Tutorial 03	PH-101	Due: 14/08/2012	DKG

Note Title 8/9/2012

3.9 A freight car of mass M contains a mass of sand m. At $t = 0$ a constant horizontal force F is applied in the direction of rolling and at the same time a port in the bottom is opened to let the sand flow out at constant rate dm/dt. Find the speed of the freight car when all the sand is gone. Assume the freight car is at rest at $t = 0$.

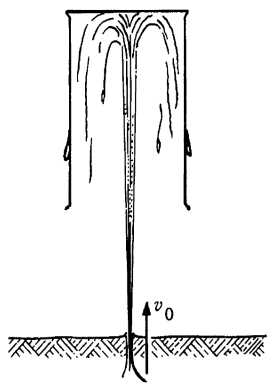
3.14 N men, each with mass m, stand on a railway flatcar of mass M. They jump off one end of the flatcar with velocity u relative to the car. The car rolls in the opposite direction without friction.

a. What is the final velocity of the flatcar if all the men jump at the same time?

b. What is the final velocity of the flatcar if they jump off one at a time? (The answer can be left in the form of a sum of terms.)

c. Does case *a* or case *b* yield the largest final velocity of the flat car? Can you give a simple physical explanation for your answer?

3.17 An inverted garbage can of weight W is suspended in air by water from a geyser. The water shoots up from the ground with a speed v_0, at a constant rate dm/dt. The problem is to find the maximum height at which the garbage can rides. What assumption must be fulfilled for the maximum height to be reached?



Ans. clue. If $v_0 = 20$ m/s, $W = 10$ kg, $dm/dt = 0.5$ kg/s, then $h_{\max} \approx 17$ m

3.18 A raindrop of initial mass M_0 starts falling from rest under the influence of gravity. Assume that the drop gains mass from the cloud at a rate proportional to the product of its instantaneous mass and its instantaneous velocity:

$$\frac{dM}{dt} = kMV,$$

where k is a constant.

Show that the speed of the drop eventually becomes effectively constant, and give an expression for the terminal speed. Neglect air resistance.

3.20 A rocket ascends from rest in a uniform gravitational field by ejecting exhaust with constant speed u. Assume that the rate at which mass is expelled is given by $dm/dt = \gamma m$, where m is the instantaneous mass of the rocket and γ is a constant, and that the rocket is retarded by air resistance with a force mbv, where b is a constant. Find the velocity of the rocket as a function of time.

Solutions Tutorial 03 DKG

Note Title 8/9/2012

3.9

Mass of the car = M

Mass of sands at time t = m(t)

Mass of the sand released in time Δt = Δm

$$p_i = (M + m(t) + \Delta m)v$$

$$p_f = (M + m(t))(v + \Delta v) + \Delta m(v + \Delta v)$$

$$p_f - p_i = (M + m(t))\,\Delta v = F\Delta t$$

$$dv = \frac{F\,dt}{M + m(t)}$$

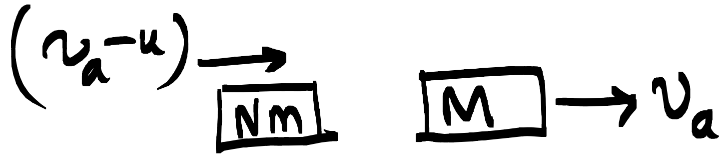
$$\frac{dm}{dt} = \alpha$$

$$m(t) = m - \alpha t$$

$$\int_0^{v_f} dv = F\int_0^{m/\alpha} \frac{dt}{M + m - \alpha t}$$

$$\Rightarrow \quad v_f = \frac{F}{\alpha}\ln\left(1 + \frac{m}{M}\right)$$

3.14 (a)



$$P_i = 0$$

$$P_f = M v_a + Nm\,(v_a - u) = 0$$

$$v_a = u\,\frac{Nm}{Nm + M}$$

(b) For first jumping,

$$m\,[u - v_1] = [M + (N-1)\,m]\,v_1$$

For 2nd person jumping

$$m\,(u - v_2) = [M + (N-2)\,m]\,v_2 - [M + (N-1)\,m]\,v_1$$

For k^{th} jump

$$m\,(u - v_k) = [M + (N-k)\,m]\,v_k - [M + (N-k+1)\,m]\,v_{k-1}$$

$$v_k = v_{k-1} + \frac{mu}{M + (N-k+1)\,m}$$

so final velocity

$$v_b = \sum_{k=1}^{N} \frac{mu}{M + (N-k+1)\,m}$$

$$v_b = \left[\frac{m}{M+Nm} + \frac{m}{M+(N-1)m} + \cdots + \frac{m}{M+m} \right] u$$

