Multivariable problem

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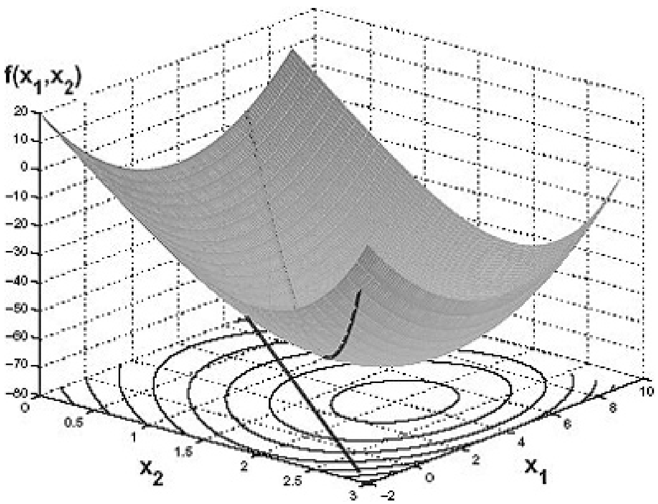
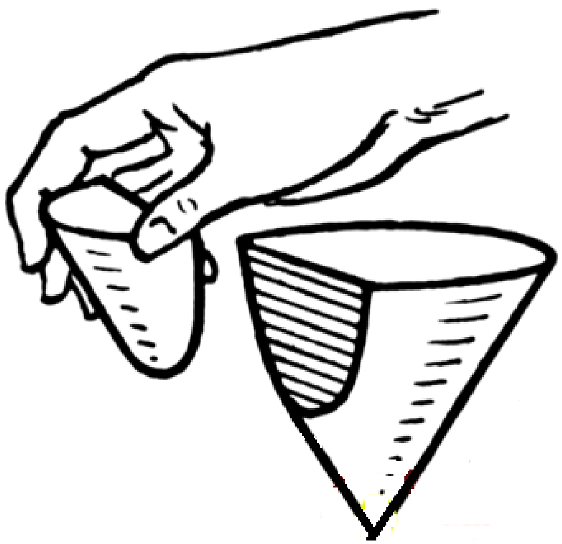
