Computing a Euclidean shortest path in the plane with Dijkstra wavefront

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under the guidance of
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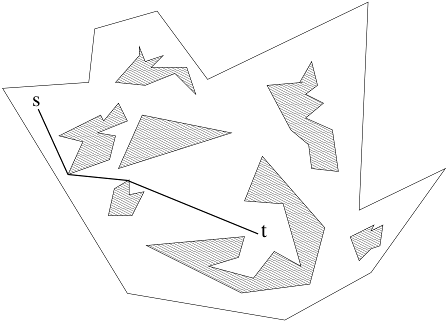
Problem Description



n is the number of vertices in the polygonal domain

m is the number of holes (obstacles)

History

- Known Lower Bound: $\Omega(n + m \lg m)$ time with $O(n)$ space
- Visibility Graph

  [Welzl, 1985]: $O(n^2)$

  [Ghosh, Mount 1991] and [Kapoor, Maheshwari 1988]: $O(n \lg n + E)$

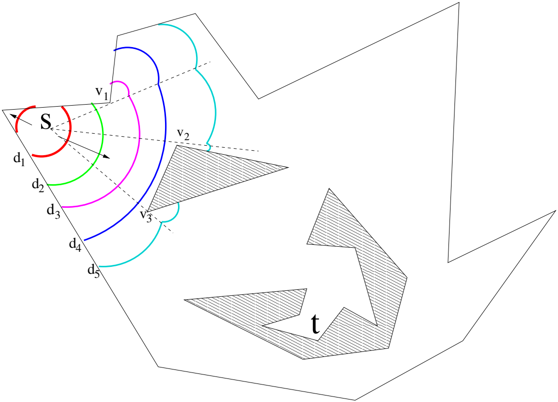
  [Storer, Reif 1994]: $O(T + mn)$
- Wavefront Propagation

  [Mitchell, 1993], [Kapoor 1999]: $O(n^{3/2+\epsilon})$ time and space

  [Hershberger, Suri 1999]: $O(n \lg n)$
- Wavefront Propagation with the restricted Visibility Graph

  This Research: $O(T + m(\lg m)(\lg n))$ time and $O(n)$ space

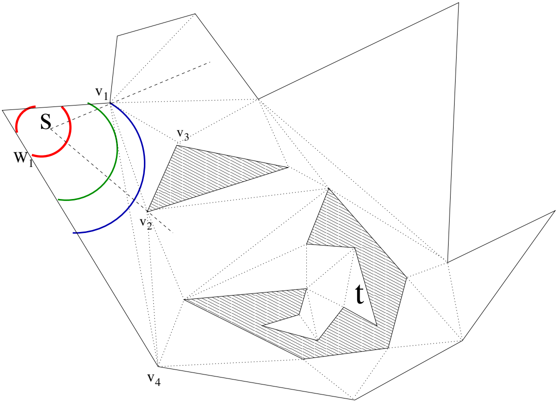
Wavefront Propagation



Wavefront, initiated at *s*, is expanded until it strikes *t*.

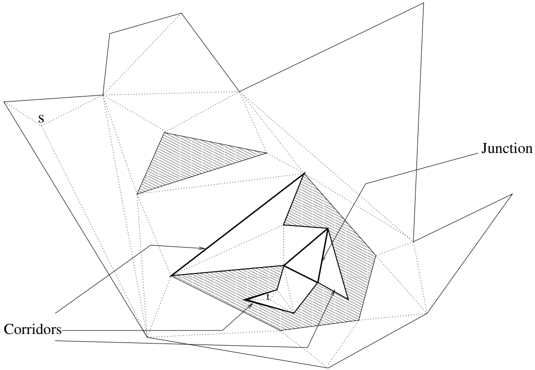
Initially wavefront comprises of single arc; with time, more arcs are added.

Triangulation to guide the wavefront



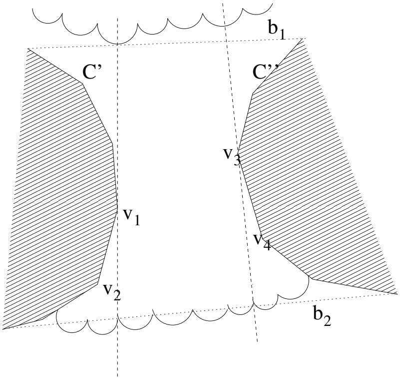
$O(T)$ is the time to triangulate the polygonal region ($O(n + m(\lg m)^{1+\epsilon})$, for a fixed positive constant $\epsilon > 0$)

