ME 101: Engineering Mechanics

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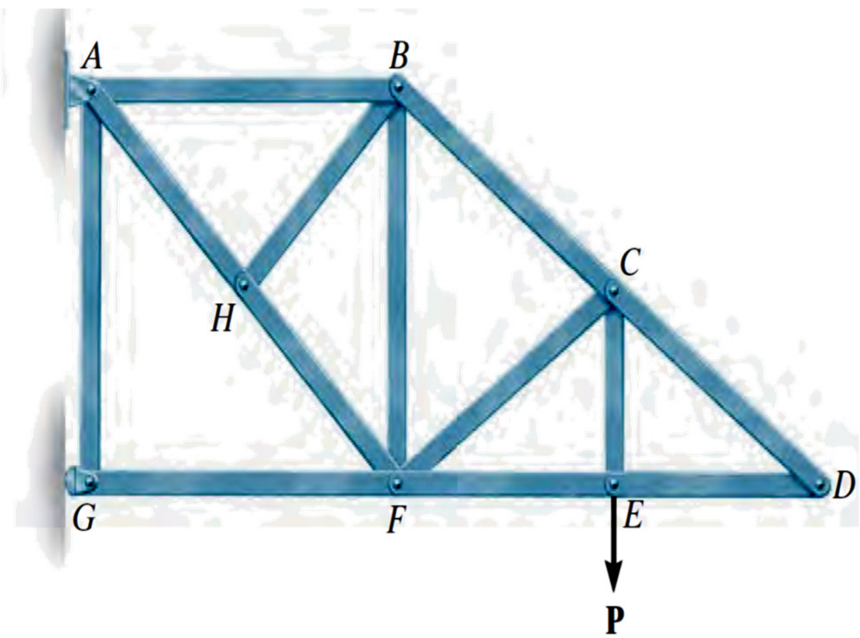
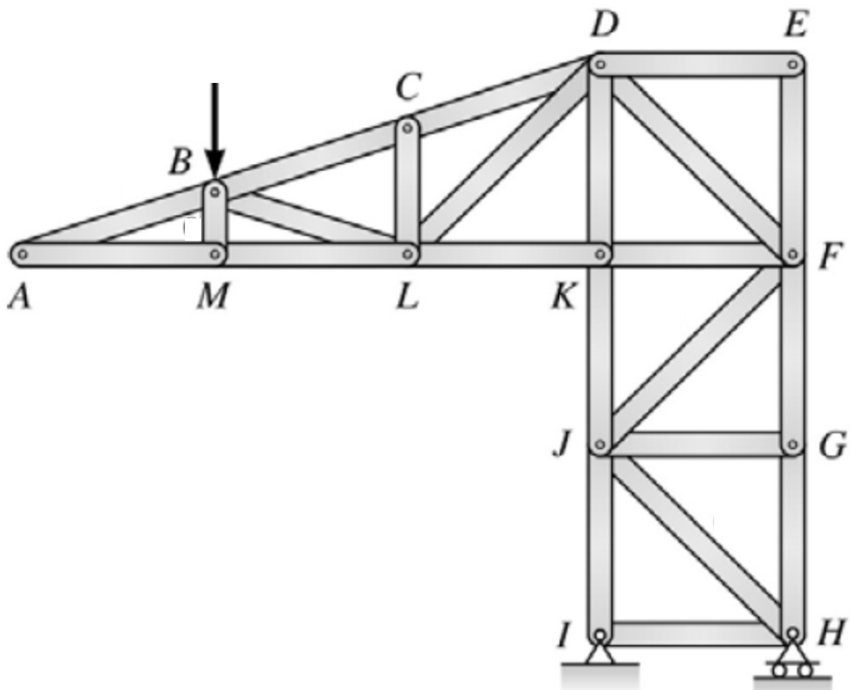