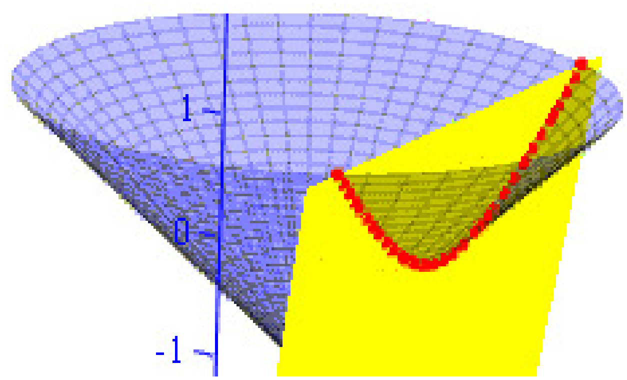
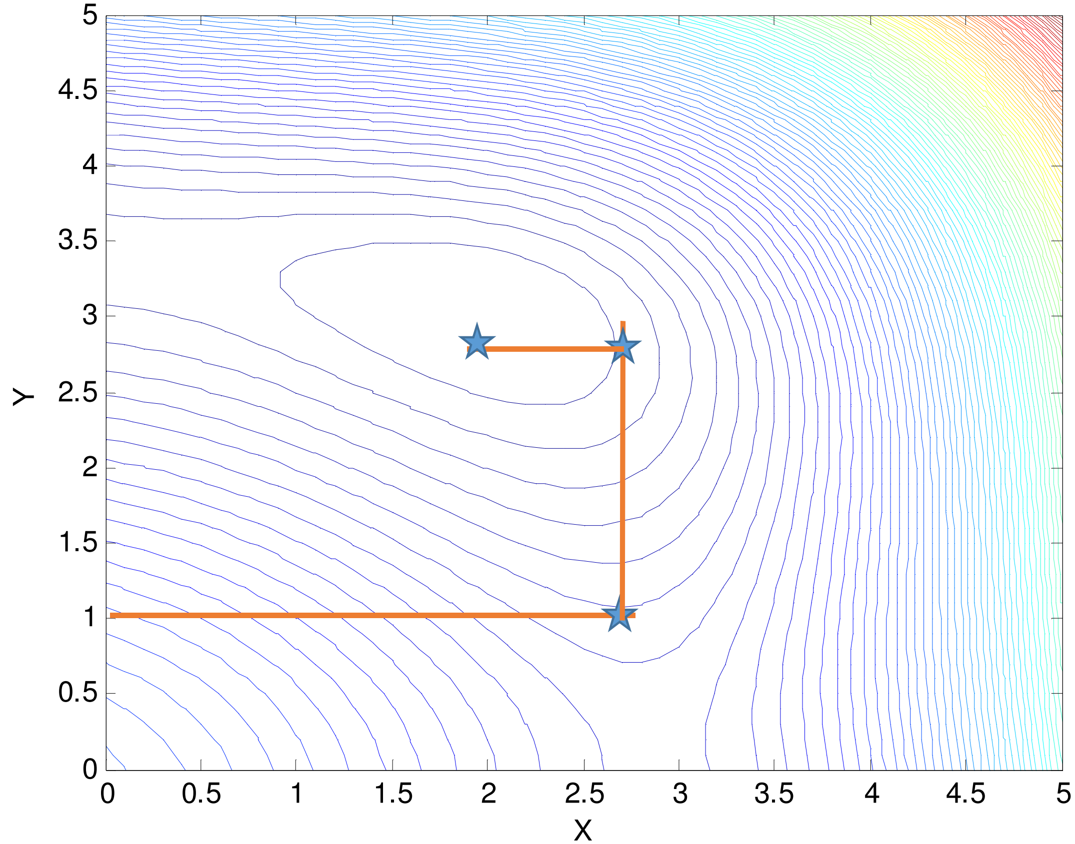
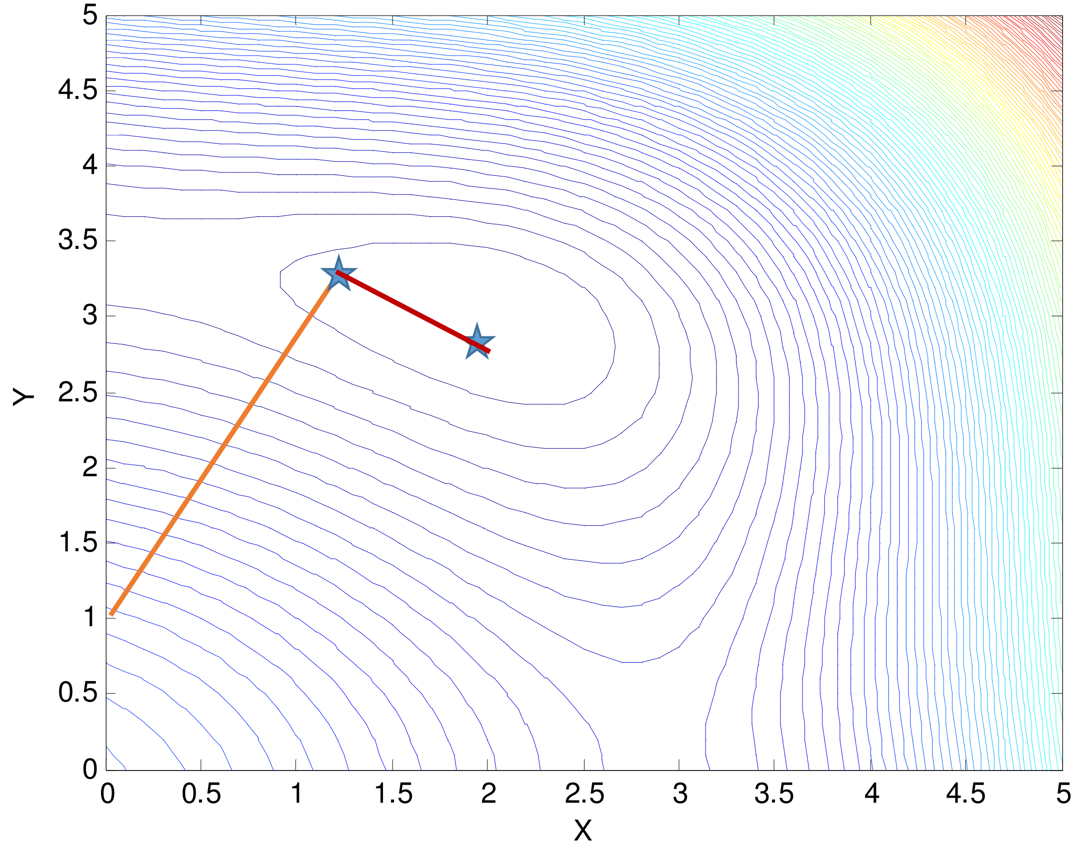

