Brief Announcement: Sweep Coverage with Mobile and Static Sensors

Barun Gorain and Partha Sarathi Mandal

Department of Mathematics
Indian Institute of Technology Guwahati, India
{b.gorain,psm}@iitg.ernet.in

Abstract. The objective of a sweep coverage problem is to find the minimum number of mobile sensors to ensure the periodic monitoring for a given set of points of interest. In this paper we have remarked on the flaw of approximation algorithms proposed in the paper [1] for sweep coverage with mobile sensors and proposed a 3-approximation algorithm to guarantee sweep coverage with mobile and static sensors.

Keywords: Sweep Coverage, TSP tour, Approximation Algorithm, WSNs.

1 Introduction

Sweep coverage concept is recently introduced in the literature where periodic patrol inspections are sufficient for a given set of points of interest (PoIs) by a set of mobile sensors [1]. A point is said to be $t-$*sweep covered* if and only if at least one mobile sensor visits the point within every t time period, where t is called *sweep period* of the point. Objective of the sweep coverage problem is to find minimum number of mobile sensors to guarantee sweep coverage for the set of PoIs. Li et al. in [1] proved that finding minimum number of mobile sensors to sweep cover a set of PoIs is NP hard and proposed a $(2+\epsilon)-$approximation and a $3-$approximation algorithm to find minimum number of mobile sensors. Each algorithm in [1] computes approximate TSP tour through all the PoIs and divides the tour into equal parts of length $\frac{vt}{2}$, where v is the uniform speed of the mobile sensors and t is the sweep period for all the PoIs. Then one mobile sensor is deployed in every partition and let the mobile sensors move back and forth to sweep cover all PoIs of the corresponding partitions. But there is a serious flaw of the approximation algorithms proposed in [1] as explained below. In general, if the distance between the PoIs is large compare to the length of the partitions then there may not exist any PoI on some of the parts. For example, assume there are only two PoIs on a plane, the distance between them is 100 meter and vt is 20 meter. Therefore, the length of the TSP tour is 200 meter and according to the algorithms mentioned in [1], the total number of mobile sensors needed is $200/\frac{vt}{2}$ = 200/10=20. But practically it is sufficient to place only two sensors to monitor two PoIs respectively and thus two sensors can guarantee sweep coverage. Hence

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the algorithms proposed by Li et al. in [1] does not provide a solution which achieve approximation factors $(2 + \epsilon)$ or 3.

In this paper we introduce a variation of sweep coverage named as GSweep coverage problem, where the PoIs are represented by vertices of a weighted graph. We propose a $3-$approximation algorithm to guarantee sweep coverage of all vertices of the graph with mobile and static sensors.

2 GSweep Coverage

Let $G = (U, E)$ be a weighted graph, where weight of an edge (u_i, u_j) for $u_i, u_j \in U$ is denoted by $|(u_i, u_j)|$. Let n be the total number of vertices in G. For any subgraph H of G, we denote $|H|$ as the sum of the edge weights of H. The definition of GSweep coverage is given below.

Definition 1. *Let $\mathcal{U} = \{u_1, u_2, \cdots, u_n\}$ be the vertices of a weighted graph, $S = \{s_1, s_2, \cdots, s_p\}$ and $M = \{m_1, m_2, \cdots, m_q\}$ be the sets of static and mobile sensors respectively. The mobile sensors move with a uniform speed v along edges of the graph. A vertex u_i is said to be $t-$GSweep covered with (mobile and/or static) sensors iff either at least one mobile sensor m_j visits u_i in every t time period or one static sensor s_j is deployed at u_i which periodically monitors u_i in every t time period.*

The objective of GSweep coverage problem is to find minimum number of sensors, combination of static and mobile sensors, such that each vertex is $t-$GSweep covered. The problem is NP hard, follows from the hardness proof given in [1].