Hourglasses and Junctions

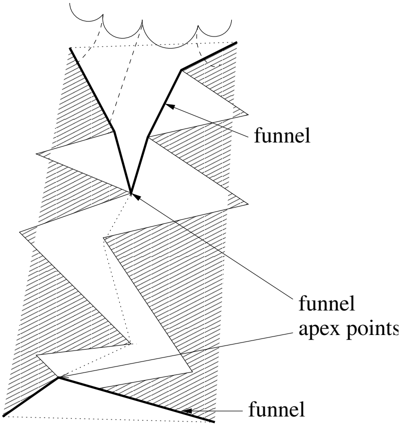


Given a triangulation, finding hourglasses and junctions takes $O(m \lg n)$ time

Open and Closed Hourglasses

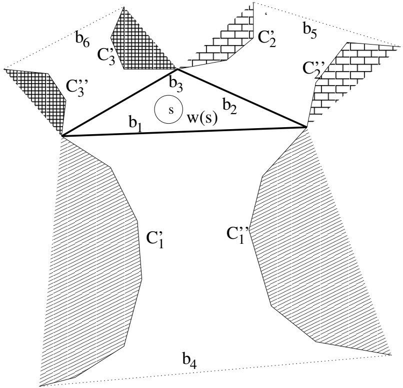


Open hourglass

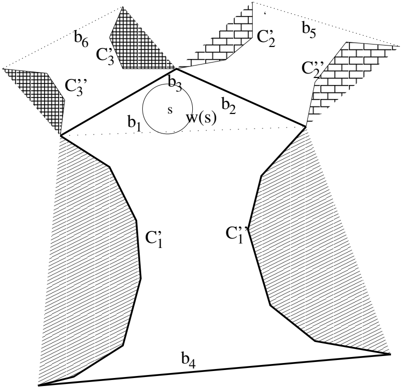


Closed hourglass

Boundary cycle and Event Points

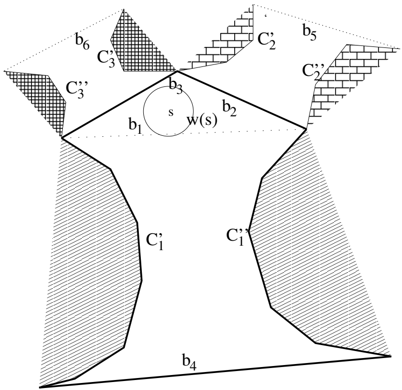


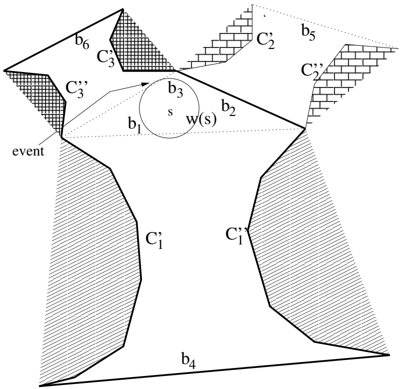
boundary cycle: b_1, b_2, b_3



boundary cycle: $C_1', b_4, C_1'', b_2, b_3$

Boundary cycle and Event Points (cont)

