vec{r}'_j \times \vec{p}'_j + \sum_j (\vec{r}_j - \vec{R}) \times \vec{p}'_j$$

$$= \sum_j \vec{r}'_j \times \vec{p}'_j - \vec{R} \times \sum_j \vec{p}'_j$$

$$= \vec{L} \quad \text{since } \sum_j \vec{p}'_j = 0$$

⑥

$$\begin{aligned} \vec{\tau} &= \sum \vec{r}_j \times \vec{F}_j \\ \vec{F}_j &= \vec{F}_j \\ \vec{\tau}' &= \sum \vec{r}_j' \times \vec{F}_j' = \sum (\vec{r}_j - \vec{R}) \times \vec{F}_j \\ &= \sum \vec{r}_j \times \vec{F}_j - \vec{R} \times \sum \vec{F}_j = \sum \vec{r}_j \times \vec{F}_j = \tau \end{aligned}$$

KK 6.6

$$N_i + N_o = Mg \quad \text{--- (1)}$$

$$f_i + f_o = M \frac{v^2}{R} \quad \text{--- (2)}$$

Take torque about CM which is zero.

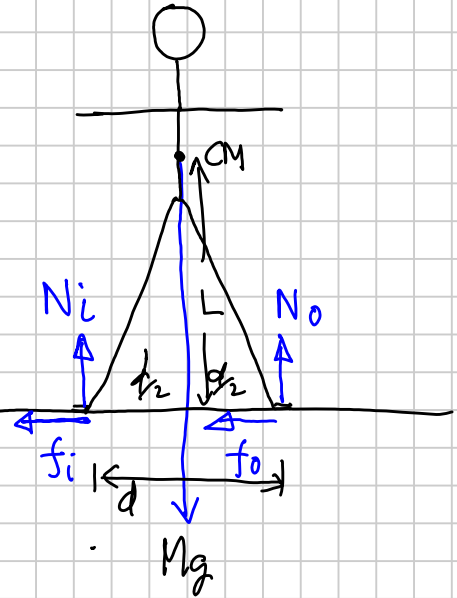
$$N_o \frac{d}{2} - N_i \frac{d}{2} - (f_i + f_o)L = 0$$

$$\Rightarrow (N_o - N_i) \frac{d}{2} - M \frac{v^2}{R} L = 0 \quad \text{--- (3)}$$

Solving (1) & (3)

$$N_i = \frac{1}{2} \left[ Mg - \frac{Mv^2 L}{R(d/2)} \right]$$

$$N_o = \frac{1}{2} \left[ Mg + \frac{Mv^2 L}{R(d/2)} \right]$$



KK 6.10

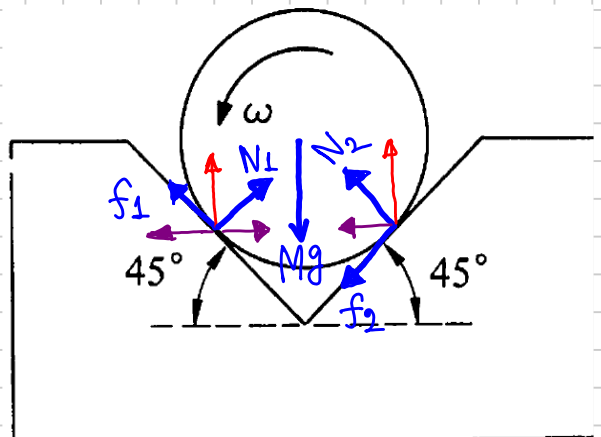
$$\text{Torque } \tau = (f_1 + f_2)R$$

$$= \mu [N_1 + N_2]R$$

Considering components of different forces:

$$(N_1 + N_2) \frac{1}{\sqrt{2}} + f_1 \frac{1}{\sqrt{2}} - f_2 \frac{1}{\sqrt{2}} = Mg$$

$$\Rightarrow N_1(1 + \mu) + N_2(1 - \mu) = \sqrt{2} Mg$$



$$f_1 \frac{1}{\sqrt{2}} + f_2 \frac{1}{\sqrt{2}} + \frac{N_2}{\sqrt{2}} = N_1 \frac{1}{\sqrt{2}}$$

$$\Rightarrow N_2 = f_1 + f_2 + N_2$$

$$\Rightarrow N_1(1 - \mu) = N_2(1 + \mu)$$

Solving

$$N_1 = \sqrt{2} \frac{Mg}{2} \left( \frac{1 + \mu}{1 + \mu^2} \right)$$

$$N_2 = \sqrt{2} \frac{Mg}{2} \left( \frac{1 - \mu}{1 + \mu^2} \right)$$

Hence  $\tau = \sqrt{2} Mg \left( \frac{\mu}{1 + \mu^2} \right) R.$

$$\tau = \sqrt{2} \times 100 \left( \frac{0.5}{1 + 0.5^2} \right) 0.1 = 5.7 \text{ N.m}$$

KK 6.13

(a) For central force angular momentum is conserved.

$$\text{as } \tau = 0$$

$$L = m v_0 r = m v_f R$$

$$\Rightarrow v_f = \frac{v_0 r}{R}$$

(b)



Angular momentum is not conserved as force is not central.

$$\tau \neq 0$$

Kinetic energy is conserved

$$\frac{1}{2} m v_f^2 = \frac{1}{2} m v_0^2 \Rightarrow v_f = v_0$$