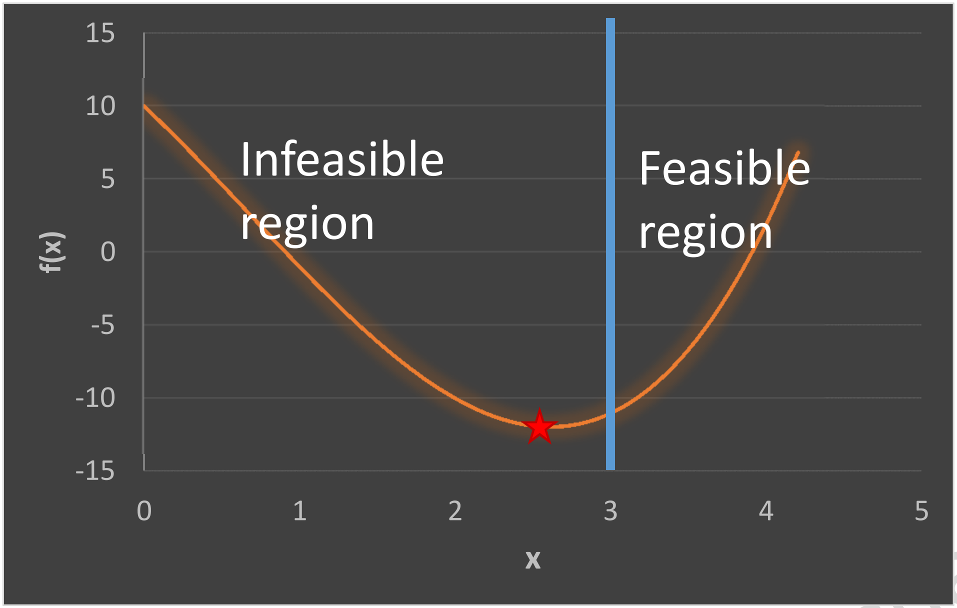
Transformation method

Rajib Bhattacharjya
Department of Civil Engineering
IIT Guwahati



Minimize

$$f(x) = x^3 - 10x - 2x^2 + 10$$

Subject to $g(x) = x \geq 3$

Or, $g(x) = x - 3 \geq 0$

The problem can be written as

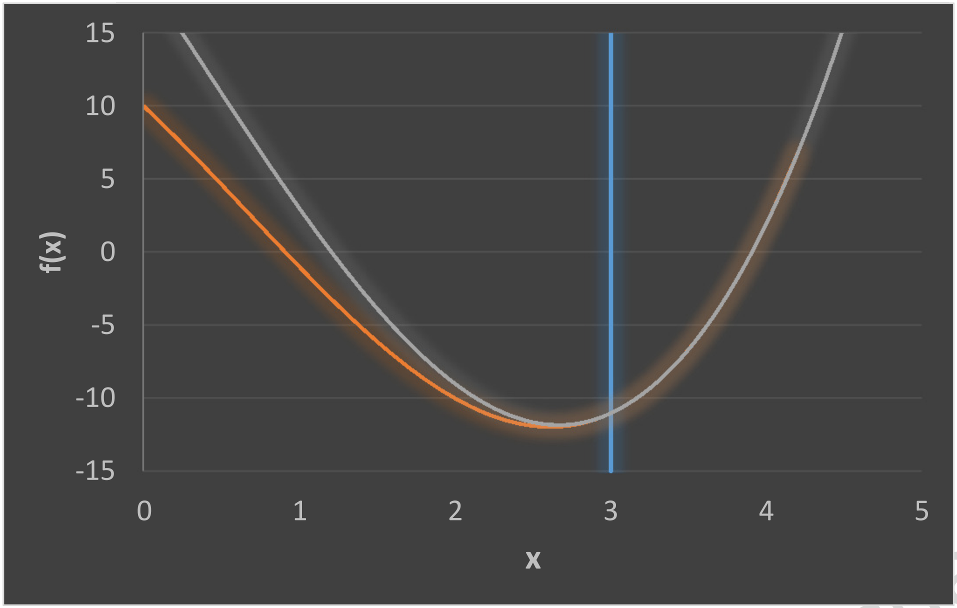
$$\text{F}(x, R) = f(x) + R\langle g(x)\rangle^2$$

Where,

$\langle g(x)\rangle = 0$ if $x \geq 3$

$\langle g(x)\rangle = g(x)$ otherwise

The bracket operator $\langle \quad \rangle$ can be implemented using $min(g, 0)$ function