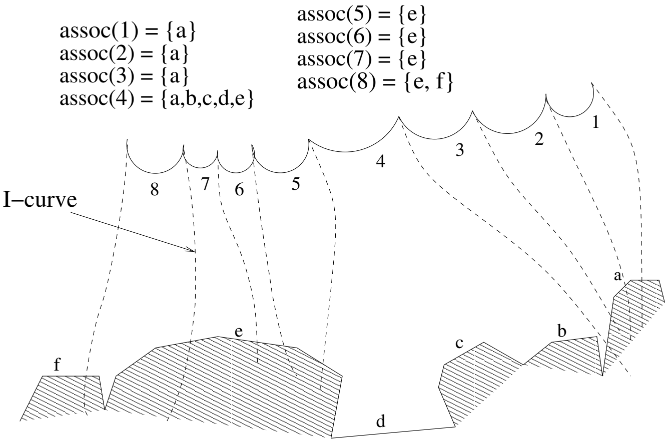




boundary cycle:
$C_1', b_4, C_1'', b_2, b_3$

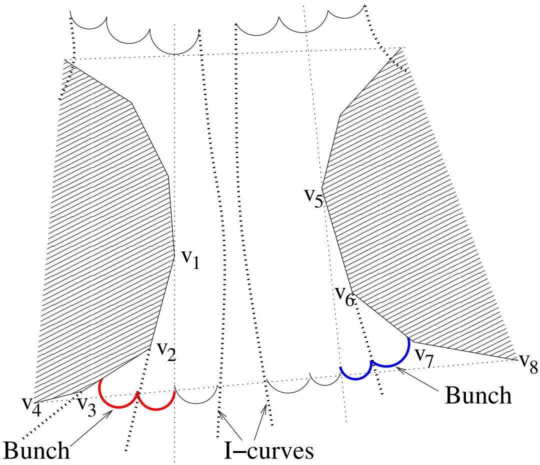
boundary cycle:
$C_1', b_4, C_1'', b_2, C_3', b_6, C_3''$

Associating wavefront segments to sections of boundary

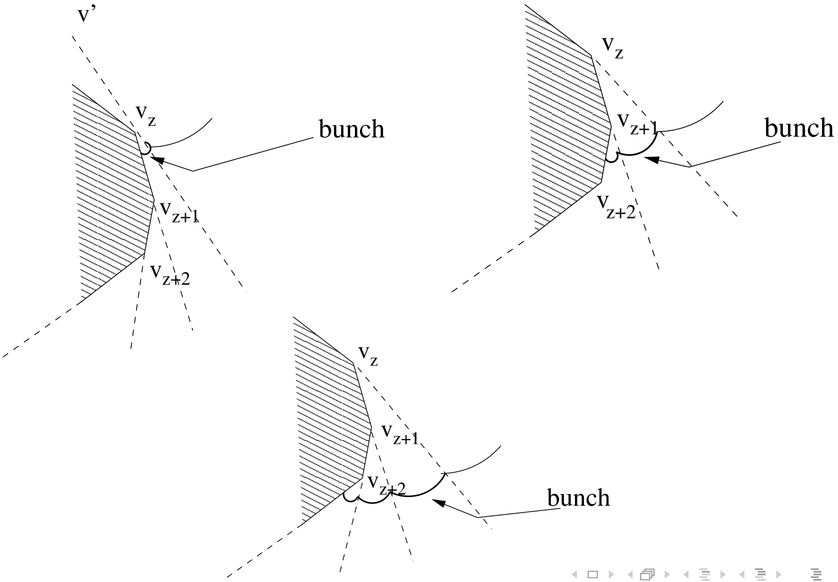


Each I-curve separates two weighted voronoi regions

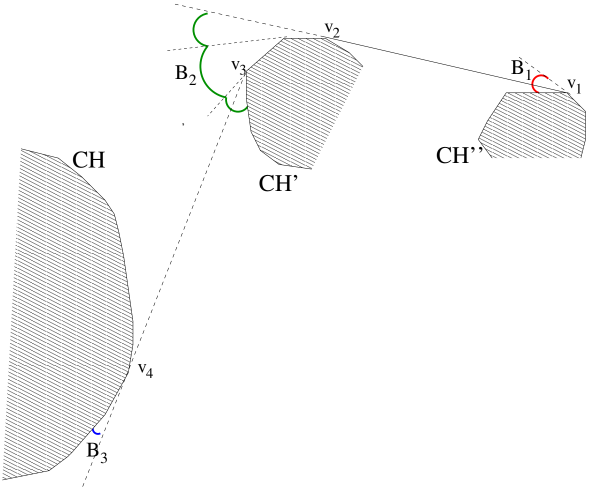
Exploiting convex chain structure: bunches



Bunch progression

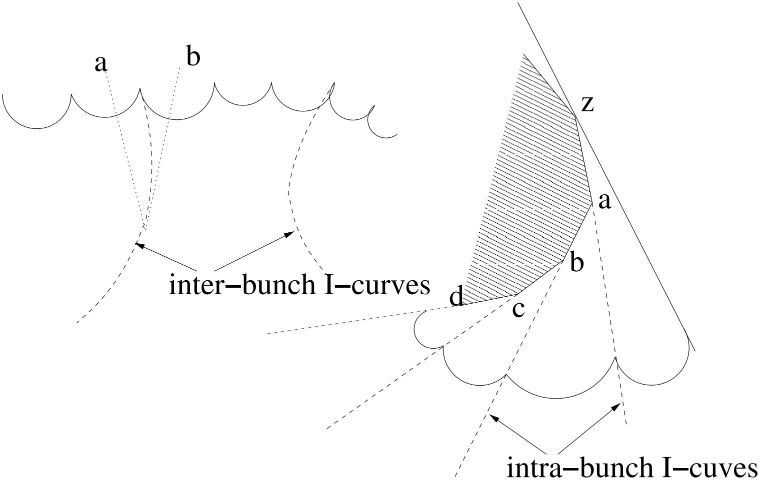


Wavefront: collection of bunches



Bunch B_1 caused bunch B_2, and bunch B_2 caused bunch B_3.