f(x1,x2)
20
10
0
−10
−20
−30
−40
−50
−60
−70
−80
x2
0
0.5
1
1.5
2
2.5
3
x1
−2
0
2
4
6
8
10



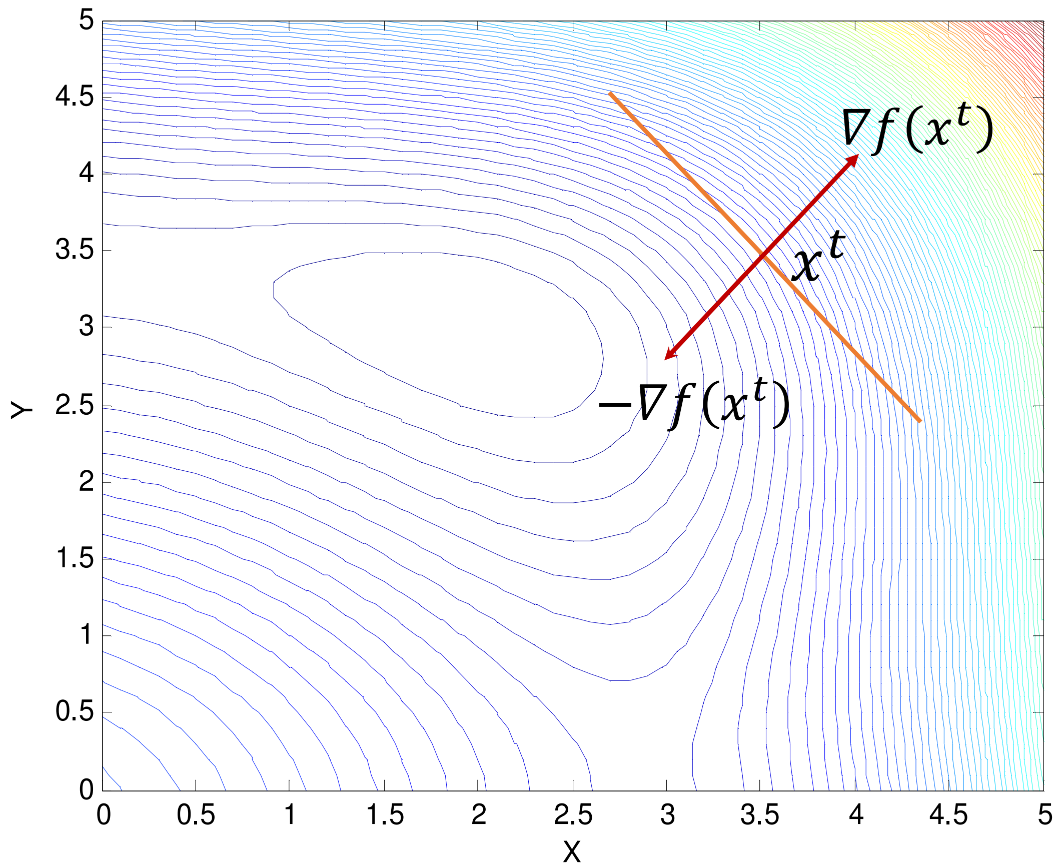

1
0
-1



5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Y
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
X



5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Y
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
X



Descent direction

A search direction d^t is a descent direction at point x^t if the condition $\nabla f(x^t).d^t < 0$ is satisfied in the vicinity of the point x^t.

$$f(x^{t+1}) = f(x^t + \alpha d^t)$$
$$= f(x^t) + \alpha \nabla^T f(x^t).d^t$$

The $f(x^{t+1}) < f(x^t)$

When $\alpha \nabla^T f(x^t).d^t < 0$

Or, $\nabla^T f(x^t).d^t < 0$

Q. Show that the Newton's method finds the minimum of a quadratic function in one iteration

A quadratic function can be written as

$$f(X) = \frac{1}{2}X^T AX + B^T X + \text{C}$$

The minimum of the function is given by

$$\nabla f(X) = AX + B = 0$$

$$\text{X} = -A^{-1}B$$

Now apply Newton's method. The iterative step gives

$$\text{X}_{i+1} = \text{X}_i - H^{-1}\nabla f(\text{X}_i)$$

In this case $H = A$

$$\text{X}_{i+1} = \text{X}_i - A^{-1}(A\text{X}_i + B)$$

$$\text{X}_{i+1} = -A^{-1}B$$

$$\frac{\partial(X^T AX)}{\partial X} = AX + A^T X$$

In this case $A = A^T$

$$\frac{\partial(X^T AX)}{\partial X} = 2AX$$

$$\frac{\partial(AX)}{\partial X} = A^T$$

$$\frac{\partial(X^T A)}{\partial X} = A$$

$$\frac{\partial(A^T X)}{\partial X} = A$$

Powell's conjugate direction method

Parallel subspace property

Given a quadratic function $f(X) = \frac{1}{2}X^T AX + B^T X + C$ of two variables and X^1 and X^2 are the two arbitrary but distinct points.

