Intersections of Solids

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Whenever two or more solids combine, a definite curve is seen at their intersection. This curve is called the *curve of intersection* (COI).
CASES OF INTERSECTION
The cases of intersection depend on the type of intersecting solids and the manner in which they intersect. Two intersecting solids may be of the same type (e.g., prism and prism) or of different types (e.g., prism and pyramid). The possible combinations are shown in Table below.
The two solids may intersect in different ways. The axes of the solids may be parallel, inclined or perpendicular to each other. The axes may be intersecting, offset or coinciding. Therefore, the following sub-cases exist:

(i) Axes perpendicular and intersecting
(ii) Axes perpendicular and offset
(iii) Axes inclined and intersecting
(iv) Axes inclined and offset
(v) Axes parallel and coinciding
(vi) Axes parallel and offset
Intersection

The type of intersection created depends on the types of geometric forms, which can be two- or three-dimensional.

Intersections must be represented on multiview drawings correctly and clearly. For example, when a conical and a cylindrical shape intersect, the type of intersection that occurs depends on their sizes and on the angle of intersection relative to their axes.

The line of intersection is determined using auxiliary views and cutting planes.
Methods — (1) Line and (2) Cutting-plane methods

*Line method:* A number of lines are drawn on the lateral surface of one of the solids and in the region of the line of intersection.

Points of intersection of these lines with the surface of the other solid are then located.

These points will lie on the required line of intersection. They are more easily located from the view in which the lateral surface of the second solid appears edgewise (i.e. as a line). The curve drawn through these points will be the line of intersection.
Cutting-plane method: The two solids are assumed to be cut by a series of cutting planes. The cutting planes may be vertical (i.e. perpendicular to the H.P.), edgewise (i.e. perpendicular to the V.P.) or oblique.

The cutting planes are so selected as to cut the surface of one of the solids in straight lines and that of the other in straight lines or circles.
Intersection of two prisms

Prisms have plane surfaces as their faces.

The line of intersection between two plane surfaces is obtained by locating the positions of points at which the edges of one surface intersect the other surface and then joining the points by a straight line. These points are called vertices.

The line of intersection between two prisms is therefore a closed figure composed of a number of such lines meeting at the vertices.
Intersection of two prisms
A vertical square prism, base 50 mm side, is completely penetrated by a horizontal square prism, base 35 mm side, so that their axes intersect. The axis of the horizontal prism is parallel to the prism, while the faces of the two prisms are equally inclined to the prism. Draw the projections of the solids, showing lines of intersection. (Assume suitable lengths for the prisms.)

**Steps:**

Draw the projections of the prisms in the required position. The faces of the vertical prism are seen as lines in the top view. Hence, let us first locate the points of intersection in that view.
Steps:

Lines 1-1 and 3-3 intersect the edge of the vertical prism at points $p_1$ and $p_3$ (coinciding with $a$). Lines 2-2 and 4-4 intersect the faces at $p_2$ and $p_4$ respectively.

The exact positions of these points along the length of the prism may now be determined by projecting them on corresponding lines in the front view. For example, $p_2$ is projected to $p_2'$ on the line 2'2'. Note that $p_4'$ coincides with $p_2'$.
Intersection of two prisms

Draw lines $p_1'p_2'$ and $p_2'p_3'$. Lines $p_1'p_4'$ and $p_3'p_4'$ coincide with the front lines. These lines show the line of intersection.

Lines $q_1'q_2'$ and $q_2'q_3'$ on the other side are obtained in the same manner.

Note that the lines for the hidden portion of the edges are shown as dashed lines. The portions $p_1'p_3'$ and $q_1'q_3'$ of vertical edges $a'a'$ and $c'c'$ do not exist and hence, must be removed or kept fainter.
Intersection of Cylinder and Cylinder
Intersection of Cylinder and Cylinder

As cylinders have their lateral surfaces curved – the line of intersection between them will also be curved. Points on this line may be located by any of the methods.

For plotting an accurate curve, certain critical or key points, at which the curve changes direction, must also be located. These are the points at which outermost or extreme lines of each cylinder pierce the surface of the other cylinder.
Intersection of Cylinder and Cylinder

Example - A vertical cylinder of 80 mm diameter is completely penetrated by another cylinder of 60 mm diameter, their axes bisecting each other at right angles. Draw their projections showing curves of penetration, assuming the axis of the penetrating cylinder to be parallel to the VP.
Intersection of Cylinder and Cylinder

Assume a series of horizontal cutting planes passing through the horizontal cylinder and cutting both cylinders.

Sections of the horizontal cylinder will be rectangles, while those of the vertical cylinder will always be circles of the same diameter as its own.

Points at which sides of the rectangles intersect the circle will be the curve of intersection. For example, let a horizontal section pass through points 2 and 12.
In the front view, it will be seen as a line coinciding with line $2' 2'$. The section of the horizontal cylinder will be a rectangle of width (i.e. the line 2-12).

The section of the vertical cylinder will be a circle.

Points $p_2$ and $p_{12}$ at which the sides $(2-2$ and $12-12)$ of the rectangle cuts the circle, lie on the curve.
These points are first marked in the top view and then projected to points $p_2'$ and $p_{12}'$ on lines $2'2'$ and $12'12'$ in the front view. Points on the other side of the axis are located in the same manner.
Intersection of Cone and Cylinder
Intersection of Cone and Cylinder

Example - A vertical cone, diameter of base 75 mm and axis 100 mm long, is completely penetrated by a cylinder of 45 mm diameter. The axis of the cylinder is parallel to HP and the VP and intersects the axis of the cone at a point 22 mm above the base. Draw the projections of the solids showing curves of intersection.
Cutting-Plane Method

Draw lines dividing the surface of the cylinder into twelve equal parts.

Assume a horizontal cutting plane passing through say, point 2. The section of the cylinder will be a rectangle of width \( w \) (i.e. the line 2-12), while that of the cone will be a circle of diameter \( ee \).
These two sections intersect at points $p_2$ and $p_{12}$. These sections are *clearly* indicated in the top view by the rectangle 2-2-12-12 and the circle of diameter ee.
In the front view, the cutting plane is seen as a line coinciding with 2' 2'. Points $p_2$ and $p_{12}$ when projected on the line 2' 2' (with which the line 12'-12' coincides) will give a point $p_2'$ (with which $p_{12}'$ will coincide). Then $p_2'$ and $p_{12}'$ are the points on the curve of intersection.
To obtain the points systematically, draw circles with centre 0 and diameters dd, ee, ff, etc. cutting lines through 1, 2 and 12, 3 and 11 etc. at points $p_1$, $p_2$ and $p_{12}$, $p_3$ and $p_{11}$ etc. Project these points to the corresponding lines in the front view.
Final views
THANK YOU