Coverage in WSNs
(Sweep Coverage on Graph)

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Introduction

• Coverage it is defined as quality of surveillance of a sensing function in WSNs.
• Is is a widely studied research area, many efforts have been made for addressing coverage problems in sensor networks, which are:
  – Point coverage
  – Area coverage,
  – Barrier coverage,
  – k-coverage, etc.
Point Coverage
Area Coverage
Barrier Coverage
Sweep Coverage

• For those coverage scenarios, the monitored objective being covered all time, featured as static coverage or full coverage.

• In some applications, patrol inspection/periodic monitoring are sufficient instead of continuous monitoring, which is featured as a sweep coverage.

• For such applications a small number of mobile sensor nodes can guarantee sweep coverage with a given sweep period.
Static coverage/Full coverage
Sweep Coverage
Sweep Coverage

Let $U = \{u_1, u_2, \ldots, u_n\}$ be a set of points on a two dimensional plane and $M = \{m_1, m_2, \ldots, m_p\}$ be a set of mobile sensor nodes. A point $u_i$ is said to be \textbf{t-sweep covered} if and only if at least one mobile sensor node visits $u_i$ within every \textbf{t time period}.

- The set $U$ is said to be \textbf{globally sweep covered} by the mobile sensor nodes of $M$ if all $u_i$ are \textbf{t-sweep covered}.
- The \textbf{time period} $t$ is called the \textbf{sweep period} of the points in $U$.

Previous Results

• The contribution of Li et al. in [1] are the following:
  – The sweep coverage problem is NP-hard.
  – It is not possible to approximate sweep coverage problem with a factor less than 2 unless P = NP.
  – Proposed a 3-approximation algorithm for sweep coverage problem.
Previous Results

Basic idea of the proposed algorithm [1]

• Find an approximated TSP tour among the set of points. (1.5 factor)
• Divide the tour into parts of length (vt/2).
• One mobile sensor is deployed at each of the parts.
• Mobile sensors moves back-and-forth to cover the points belonging to the corresponding parts.
Previous Results
Correctness

• Li et al. proved that the proposed algorithm is a 3 factor approximation algorithm.

• But the statement is wrong

Counter Example:

\[ v=20, \ t=1 \]

• Number of mobile sensors needed according to [1] is 20 Optimal solution is 2 as two sensor at two points will be sufficient
Sweep Coverage Problem

- We introduce a variation of sweep coverage named as GSweep coverage problem, where the Pols are represented by vertices of a weighted graph.
- We propose a 3-approximation algorithm to guarantee sweep coverage of all vertices of the graph.
- We generalize the above algorithm to solve the problem with approximation factor \(O(\log \rho)\) when vertices of the graph have different sweep periods and processing times, where \(\rho\) is the ratio of the max and min sweep periods.
GSweep coverage problem

• Let $G = (U, E)$ be a weighted graph, where weight of an edge $(u_i, u_j)$ for $(u_i, u_j) \in U$ is denoted by $|(u_i, u_j)|$. Let $n$ be the total number of vertices in $G$. For any subgraph $H$ of $G$, we denote $|H|$ as the sum of the edge weights of $H$.

• **Definition (GSweep coverage):** Let $U = \{u_1, u_2, \cdots, u_n\}$ be the vertices of a weighted graph $G = (U, E, w)$ and $M = \{m_1, m_2, \cdots, m_n\}$ be the set of mobile sensors. The mobile sensors move with a uniform speed $v$ along the edges of the graph. For given $t > 0$, find the minimum number of mobile sensors such that each vertex of $G$ is $t$-sweep covered.

• The problem is NP hard, follows from the hardness proof given in [1].
**Algorithm 1: GSweepCoverage**

1. for \( k = 1 \) to \( n \) do
2. Find the minimum spanning forest \( F_k \) on \( G \) with \( n - k \) edges. Let \( C_1, C_2, \ldots, C_k \) be the connected components of \( F_k \).
3. \( N_k = 0 \)
4. for \( j = 1 \) to \( k \) do
5. if \( C_j \) is a component having more than one vertex then
6. \( N_k = N_k + \left\lceil \frac{2w(C_j)}{v_f} \right\rceil \)
7. else
8. \( N_k = N_k + 1 \)
9. end if
10. end for
11. end for
12. Let \( J \) be the index \( \in \{1, 2, \ldots, n\} \) such that \( N_J = \min\{N_1, N_2, \ldots, N_n\} \)