Minimize

$$f(x) = x^3 - 10x - 2x^2 + 10$$

Subject to $g(x) = x \geq 3$

Or, $g(x) = x - 3 \geq 0$

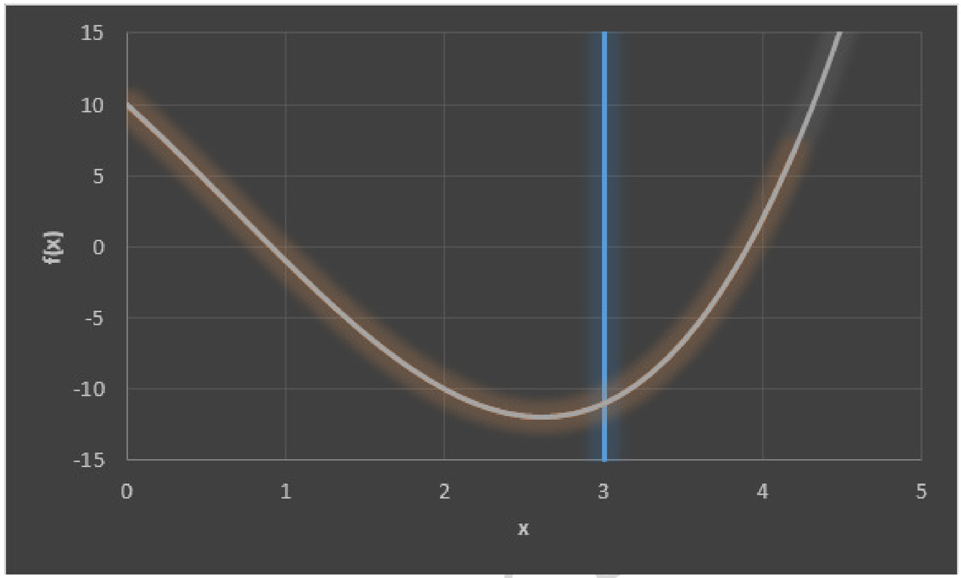
The problem can be written as

$$\mathrm{F}(x, R) = (x^3 - 10x - 2x^2 + 10) + R\langle x - 3\rangle^2$$

$$\mathrm{F}(x, R) = (x^3 - 10x - 2x^2 + 10) + R\left(min(x - 3{,}0)\right)^2$$

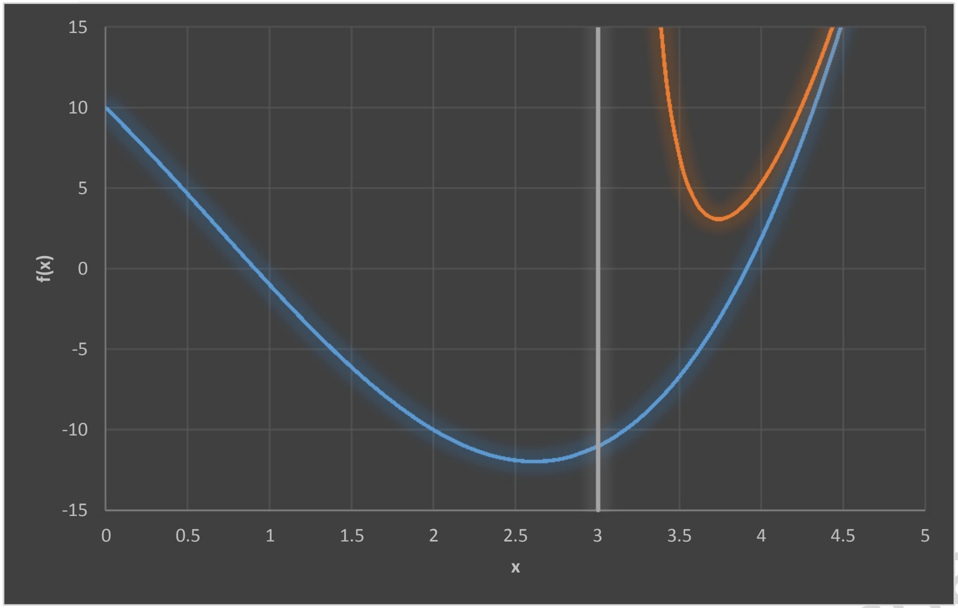
Minimize $\mathrm{F}(x,R) = (x^3 - 10x - 2x^2 + 10) + R\big(min(x-3,0)\big)^2$

R 0



By changing R value, it is possible to avoid the infeasible solution

The minimization of the transformed function will provide the optimal solution which is in the feasible region only



