Optimization Formulation

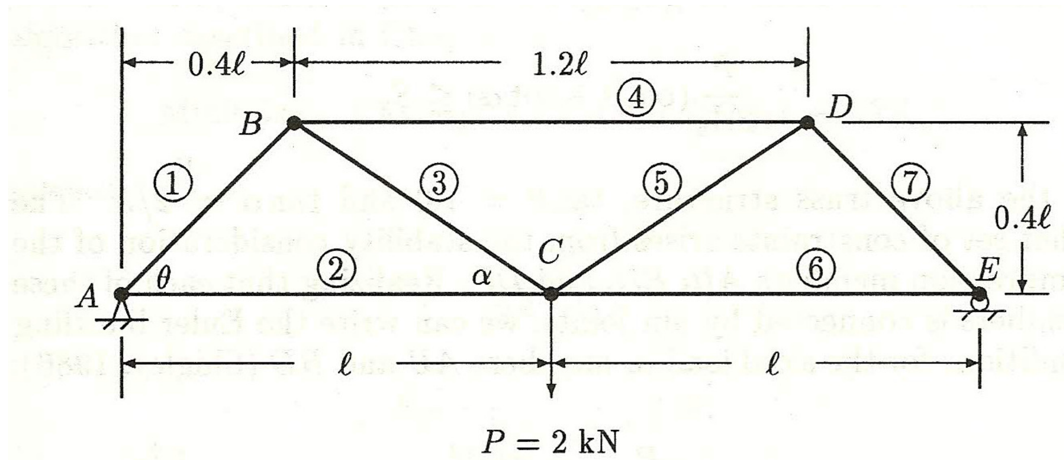
An Example

Objectives

Topology: Optimal connectivity of the structure

Minimum cost of material: optimal cross section of all the members

We will consider the second objective only



The design variables are the cross sectional area of the members, i.e. A1 to A7

Using symmetry of the structure
A7=A1, A6=A2, A5=A3

You have only four design variables, i.e., A1 to A4

Optimization formulation

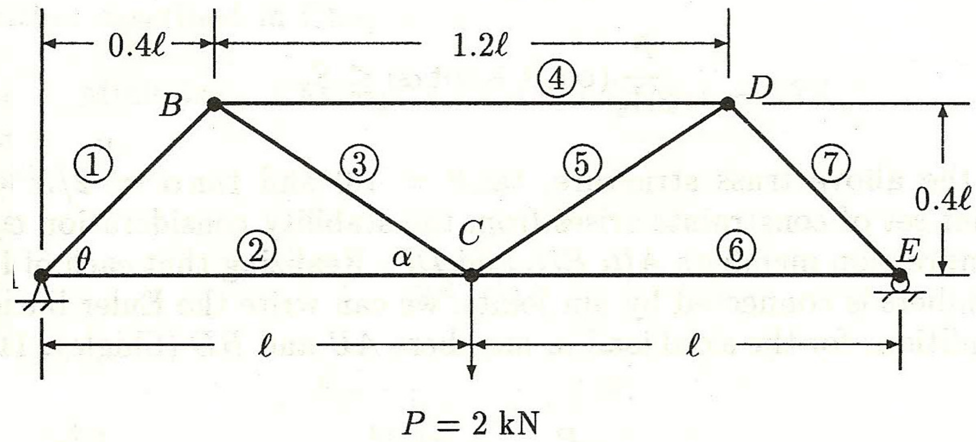
Objective

$$\text{Minimize} \quad 1.132A_1\ell + 2A_2\ell + 1.789A_3\ell + 1.2A_4\ell$$

What are the constraints?
One essential constraint is non-negativity of design variables, i.e.
A1, A2, A3, A4 >= 0

Is it complete now?