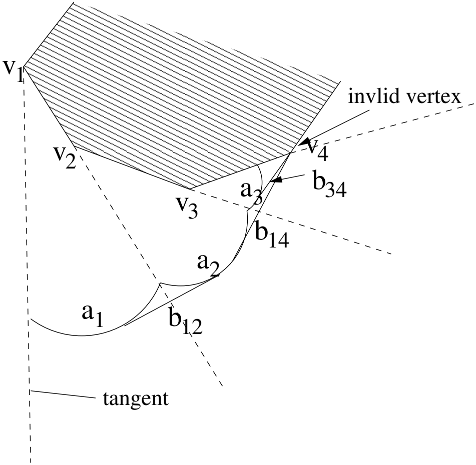
Inter- and Intra-bunch I-curves

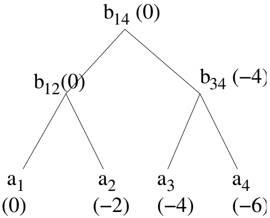


I-curve is a weighted voronoi curve, separating the voronoi regions

Diverging intra-bunch I-curves

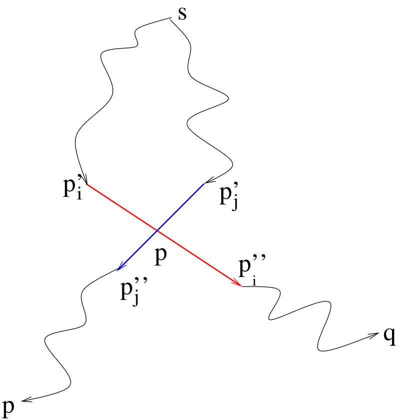
Convex hull approximation of a bunch

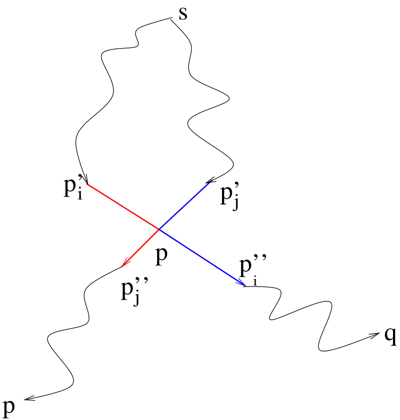




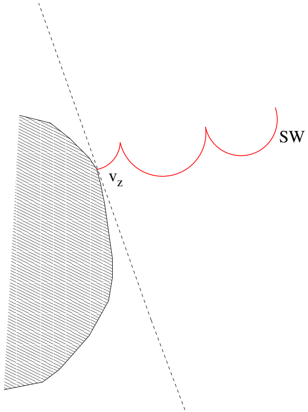
Eager initiation of segments in a bunch

Non-crossing property of shortest paths

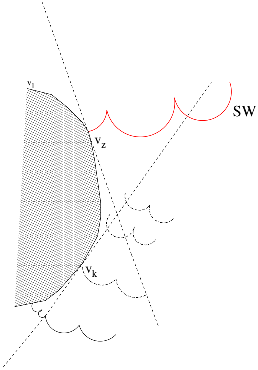




Initiating a new bunch

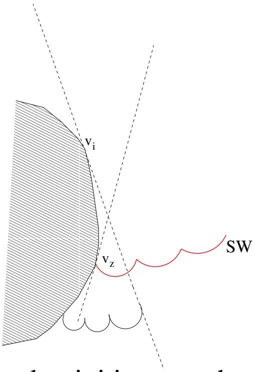


Initiates a new bunch from v_z



No need to initiate bunch from v_z

Initiating a new bunch (cont)



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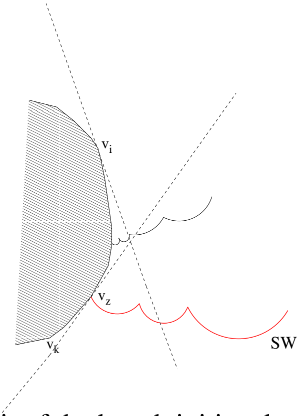