If Y^1 is the solution of the problem

Min $f(X^1 + \lambda d)$

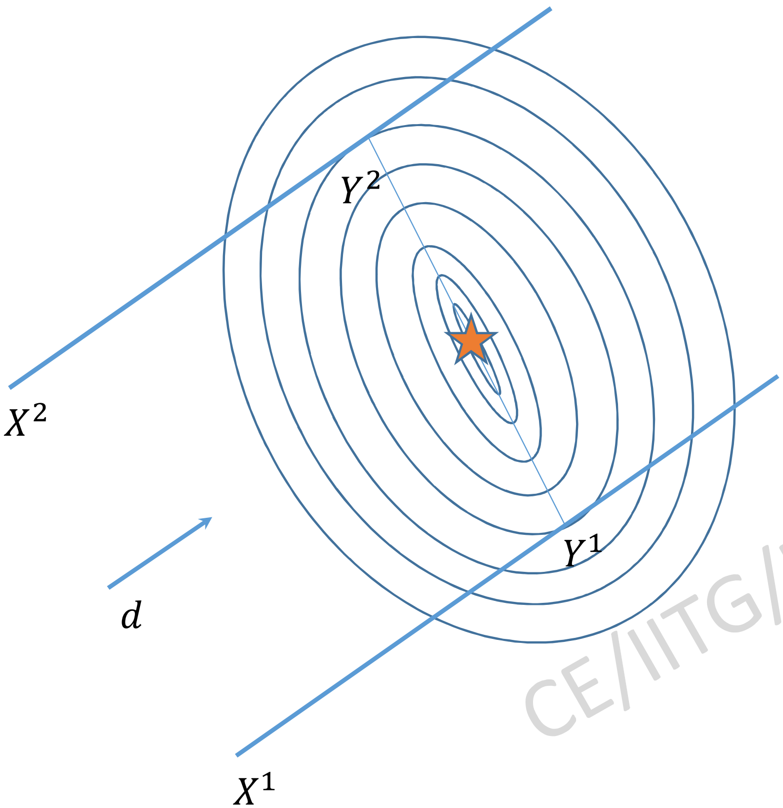
If Y^2 is the solution of the problem

Min $f(X^2 + \lambda d)$

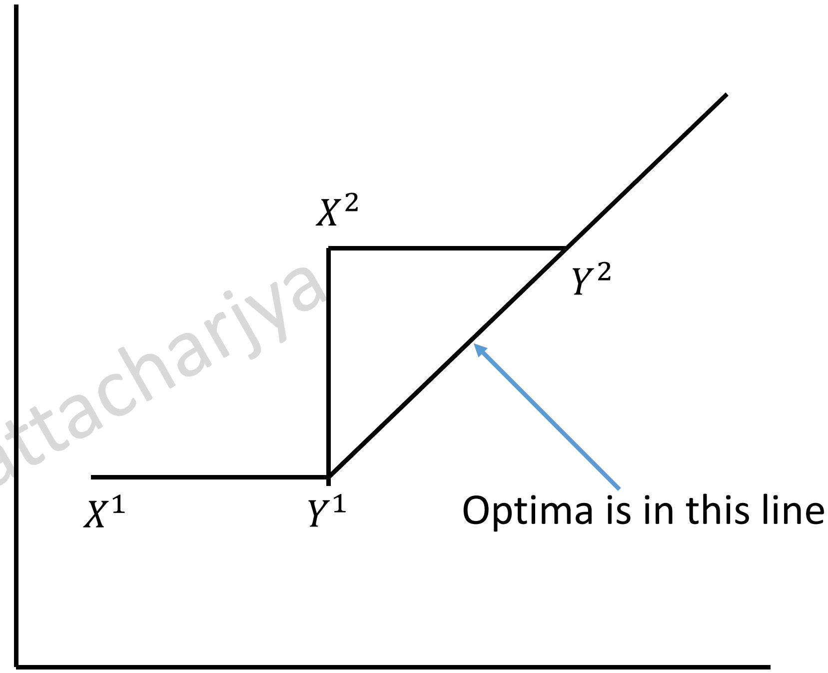
Then the direction $(Y^2 - Y^1)$ is conjugate to d, or other words, the quantity

$$(Y^2 - Y^1)^T Ad = 0$$

For quadratic function minimum lies on the direction $(Y^2 - Y^1)$



X^2
Y^2
Y^1
d
X^1



X^2
Y^2
X^1
Y^1
Optima is in this line