13. Let \( C_1, C_2, \ldots, C_J \) be the connected components of \( F_J \).
14. for \( i = 1 \) to \( J \) do
15. if \( C_i \) is a component having more than one vertex then
16. Find a tour \( T_i \) on \( C_i \) by doubling each edge of \( C_i \). Partition the tour into \( \left\lceil \frac{w(T_i)}{v_f} \right\rceil \) parts and deploy one mobile sensor at each of the partitioning points.
17. else
18. Deploy one mobile sensor at the vertex of \( C_i \).
19. end if
20. end for
21. All mobile sensors start moving at the same time along the respective tours having more than one vertex in same direction. If a mobile sensor is deployed on a tour containing only one vertex then it periodically monitors the vertex like a static sensor.
Example
For $k=1$, the weight of the Forest $F_1$ is 22.

Number of sensor nodes needed is $\lceil \frac{44}{8} \rceil = 6$.
For $k=2$, the weight of the Forest $F_2$ is 14.

Number of sensor nodes needed is $\lceil \frac{28}{8} \rceil + 1 = 5$. 

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For $k=3$, the weight of the Forest $F_3$ is 9.
Number of sensor nodes needed is $\lceil 18/8 \rceil + 2 = 5$.
For $k=4$, the weight of the Forest $F_4$ is 9. Number of sensor nodes needed is $\lceil \frac{8}{8} \rceil + 3 = 4$. 

$4^{th}$ Iteration, $vt=8$
For $k=5$, The weight of the Forest $F_5$ is 9
Number of sensor nodes needed is 5
Lemma: Let opt be the minimum number of sensors needed in the optimal solution. Let $opt'$ be the minimum number of paths of length $\leq vt$ which span $U$ on $G$. Then $opt \geq opt'$.

Proof: Let us assume that $opt < opt'$. Let $P_1, P_2, \ldots, P_{opt}$ be the movement paths of the sensors in optimal solution in a time interval $[t_0, t+t_0]$. Then $|P_i| \leq vt$.

- According to the definition of Gsweep coverage, all vertices of $G$ are covered by the set of paths $P_1, P_2, \ldots, P_{opt}$.
- Hence $P_1, P_2, \ldots, P_{opt}$ is a collection of paths with $|P_i| \leq vt$ which spans $U$ on $G$, which contradicts the fact that $opt < opt'$.
- Therefore $opt \geq opt'$. 

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Analysys

**Theorem:** The Algorithm is a 3 factor approximation algorithm.

**Proof:**
- Let $\text{opt}$ be the number of mobile sensors needed in optimal solution.
- Let $\text{opt}'$ be the minimum number of paths of length $\leq vt$ which span $U$ on $G$.
- We know $\text{opt} \geq \text{opt}'$ (from previous lemma)
- Let $\text{Min}_\text{path}$ be the sum of the edge weights of $\text{opt}'$ number of paths of length $\leq vt$ which span $U$. 
Proof (continue..)

- Algorithm 1 chooses the minimum over all $N_k$ for $k = 1$ to $n$
- $N \leq N_k$ for $k=1$ to $n$.
- Consider the iteration when $k=\text{opt'}$.
- Then $|F_k| \leq \text{Min}_\text{path}$ and $\text{Min}_\text{path} \leq k.vt$
- Total length of the tours after doubling the edges of $F_k$ is $2|F_k|$.
- Since $\left\lceil |T_i|/vt \right\rceil \leq (|T_i|/vt)+1$, therefore total number of sensor nodes needed is $N_k = 2(|F_k|/vt) + k \leq 2k+k = 3k \leq 3 \text{ opt}$.

Therefore approximation factor of our algorithm is 3.
The Algorithm is a 3 factor approximation algorithm.

Proof: Let \( opt \) be the minimum number of mobile sensors required in the optimal solution. Let \( opt' \) be the minimum number of paths of weight \( \leq vt \) which span \( U \) on \( G \) and \( Min\_path \) be sum of the weight of all the paths. Then by Lemma 1,

\[
\text{opt}' \leq \text{opt}
\]

and

\[
Min\_path \leq \text{opt}' \times vt
\]

Again, these \( opt' \) number of paths of weights \( \leq vt \) forms a spanning forest with \( opt' \) disjoint connected components and \( F_{opt'} \) is the minimum spanning forest with \( opt' \) connected components. Therefore,

\[
w(F_{opt'}) \leq Min\_path
\]

The Algorithm 1 chooses the minimum over all \( N_k \) for \( k = 1 \) to \( n \). Let us consider the iteration of the algorithm when \( k = opt' \).