Split of the bunch initiated at v_i

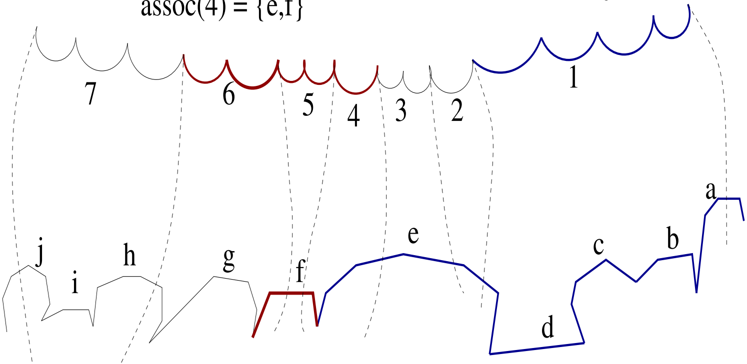
The total number of bunches at any point of execution of the algorithm are $O(m)$

Building and maintaining all BHTs during the entire algorithm takes $O(m \lg n + n)$

Associations & Boundary-sections & Waveform-sections

assoc(1) = {a,b,c,d,e}
assoc(2) = {e}
assoc(3) = {e}
assoc(4) = {e,f}

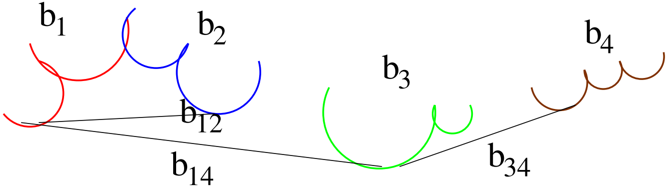
assoc(5) = {f}
assoc(6) = {f,g,h}
assoc(7) = {h,i,j}

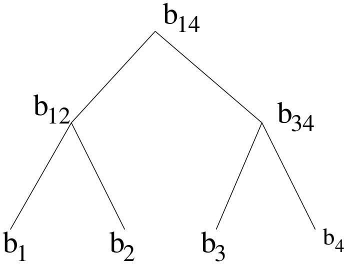


waveform−section(e) = {1,2,3,4}
waveform−section(f) = {4,5,6}

boundary−section(1) = {a,b,c,d,e}
boundary−section(6) = {f,g,h}
boundary−section(7) = {h,i,j}

Convex hull approximation of a section of wavefront

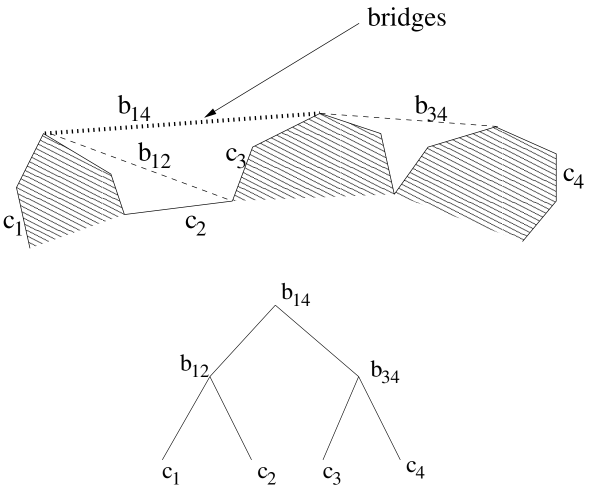




The number of waveform-sections created/updated are $O(m)$

The shortest distance computation between a waveform-section and a bunch takes $O((\lg m)(\lg n))$ amortized time