Member	Force	Member	Force
AB	$-\frac{P}{2}\csc\theta$	BC	$+\frac{P}{2}\csc\alpha$
AC	$+\frac{P}{2}\cot\theta$	BD	$-\frac{P}{2}(\cot\theta+\cot\alpha)$



First set of constraints

$$\frac{P\csc\theta}{2A_1} \le S_{yc},$$

$$\frac{P\cot\theta}{2A_2} \le S_{yt},$$

$$\frac{P\csc\alpha}{2A_3} \le S_{yt},$$

$$\frac{P}{2A_4}(\cot\theta+\cot\alpha) \le S_{yc}$$

Another constraint may be the minimization of deflection at C

$$\frac{P\ell}{E}\left(\frac{0.566}{A_1}+\frac{0.500}{A_2}+\frac{2.236}{A_3}+\frac{2.700}{A_4}\right) \le \delta_{\max}$$

Another constraint is buckling of compression members

$$\frac{P}{2\sin\theta} \le \frac{\pi E A_1^2}{1.281\ell^2}$$

$$\frac{P}{2}(\cot\theta+\cot\alpha) \le \frac{\pi E A_4^2}{5.76\ell^2}$$

$$\text{Minimize} \quad 1.132A_1\ell + 2A_2\ell + 1.789A_3\ell + 1.2A_4\ell$$

subject to

$$S_{yc} - \frac{P}{2A_1 \sin\theta} \geq 0,$$

$$S_{yt} - \frac{P}{2A_2 \cot\theta} \geq 0,$$

$$S_{yt} - \frac{P}{2A_3 \sin\alpha} \geq 0,$$

$$S_{yc} - \frac{P}{2A_4}(\cot\theta + \cot\alpha) \geq 0,$$

$$\frac{\pi E A_1^2}{1.281\ell^2} - \frac{P}{2\sin\theta} \geq 0,$$

$$\frac{\pi E A_4^2}{5.76\ell^2} - \frac{P}{2}(\cot\theta + \cot\alpha) \geq 0,$$

$$\delta_{\max} - \frac{P\ell}{E}\left(\frac{0.566}{A_1} + \frac{0.500}{A_2} + \frac{2.236}{A_3} + \frac{2.700}{A_4}\right) \geq 0,$$