After doubling edges in step 16 of the algorithm, the total weight of the movement paths of the mobile sensors is \( \leq 2w(F_k) \). Since \( \left\lceil \frac{T_k}{vt} \right\rceil \leq \frac{T_k}{vt} + 1 \), the number of mobile sensors needed in our solution is

\[
N \leq \frac{2w(F_k)}{vt} + k
\]

\[
\leq \frac{2\cdot Min\_path}{vt} + k \quad \text{from Equation (3)}
\]

\[
\leq 2k + k \quad \text{from Equation (2)}
\]

\[
= 3k
\]

\[
\leq 3opt \quad \text{from Equation (1)}
\]
Sweep Coverage with Different Sweep Periods and Processing Time

• In practice some finite amount of time i.e., processing time is required by a mobile sensor to process some tasks such as monitoring, sampling or exchanging data at each of the vertices during its visits.

• Sweep periods may be different for different vertices as per requirement of applications.
Sweep Coverage with Different Sweep Periods and Processing Time

• **Problem 2.** Let $U = \{u_1, u_2, \cdots, u_n\}$ be the vertex set of a weighted graph $G = (U, E, w)$, 
$\{t_1, t_2, \cdots, t_n\}$ and $\{\tau_1, \tau_2, \cdots, \tau_n\}$ be the corresponding set of sweep periods and processing times of the vertices. Let $M = \{m_1, m_2, \cdots, m_n\}$ be the set of mobile sensors which can move with a uniform speed $v$ along the edges of the graph. Find the minimum number of mobile sensors such that each $u_i$ is $t_i$-sweep covered.
Proposed Algorithm

• The proposed algorithm execute in two different phases.

• In the 1\textsuperscript{st} phase, we will compute the number of mobile sensors required for sweep coverage.

• In the 2\textsuperscript{nd} phase, initial positions and movement schedules of mobile sensors are computed.
Finding Number of Mobile sensors

• For the given graph $G=(U,E,w)$, compute the complete graph $G’=(U, E’,w’)$, where
  $$w'(u_i,u_j)=d(u_i,u_j)+v(\tau_i+\tau_j)/2$$
  Where $d(u_i,u_j)$ is the shortest path between $u_i,u_j$ with respect to $w$.

• Let $\ell=(t_{\text{max}}/t_{\text{min}})$, where $t_{\text{min}}$ and $t_{\text{max}}$ are the minimum and maximum sweep periods among the vertices.
Finding Number of Mobile sensors

- Let $U_i = \{u_j | 2^{i-1} t_{\text{min}} \leq t_j < 2^i t_{\text{min}} \}$
- $\{U_i | i = 1 \text{ to } \lceil \log q \rceil \}$ is a partition of U.
- Let $G'_i$ be the induced subgraph of $G'$ for the vertex set $U_i$.

- For each $i, i = 1 \text{ to } \lceil \log q \rceil$, Apply step 1 to 12 of Algorithm 1 on $G'_i$ to find the number of mobile sensors required for $(2^{i-1} \cdot t_{\text{min}})$-sweep coverage for all the vertex in $U_i$. 

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Movement strategy of the mobile sensors

• Let $C_1, C_2, \ldots, C_{j_i}$ be the connected components for $G’i$.

• Find tours $T_1, T_2, \ldots, T_{j_i}$ after doubling the edges.

• If $T_k$ contains only one vertex, deploy one mobile sensor which acts like a static sensor.
Movement strategy of the mobile sensors

- If $T_k$ contains more than one vertex, compute $T'_k$ by replacing vertices and edges of $T_k$.
- Let $u_{i1}, u_{i2}, ..., u_{ih}$ be the vertices of $T_k$ in the clockwise direction.
- Replace each $u_{ij}$ by two vertices $u'_{ij}$ and $u''_{ij}$ and introduce an edge $(u'_{ij}, u''_{ij})$ with $w'(u'_{ij}, u''_{ij}) = vT_l$.
- Each edge $(u_{ij}, u_{ij+1})$ is replaced by $(u''_{ij}, u'_{ij+1})$.
Movement strategy of the mobile sensors

\[ u_{i_1} \quad u_{i_2} \quad u_{i_3} \]

\[ u''_{i_1} \quad u''_{i_2} \quad u''_{i_3} \]

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Movement strategy of the mobile sensors

• Partition $T'_k$ into $\lceil w'(T'_k)/vt \rceil$ parts of weight at most $vt$.

• Deploy one mobile sensor at each of the partitioning points.