Minimize

$$f(x) = x^3 - 10x - 2x^2 + 10$$

Subject to $g(x) = x \geq 3$

Or, $g(x) = x - 3 \geq 0$

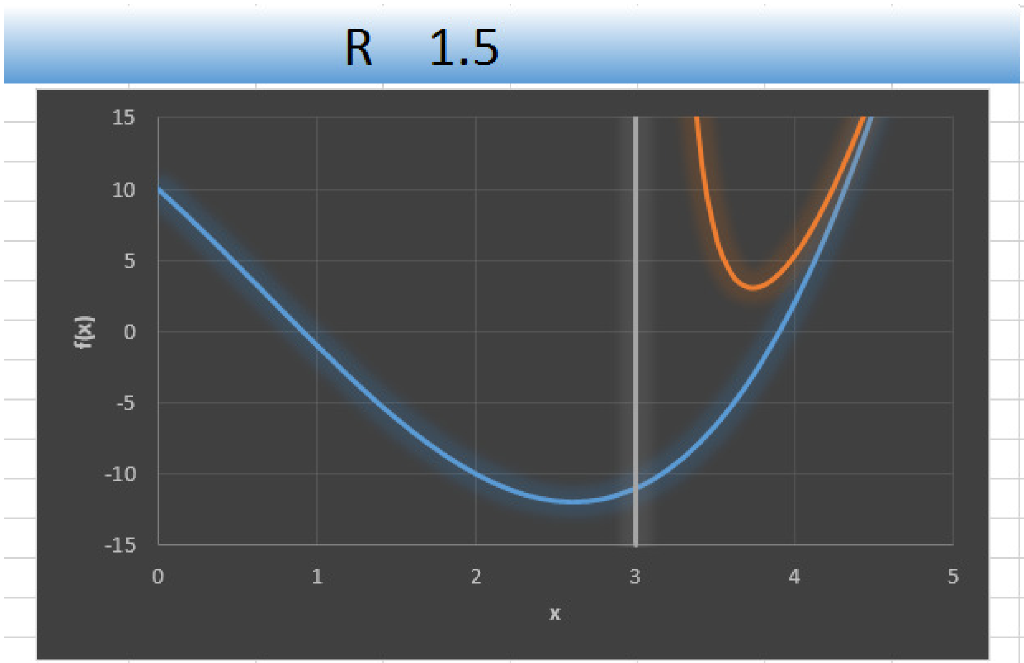
The problem can also be converted as

$$F(x, R) = (x^3 - 10x - 2x^2 + 10) + R\frac{1}{g(x)}$$

This term is added in feasible side only

$$F(x, R) = (x^3 - 10x - 2x^2 + 10) + R\frac{1}{(x-3)}$$

Minimize $\mathrm{F}(x, R) = (x^3 - 10x - 2x^2 + 10) + R\frac{1}{(x-3)}$



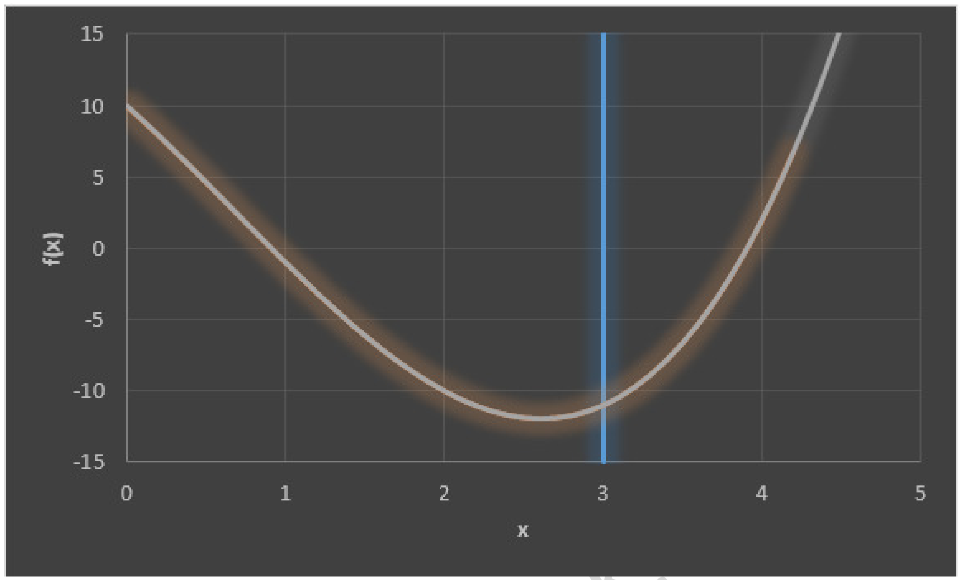
By changing R value, it is possible to avoid the infeasible solution

The minimization of the transformed function will provide the optimal solution which is in the feasible region only