$$10 \times 10^{-6} \leq A_1, A_2, A_3, A_4 \leq 500 \times 10^{-6}.$$

An optimization problem

Minimize $F = (x-p)^2 + (y-q)^2$

Subject to $a_1 x + b_1 y \leq d_1$

$$a_2 x + b_2 y \leq d_2$$

$$x, y \geq 0$$

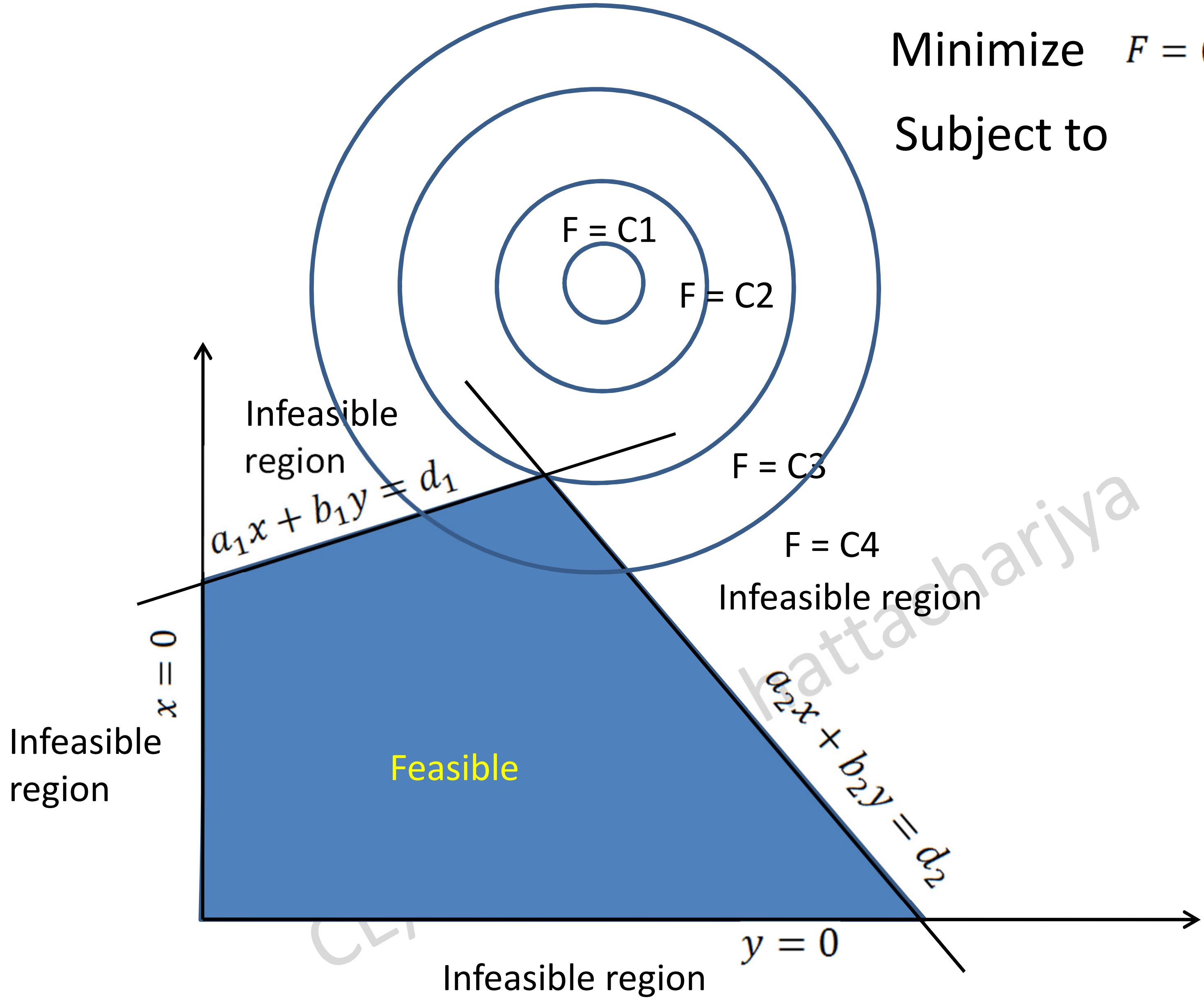
Minimize $F = (x-p)^2 + (y-q)^2$

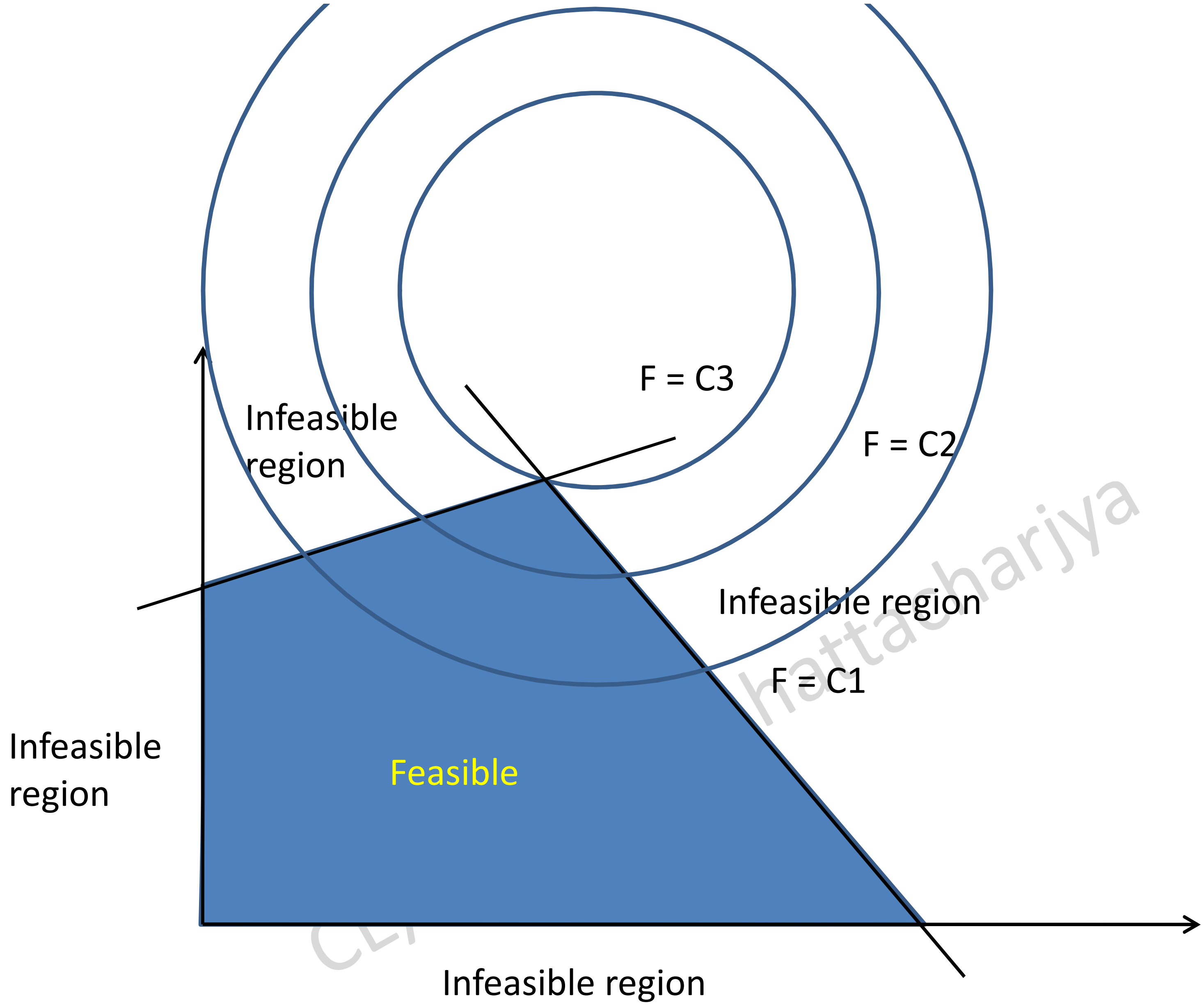
Subject to

$$a_1 x + b_1 y \leq d_1$$

$$a_2 x + b_2 y \leq d_2$$

$$x, y \geq 0$$