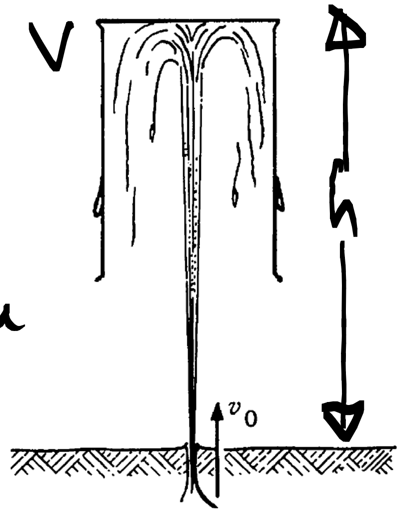
(c) $$v_a = u \frac{Nm}{Nm+M}$$

$$v_a = \left[\frac{m}{M+Nm} + \frac{m}{M+Nm} + \cdots + \frac{m}{M+Nm} \right] u$$

Hence $V_b > V_a$

$\boxed{3.17}$ water speed at height h

$$v = \sqrt{v_0^2 - 2gh}$$



Assume water bounces elastically from the can and amount of water hitting the surface is Δm in time Δt

$$\Delta p = \Delta m (2V)$$

$$F_{ext} = W = \frac{dp}{dt} = 2u \frac{dm}{dt}$$

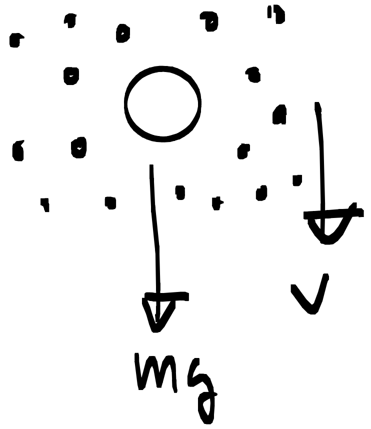
$$\Rightarrow v = W / \left(2 \frac{dm}{dt}\right)$$

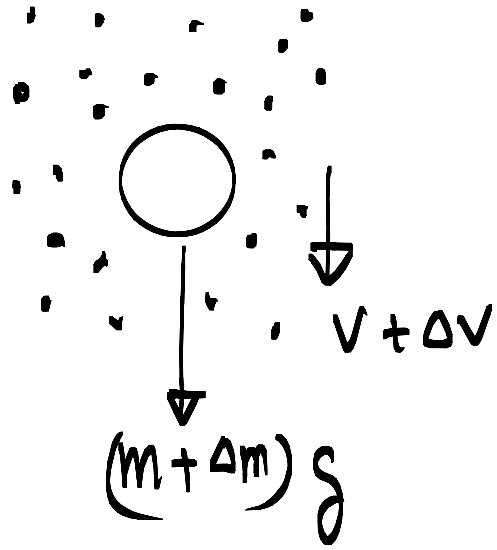
$$\Rightarrow h = \frac{1}{2g} \left[v_0^2 - \frac{W^2}{4 \left(\frac{dm}{dt}\right)^2} \right]$$

if $v_0 = 20$ m/s, $W = 10$ kg (?), $\frac{dm}{dt} = 0.5$ kg/s

$h \simeq 15$ m (No idea how 17m is coming)

$\boxed{3.18}$





$$P_i = mv$$

$$P_f = (m + \Delta m)(v + \Delta v)$$

$$\Delta p = m\Delta v + \Delta m\, v$$

$$F = \frac{dp}{dt} = m\frac{dv}{dt} + \frac{dm}{dt} v$$

or $$m\frac{dv}{dt} + kMv^2 = mg$$

$$\Rightarrow \quad \frac{dv}{dt} = g - kv^2$$

As v increases $\frac{dv}{dt}$ decreases

$$\left(\frac{dv_{term}}{dt}\right) = 0$$

$$g = kv_{term}^2 \Rightarrow v_{term} = \sqrt{g/k}$$

$\boxed{3.20}$ $$F^{ext} = m\frac{dv}{dt} - u\frac{dm}{dt} \quad \text{(Rocket eqn)}$$

$$m\frac{dv}{dt} - u\frac{dm}{dt} = -mg - mbv$$

$$m\frac{dv}{dt} - u\gamma m = -mg - mbv$$

$$\frac{dv}{dt} + bv = \gamma u - g$$

Solving with $v(t=0) = 0$

$$v(t) = \left(\frac{\gamma u - g}{b}\right)\left(1 - e^{-bt}\right)$$