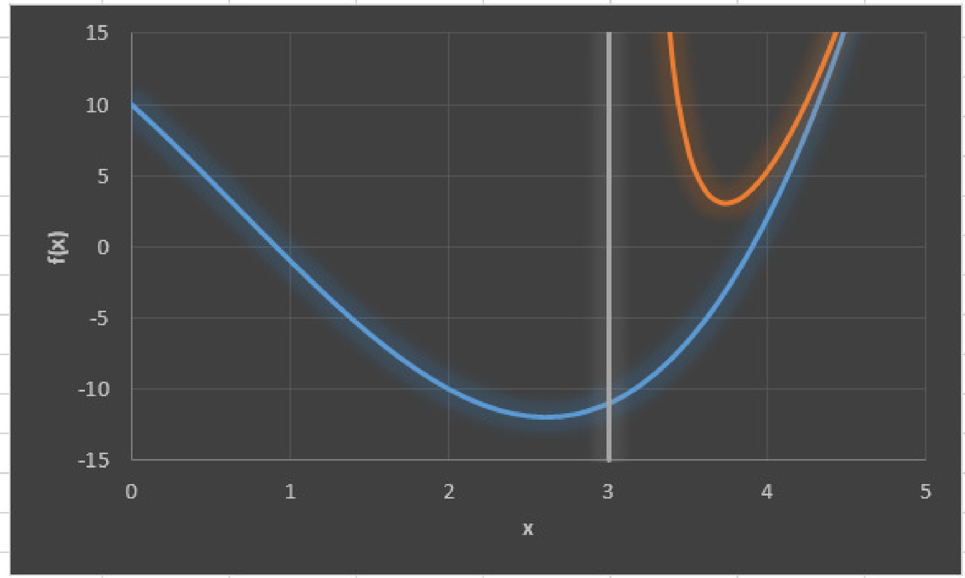
Exterior penalty method

Interior penalty method

R 0



R 1.5



The transformation function can be written as

$$\mathrm{F}(X, R) = f(X) + \Psi\big(g(X), h(X)\big)$$