We propose Algorithm 1: GSweepCoverage to find minimum number of sensors for GSweep coverage problem. First two steps of the algorithm executes n iterations for finding the best possible solution i.e., number of sensors. In kth iteration $(1 \leq k \leq n)$, the minimum spanning forest F_k with k connected components $C_1, C_2, \cdots, C_k$ is computed. After that k disjoint tours $T_1, T_2, \cdots, T_k$ are found by doubling all edges of each component. Partition each T_i into $\left\lceil \frac{|T_i|}{vt} \right\rceil$ parts of length vt. Total number of partitions for an iteration is equal to the number of sensors required for that iteration. Minimum over the number of sensors of all iterations is chosen as the solution of our Algorithm 1. Initial positions of mobile and static sensors are calculated in steps 5-13 of the algorithm with movement scheduling of the mobile sensors.

Lemma 1. *Let opt be the minimum number of sensors needed in the optimal solution. Let opt′ be the minimum number of paths of length $\leq vt$ which span U on G. Then opt $\geq$ opt′.*

Proof. Let us assume that $opt < opt'$. Consider the path of movements by the mobile and static sensors in the optimal solution during any time period $[t_0, t_0 + t]$, where the path for a static sensor is of length zero. Let $P_1, P_2, \cdots, P_{opt}$ be the movement paths of the sensors with $|P_i| \leq vt$. Since each vertex is visited by a sensor at least once in time period t therefore $\bigcup P_i$ spans all the vertices of U. Hence, $\{P_1, P_2, \cdots, P_{opt}\}$ is a collection of paths with $|P_i| \leq vt$ that spans U, which contradicts the fact that $opt < opt'$. Therefore $opt \geq opt'$. □

Algorithm 1: GSweepCoverage

1: **for** $k = 1$ *to* n **do**
2: Find the minimum spanning forest F_k on G with $n - k$ edges. Let $C_1, C_2, \cdots, C_k$ be the connected components of F_k. $N_k = \sum_{i=1}^{k} \left\lceil \frac{2|C_i|}{vt} \right\rceil$.
3: **end for**
4: Let j be the index $\in [1, 2, \cdots, n]$ such that $N_j = \min\{N_1, N_2, \cdots, N_n\}$
5: Let $C_1, C_2, \cdots, C_j$ be the connected components of F_j.
6: **for** $i = 1$ to j **do**
7: **if** C_i is a component having more than one vertices **then**
8: Find a tour T_i on C_i by doubling each edges of C_i. Partition the tour into $\left\lceil \frac{|T_i|}{vt} \right\rceil$ parts and deploy one mobile sensor at each of the partitioning points.
9: **else**
10: Deploy one static sensor at the vertex of C_i.
11: **end if**
12: **end for**
13: All mobile sensors start moving at the same time along the respective tours in same direction.

Theorem 1. *The Algorithm 1 is a 3 factor approximation algorithm.*

Proof. Let *opt* be the minimum number of sensors required in the optimal solution and *opt*′ be the minimum number of paths of length $\leq vt$ which span U on G. Then by Lemma 1, $opt' \leq opt$.

Algorithm 1 chooses the minimum over all N_k for $k = 1$ to n. Let us consider the iteration of the algorithm when $k = opt'$. Let Min_path be the total sum of the length of the edges in *opt*′. Then $Min\_path \leq k \times vt$. Again as there are k disjoint connected components in *opt*′ and F_k is the minimum spanning forest with k connected components, we have $|F_k| \leq Min\_path$. After doubling edges in step 8, the total length of the movement paths of the sensors is $\leq 2|F_k|$. Since $\left\lceil \frac{|T_i|}{vt} \right\rceil \leq \frac{|T_i|}{vt} + 1$, the number of sensors needed in our solution is $N \leq \frac{2|F_k|}{vt} + k \leq \frac{2Min\_path}{vt} + k \leq 3k \leq 3opt$. Therefore the approximation factor of the Algorithm 1 is 3. □

3 Conclusion

In this paper we overcome the limitation of a previous study [1] on sweep coverage. The key argument is that when the graph is sparse, it is better to provide sweep coverage with a combination of static and mobile sensors, instead of using only mobile sensors as was done in the the previous study. We have proposed a 3-approximation algorithm to solve this NP hard problem. Our solution overcomes the flaw of the previous solution and it is more efficient in terms of cost and energy utilization for sweep coverage.

Reference

1. Li, M., Cheng, W.-F., Liu, K., Liu, Y., Li, X.-Y., Liao, X.: Sweep coverage with mobile sensors. IEEE Trans. Mob. Comput. 10(11), 1534–1545 (2011)