Convex hull approximation of a section of boundary



The number of boundary-sections created/updated are $O(m)$

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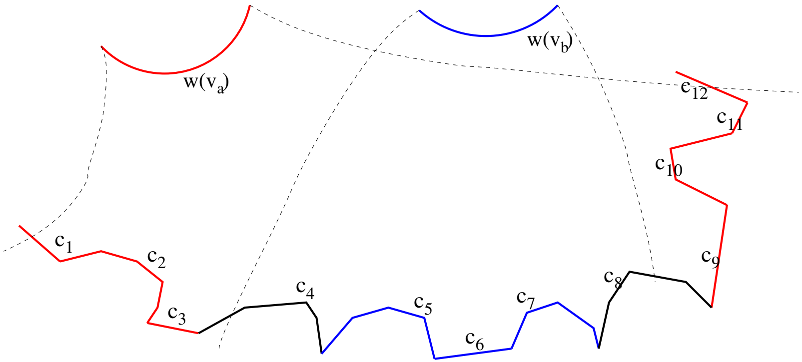
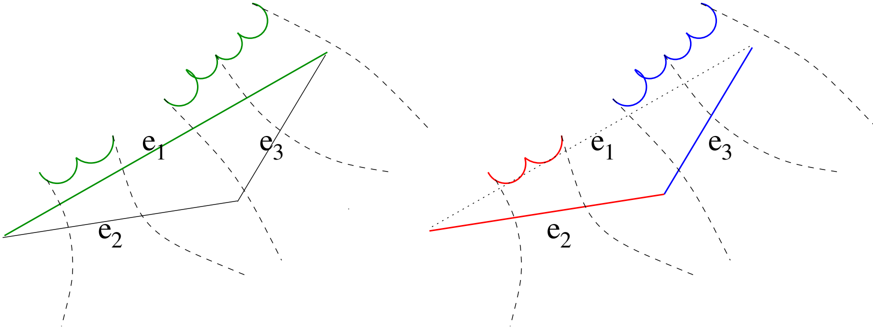
Contiguity property of associations



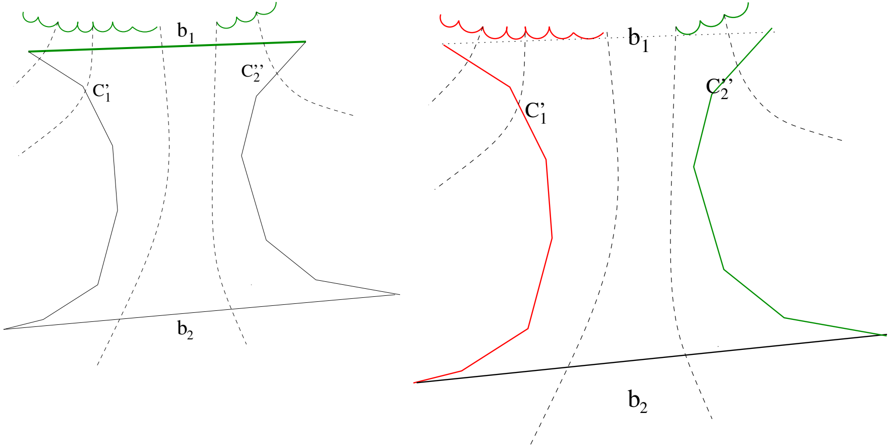
Wavefront split



The total number of splits at all junctions are $O(m)$

The total time complexity of splits at all junctions together $O(m(\lg m)(\lg n))$

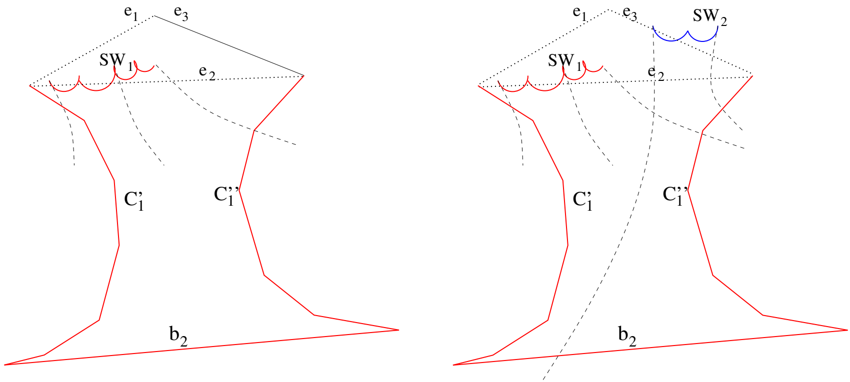
Wavefront split (cont)



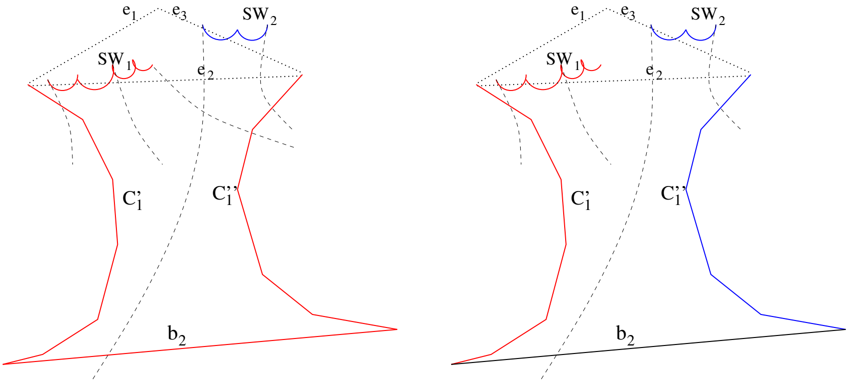
The total number of splits at all hourglasses are $O(m)$