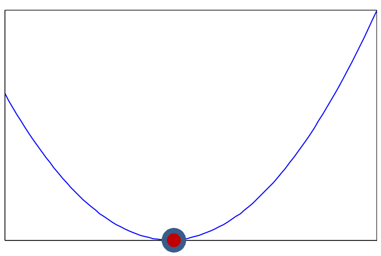



F = C3
Infeasible region
F = C2
Infeasible region
F = C1
Infeasible region
Feasible
Infeasible region

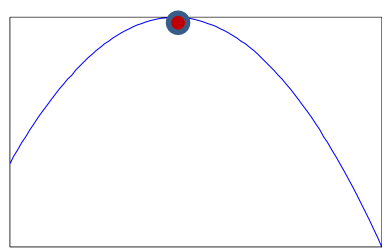
Single variable optimization

Stationary points

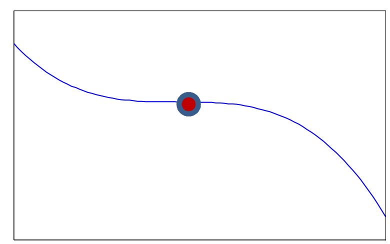
For a continuous and differentiable function *f(x), a stationary point x* is a point at which the* slope of the function is zero, *i.e.* $f'(x) = 0$ *at* $x = x^*$*,*