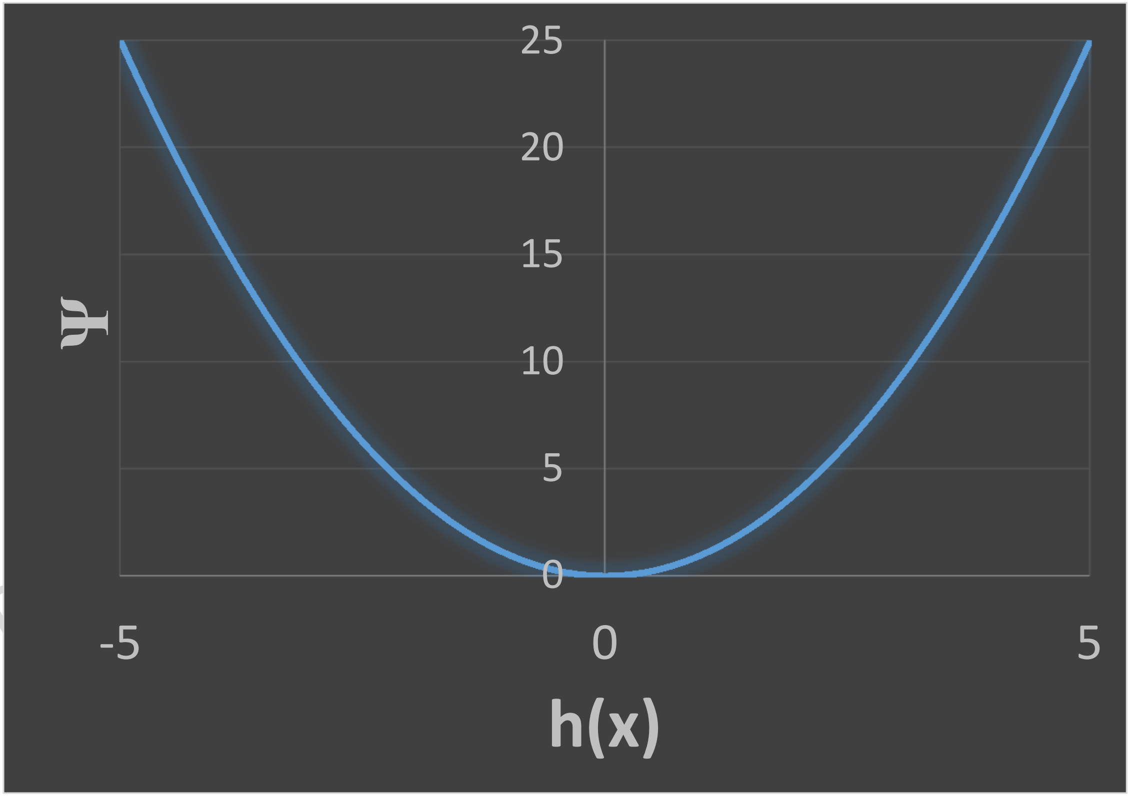
This term is called Penalty term

R Is called penalty parameters

Penalty terms

Parabolic penalty

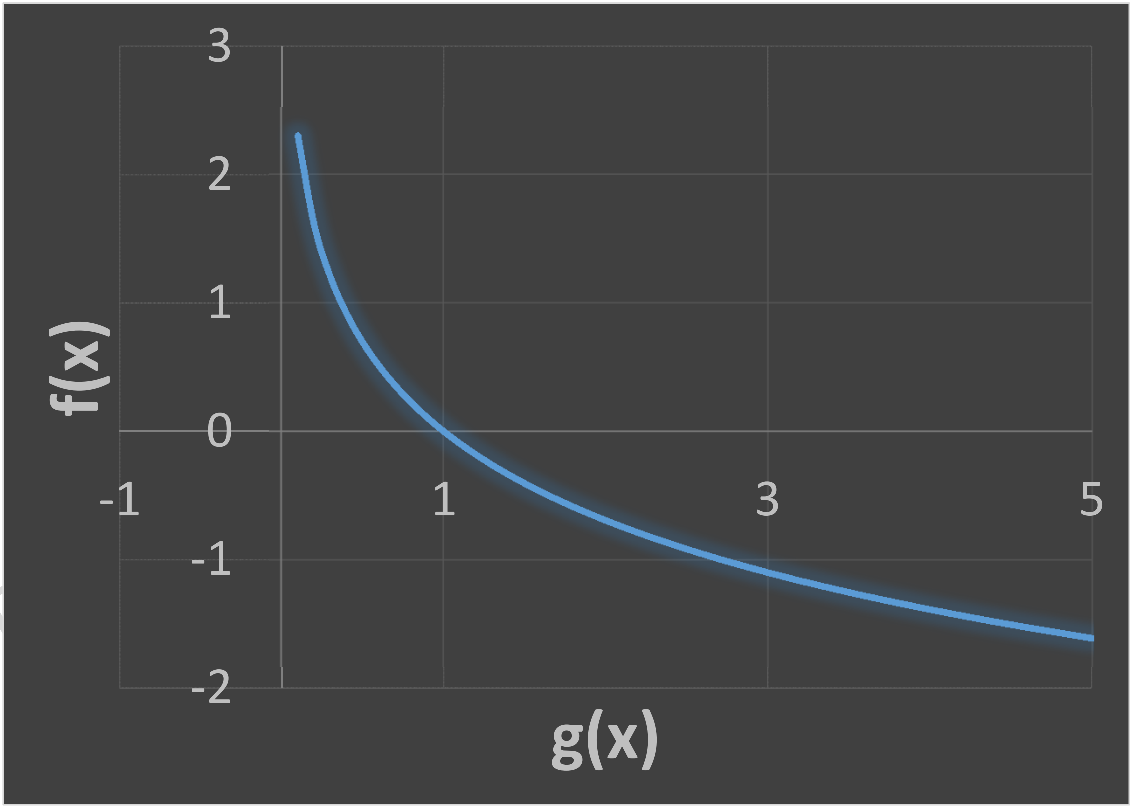
$$\Psi = R[h(x)]$$