• The mobile sensor starts their movement along $T'_k$ at the same time in the same direction.
**Partition on the tour** $T'_k$

- The mobile sensor deployed at $P$ will wait for a time taken to move from $P$ to $u_{i1}''$. Then it moves with velocity $v$ along the tour.

- The mobile sensor deployed at $Q$ will start moving along the edges $(u_{i2}, u_{i3})$ in $G'$. 

![Diagram of a graph with nodes and edges labeled $P$, $Q$, $S$, and $R$.]
Analysis

• **Theorem:** According to the movement strategy of the mobile sensors each vertex $u_i$ in $U$ is $t_i$-GSweep covered with processing time $T_i$.

**Proof:**

• If $u_i$ belongs to a component with one vertex, then $t_i$ sweep coverage is trivial by the mobile sensor deployed at $u_i$.

• Now if $u_i$ belongs to a component with more than one vertex, then according to the proposed algorithm for Problem 2, $u_i \in U_j$ for some $j = 1$ to $\lceil \log D \rceil$.

• By Theorem 1, $u_i$ is sweep covered with sweep period $2^{j-1} \cdot t_{min} \leq t_i$. Therefore $u_i$ is $t_i$ sweep covered.
Analysis

• **Theorem:** The approximation factor of the proposed algorithm for Problem 2 is $6 \lceil \log k \rceil$.

**Proof:**

• Let OPT be the optimal solution for problem 2.
• Let $OPT_1, OPT_2, \ldots, OPT_{\lceil \log k \rceil}$ be the optimal solutions on $G'_1, G'_2, \ldots, G'_{\lceil \log k \rceil}$.
• Then $OPT \leq OPT_j$, for $j = 1$ to $\lceil \log k \rceil$. 
Proof (cont..)

- Let \( \text{OUT}_1, \text{OUT}_2, \ldots, \text{OUT}_{\lfloor \log \varrho \rfloor} \) be the number of mobile sensors required for our algorithm.
- Then, \( \text{OUT}_j \leq 6 \text{OPT}_j, j = 1 \text{ to } \lfloor \log \varrho \rfloor \), as the length of the partition in \( \text{OPT}_j \) is at most twice of the length of the partition in \( \text{OUT}_j \) and the Algorithm 1 is a 3-approximation algorithm.
- Therefore, total number of mobile sensors required is
- \( \text{OUT}_1 + \text{OUT}_2 + \ldots + \text{OUT}_{\lfloor \log \varrho \rfloor} \leq 6 \lfloor \log \varrho \rfloor \text{OPT} \).
- Hence the proof follows.
Sweep Coverage with Mobile Sensors Having Different Speeds

• Problem 3: Let \( U = \{u_1, u_2, \cdots, u_n\} \) be the vertex set of a weighted graph \( G = (U, E, w) \). Let \( M = \{m_1, m_2, \cdots, m_n\} \) be the set of mobile sensors with velocities \( \{v_1, v_2, \cdots, v_n\} \) respectively. For a given \( t>0 \), Find the minimum number of mobile sensors such that each \( u_i \) is t-sweep covered.
Sweep Coverage with Mobile Sensors Having Different Speeds

• **Theorem:** No polynomial time constant factor approximation algorithm exists to solve the sweep coverage problem by mobile sensors with different velocities unless P=NP.

**Proof:**

We will prove this Theorem using reduction from Metric-TSP problem.
Proof(Cont..)

• If possible let there is a $k$ factor approximation algorithm $A$.

• Consider an instance $(G_1, L)$ of metric TSP problem, where $L$ is weight of the minimum weight TSP tour in a complete weighted graph $G_1 = (U_1, E_1, w_1)$ with $n (> k)$ Vertices which satisfy triangular inequality.
Proof (Cont..)

• Construct a complete graph \( G_2 = (U_2, E_2, w_2) \) with \( n^2 \) vertices as follows:

For each vertex \( u_i \) in \( U_1 \), consider \( n \) vertices \( u_{i1}, u_{i2}, \ldots, u_{in} \) in \( U_2 \).

\[
\begin{align*}
w_2(u_{ik}, u_{jl}) &= w_1(u_i, u_j) - \frac{1}{(n+1)^2} \\
w_2(u_{ik}, u_{il}) &= \frac{1}{(n-1)(n+1)^2}
\end{align*}
\]
Proof (Cont..)

• Claim: G1 has a tour with weight at most \( l \) iff G2 has a tour with weight at most \( l \).