The total time complexity of splits at all hourglasses together $O(m(\lg m)(\lg n))$

Wavefront Merger



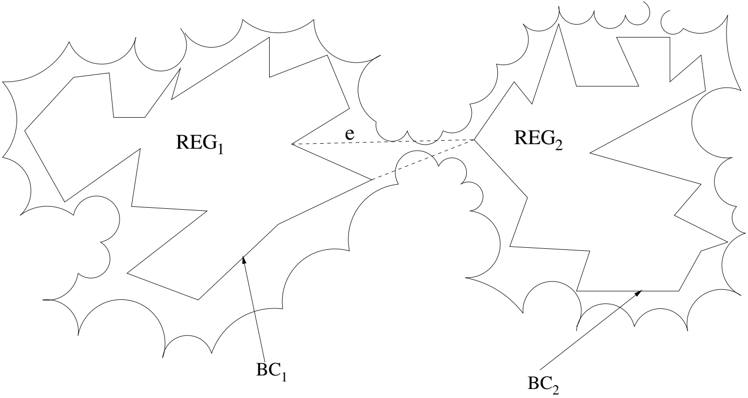
Wavefront Merger (cont)



The total number of merges during the entire algorithm are $O(m)$

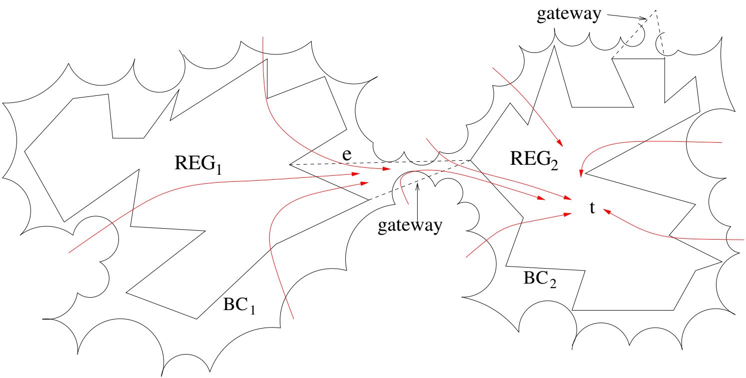
The total processing involved in the merge procedure during the entire algorithm is $O(m(\lg m)(\lg n))$

Boundary splits

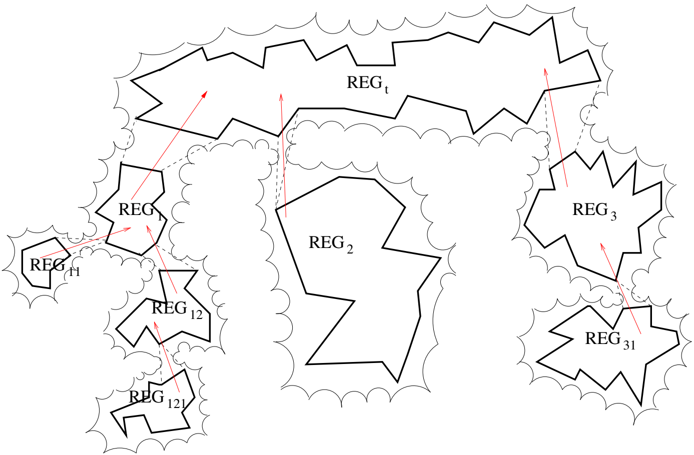


Every wavefront merger causes a boundary split, and vice versa.