Penalty terms

Log penalty

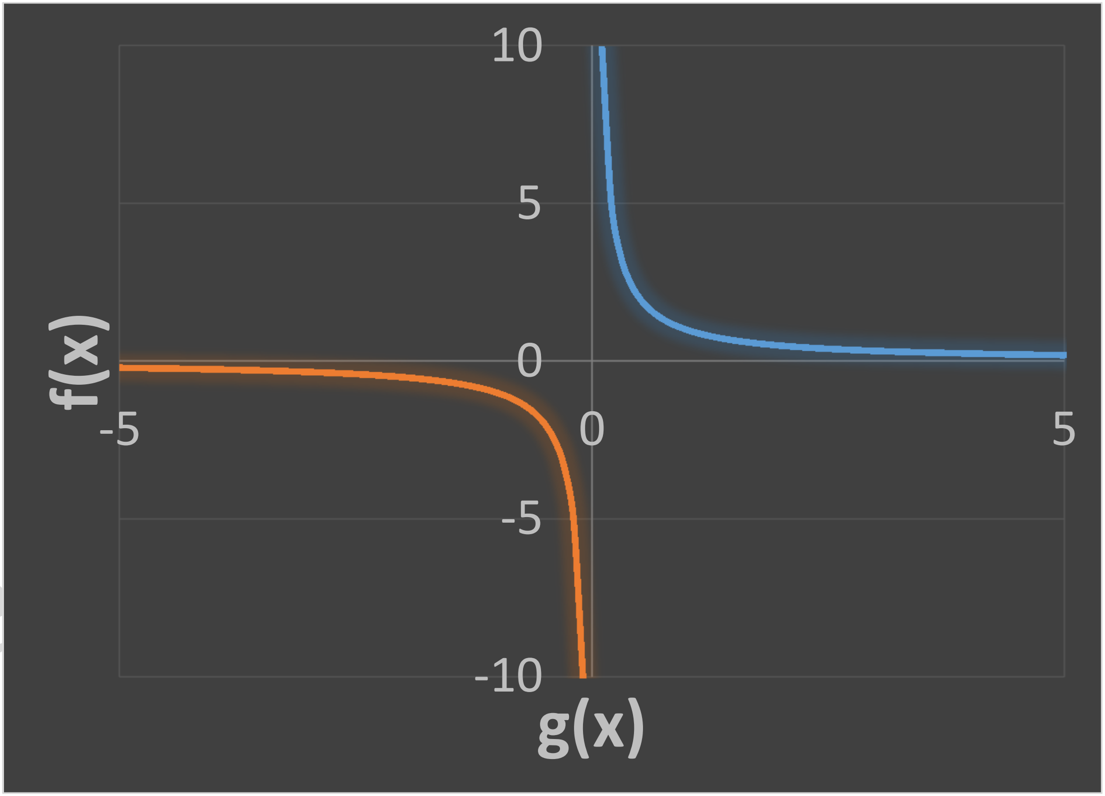
$$\Psi = -R ln[g(x)]$$



Penalty terms

Inverse penalty

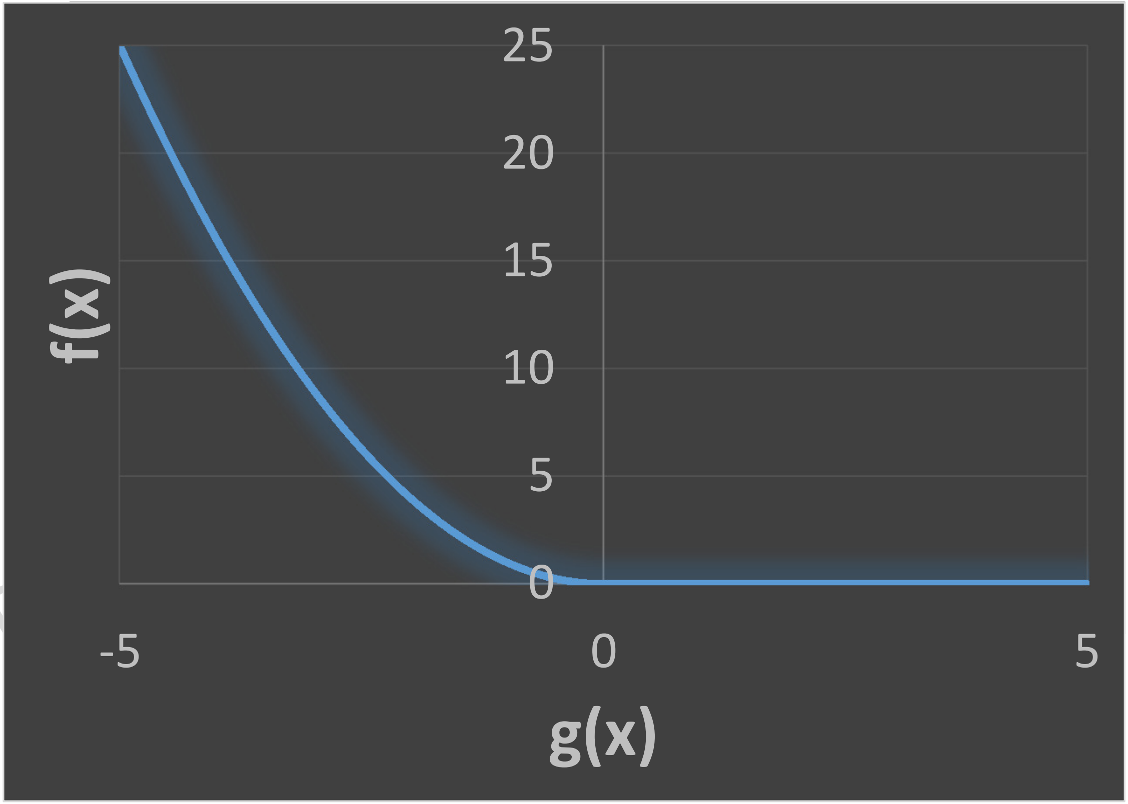
$$\Psi = -R\left[\frac{1}{g(x)}\right]$$



