

- On an $m \times n$ chocolate bar, first player (A) chooses a cell, and takes away everything below and to the right of it; the second player (B) would do the same thing with another remaining cell; this process is continued; the loser is the player who has no choice but to take the poisonous cell, located at $(1, 1)$. The claim is that the first player always has a winning strategy. (This implicitly means that the second player B cannot have a winning strategy.)
- * Assumption: The only difference between A and B is that player A plays first. That is the capabilities of both the players same.
- * In a two-player game with players A and B , player A has a *winning strategy* if, no matter what player B does, there is always a sequence of moves that player A can do to counter player B 's moves and assure that player A wins.

- The game is guaranteed to terminate as at least one cell is taken away in every turn and there are a finite number of cells on the chocolate bar.

Hence, one of the players takes away cell $(1, 1)$.

Significantly, one of the players is guaranteed to win.

- Suppose A takes bottom-right cell (m, n) in its first turn; with chocolate X_1 remaining after A 's first turn, either A wins or B wins. If A wins, A has a winning strategy with that move in its first turn; otherwise, see below.
- * Suppose B chooses cell (i, j) in X_1 , and let X_2 be the remaining chocolate after that move; i.e., B has a winning strategy with X_2 at the end of its first turn.

Noting that $X_2 \subset X_1$, if B can win with X_2 after its first turn, A can also win with X_2 after its first turn by *stealing the winning strategy B has*, i.e., by A choosing cell (i, j) in its first round.

- In the above proof, we had not given a winning strategy itself but argued that there exists a winning strategy. Hence, it is a non-constructive existence proof.

¹munch or chew noisily or vigorously