Let \( T_1: u_{i1}, u_{i2}, \ldots, u_{in}, u_{i1} \) be a tour of G with weight \( \leq l \).

Construct \( T_2: u_{i1}^1, u_{i1}^2, \ldots, u_{i1}^n, u_{i2}^1, u_{i2}^2, u_{i2}^n, \ldots, u_{in}^1, u_{in}^2, \ldots, u_{in}^n, u_{i1}^1 \).

Note that \( w_2(T_2) \leq l \).
**Proof (Cont..)**

- Claim: $G_1$ has a tour with weight at most $l$ iff $G_2$ has a tour with weight at most $l$.

Conversely, let $T_2': x_1, x_2, \ldots, x_{n_2}, x_1$ be a tour in $G_2$ with $w_2(T_2') \leq l$.

- Construct a tour $T_1'$ from $T_2'$ as follows:

  Delete all the edges of type $(u_{ik},u_{il})$ from $T_2'$.

  For each of the remaining edges of form $(u_{ij},u_{ki})$, consider the corresponding edge in $G_1$ and construct the subgraph $\mathcal{E}$ of $G_1$.

- For example, if edges $(u_{k_1 i}, u_{l_1 j})$ and $(u_{k_2 i}, u_{l_2 j})$ for $k_1, k_2$ and $l_1, l_2$ are in $T_0$ 2, then we consider the edge $(u_i, u_j)$ twice in $\mathcal{E}$. 
Proof(Cont..)

• Claim: G1 has a tour with weight at most \( l \) iff G2 has a tour with weight at most \( l \).

• Construct a tour \( T_1' \) from \( E \) by short cutting.
• Note that, \( w_1(T_1') \leq l \)
• Hence we can say that if $L$ is the weight of the optimal tour in $G_1$, then the weight of the optimal tour in $G_2$ is also $L$, otherwise by the above fact, a tour of $G_1$ with weight less than $L$ can be found.
Proof (Cont..)

- We consider an instance of the sweep coverage problem by mobile sensors with different velocities as follows:
  - We take G2 as the graph, sweep period \( t = 1 \), and a set of \( n^2 \) mobile sensors with velocity \( L \) for one and zero for remaining \( n^2 - 1 \) mobile sensors.
  - Clearly, the optimal solution of the problem is one, since using the mobile sensor with velocity \( L \), 1-sweep coverage of each of \( n^2 \) vertices can be guaranteed.
Proof(Cont..)

- As A is a \textit{k factor approximation algorithm}, A returns
- the number of mobile sensors required for sweep coverage of \textit{G2 is k in worst case}.
- \textit{Among these k mobile sensors}, one is with velocity \textit{L and remaining are} with velocities zero.
- Therefore, the mobile sensor with velocity \textit{L moves along a tour covering n2 − k + 1 vertices}.
- Remaining \textit{k − 1 mobile sensors cover remaining k − 1 vertices one for each}.

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Proof(Cont..)

• Hence at least one vertex from each of the set \{u_1i, u_2i, \cdots, u_{ni}\} for different \(i\) is visited by the mobile sensor with velocity \(L\) and weight of the tour is at most \(L\).

• From this tour, a tour \(T\) of weight at most \(L\) of \(G_1\) can be constructed in the same way by constructing an Eulerian graph of \(G_1\) and short cutting as explained before.
Proof (Cont..)

• Since $L$ is the weight of the optimal tour in $G_1$, $w(T) = L$.

• Hence optimal solution for TSP problem on $G_1$ can be computed in polynomial time by applying algorithm A on $G_2$, which is not possible unless $P=NP$.

• Hence statement of the theorem follows.
Conclusion

• In this paper we overcome the limitation of a previous study [1] on sweep coverage.
• The key argument is that when the graph is sparse, it is better to provide sweep coverage with a mixture of static and mobile sensors, instead of using only mobile sensors as the previous study did.
• We propose a 3-approximation algorithm to solve this NP hard problem for any positively weighted graph.
Conclusion

• We have generalized the above problem with different sweep periods and introducing different processing times for the vertices of the graph.

• Our proposed algorithm for this generalized problem achieves approximation factor $O(\log \mathcal{U})$, where $\mathcal{U} = \frac{t_{\text{max}}}{t_{\text{min}}}$, $t_{\text{min}}$ and $t_{\text{max}}$ are the minimum and maximum sweep periods among the vertices.

• If velocities of the mobile sensors are different, we have proved that it is impossible to design any constant factor approximation algorithm to solve the sweep coverage problem unless P=NP.
THANK YOU