Penalty terms

Bracket operator

$$\Psi = -R\langle g(x)\rangle$$



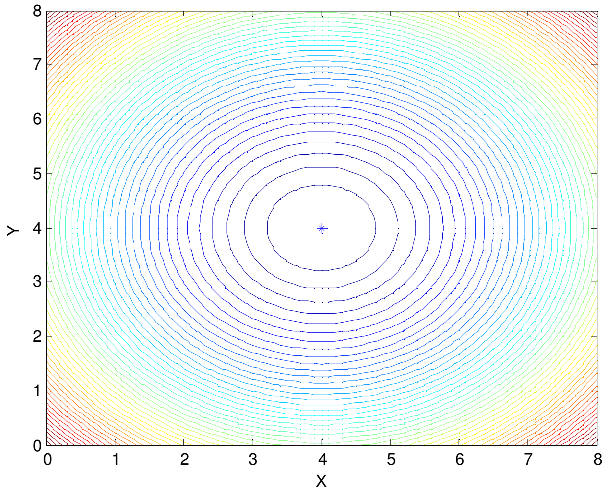
Take an example

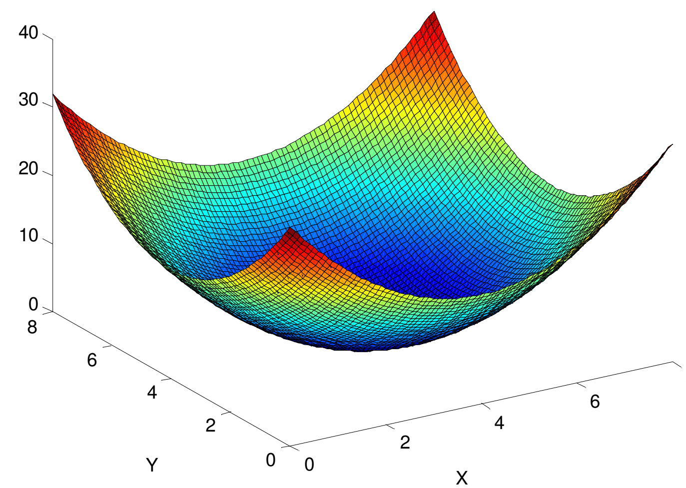
Minimize $f = (x_1 - 4)^2 + (x_2 - 4)^2$

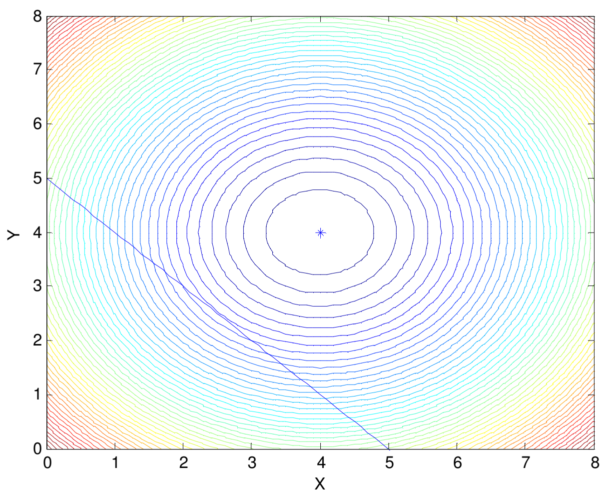
Subject to $g = x_1 + x_2 - 5$

The transform function can be written as

Minimize $\mathrm{F} = (x_1 - 4)^2 + (x_2 - 4)^2 + R(x_1 + x_2 - 5\,)^2$



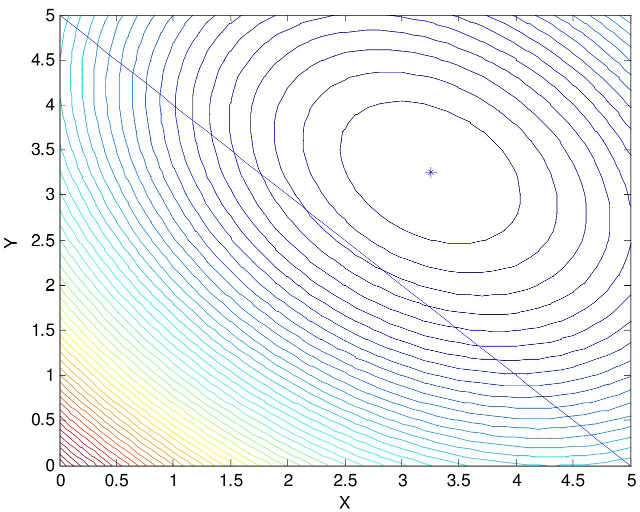




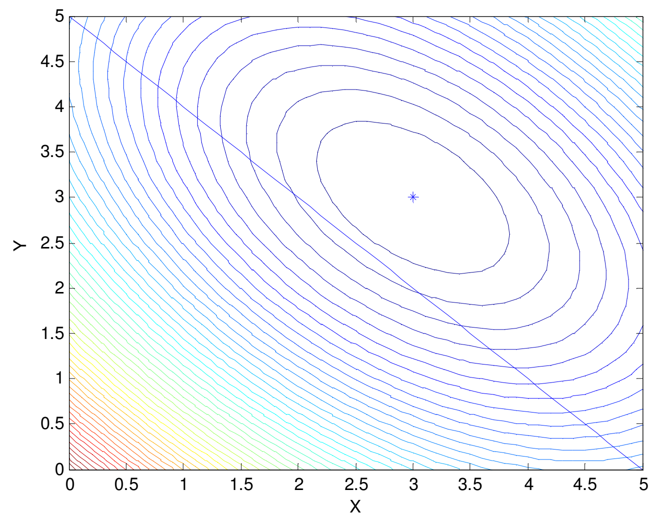
Minimize $f = (x_1 - 4)^2 + (x_2 - 4)^2$