Minima



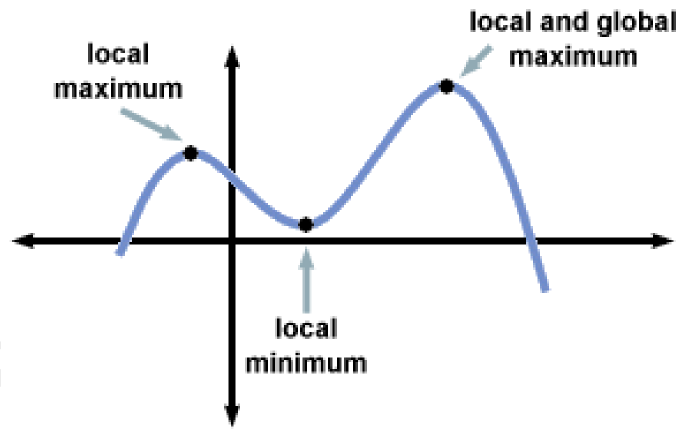
Maxima



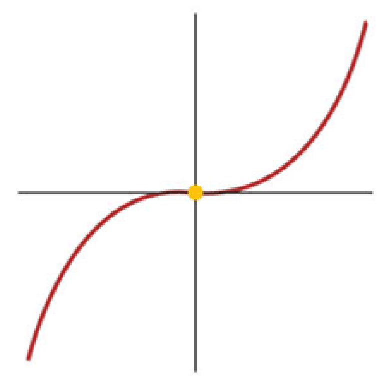
Inflection point

Relative minimum and maximum

- A function is said to have a *relative or local minimum at* $x = x^*$ *if* $f(x^*) \leq f(x^*+h)$ *for all* sufficiently small positive and negative values of h, *i.e. in the near vicinity of the point* x^*.
- Similarly, a point x^* *is called a relative or local maximum if* $f(x^*) \geq f(x^*+h)$ for all values of h *sufficiently close to zero.*
- *A point* x^* *is said to be an inflection point if the function value increases locally as* x^* *increases and decreases locally as* x^* *reduces*

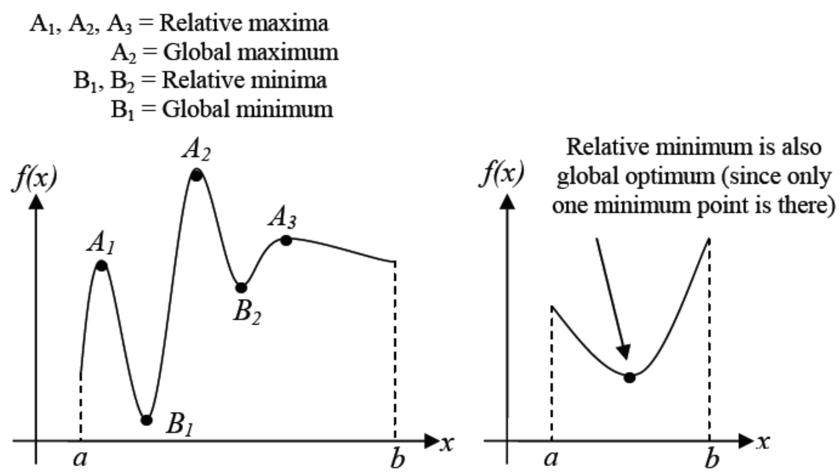


Inflection Point



Global minimum and maximum

- A function is said to have a *global or absolute minimum at x = x* if f (x*) ≤ f(x) for all x in the domain over which f(x) is defined.*
- A function is said to have a *global or absolute maximum at x = x* if f (x*) ≥ f(x) for all x in the domain over which f(x) is defined.*



Necessary and sufficient conditions for optimality

Necessary condition for a point to be stationary is

$f'(x)=0$

Sufficient condition

Suppose at point *x** , the first derivative is zero and first nonzero higher derivative is denoted by *n,* then

1. *If n is odd, x* is an inflection point*
2. *If n is even, x* is a local optimum*
   1. *If the derivative is positive, x* is a local minimum*
   2. *If the derivative is negative, x* is a local maximum*