Boundary splits (cont)



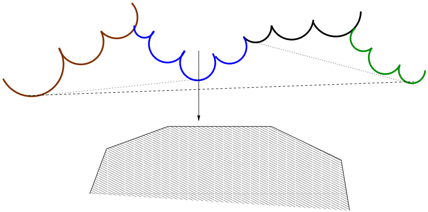
Ordering wavefront mergers at gateways



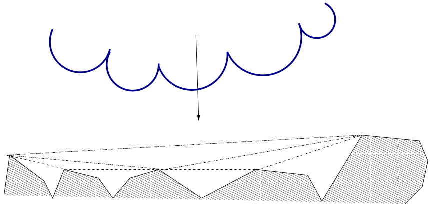
The total number of gateways/boundary splits are $O(m)$

The amortized time involved in boundary splits including orienting the gateways is $O((\lg m)(\lg n))$

Type-I and Type-II Events: Shortest distance computations between sections of wavefront and sections of boundary



Type-I

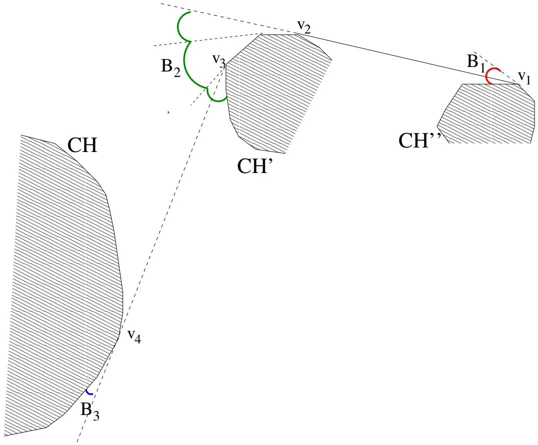


Type-II

The total number of Type-I and Type-II events are $O(m)$.

Time in determining and handling all Type-I and Type-II events together is $O(m(\lg m)(\lg n))$.

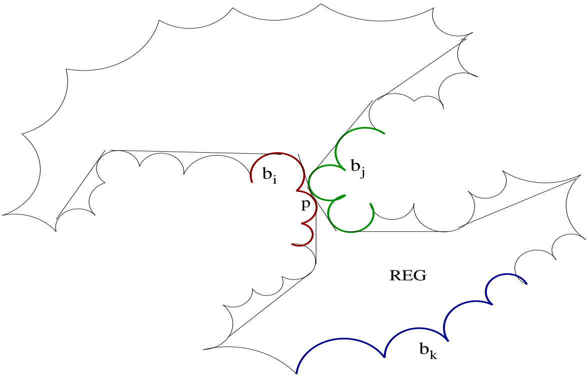
Type-III Event: Initializing bunches



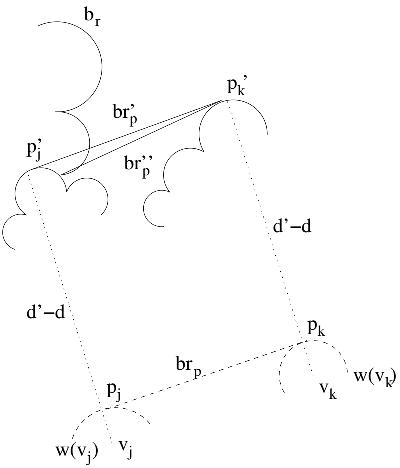
The total number of Type-III events are $O(m)$.

Time in determining and handling all Type-III events together is $O(m(\lg m)(\lg n))$.

Type-IV Event: collision of bunches within a waveform-section tree

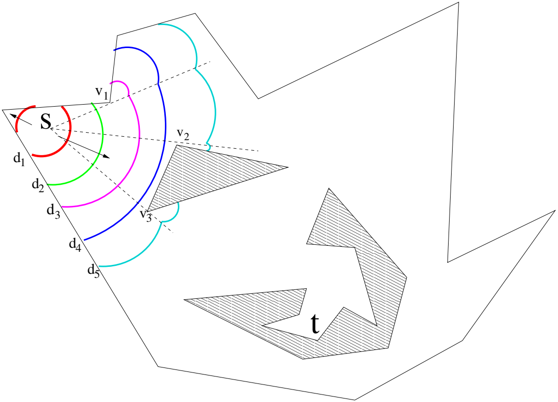


Bridge maintainance in waveform-section trees: Dirty bridges



The total number of Type-IV events are $O(m)$.
Time in determining and handling all Type-IV events together is $O(m(\lg m)(\lg n))$.

Summary



Provides a solution with $O(T + m(\lg m)(\lg n))$ time and $O(n)$ space to Problem 21 of The Open Problems Project (TOPP) of Computational Geometry, which intends for a solution with $O(n + m \lg m)$ time and $O(n)$ space.

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- Improved the time complexity of shortest path computation in polygonal domains.

E. Welzl.
Constructing the visibility graph for n line segments in $O(n^2)$ time.
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An output-sensitive algorithm for computing visibility graphs.
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