Subject to $g = x_1 + x_2 - 5$

Minimize $F = (x_1 - 4)^2 + (x_2 - 4)^2 + R(x_1 + x_2 - 5\,)^2$



$R = 0.5$



$R = 1$

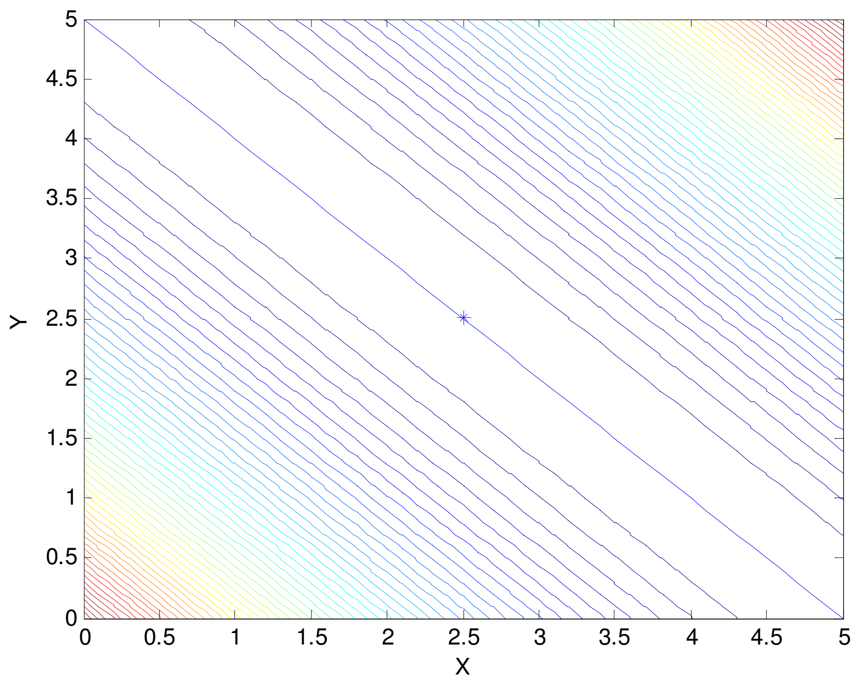
Optimal solution is

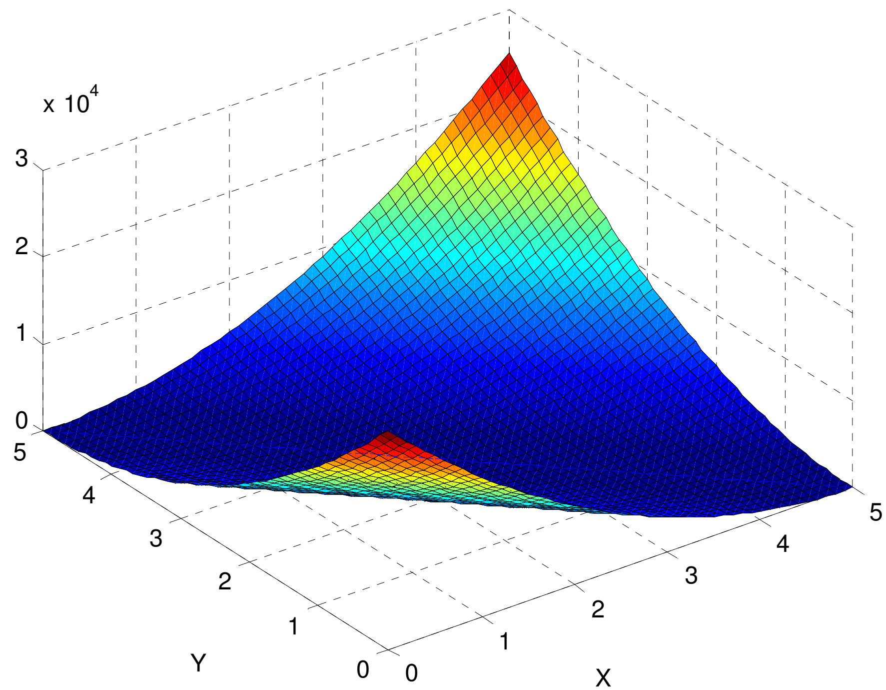
3.250	3.250

Optimal solution is

3.000	3.000

R	x1	x2	f(x)	h(x)	F
0	4.000	4.000	0.000	3.000	0.000
0.5	3.250	3.250	1.125	1.500	2.250
1	3.000	3.000	2.000	1.000	3.000
5	2.636	2.636	3.719	0.273	4.091
10	2.571	2.571	4.082	0.143	4.286
20	2.537	2.537	4.283	0.073	4.390
30	2.525	2.525	4.354	0.049	4.426
50	2.515	2.515	4.411	0.030	4.455
100	2.507	2.507	4.455	0.015	4.478
200	2.504	2.504	4.478	0.007	4.489
500	2.501	2.501	4.491	0.003	4.496
1000	2.501	2.501	4.496	0.001	4.498
10000	2.500	2.500	4.500	0.000	4.500





$R = 1000$

Optimal solution is 2.501 2.501

Thanks