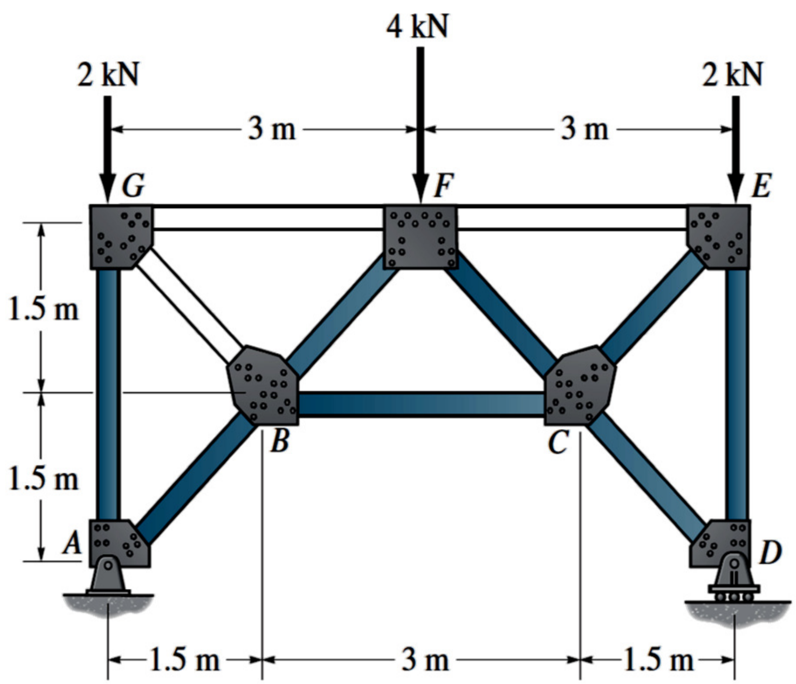
Q. No. 1 For the trusses shown below, indicate the members, which carry zero force.

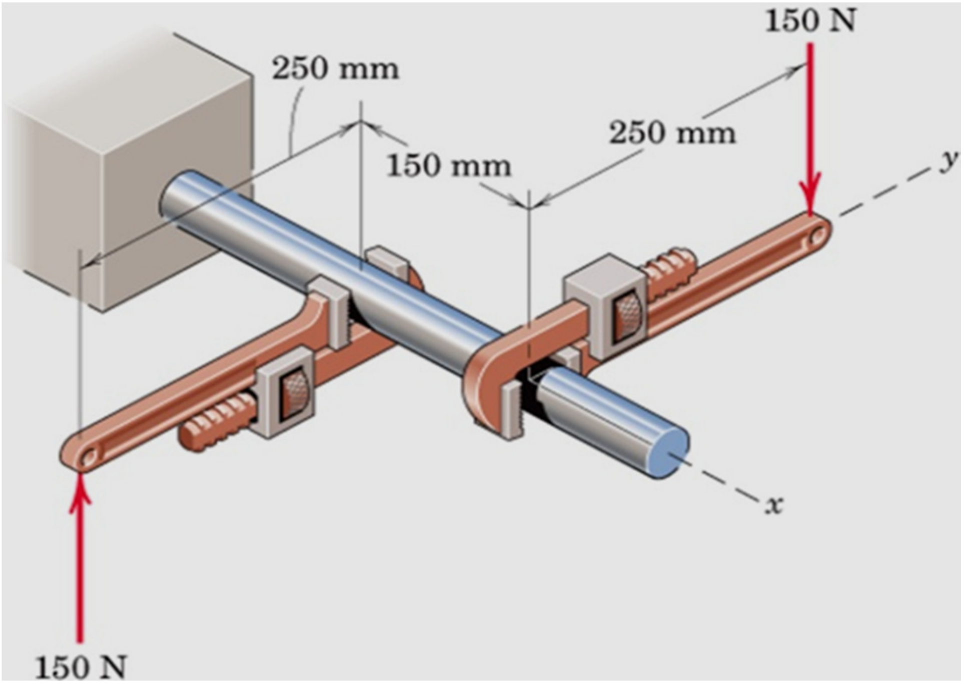




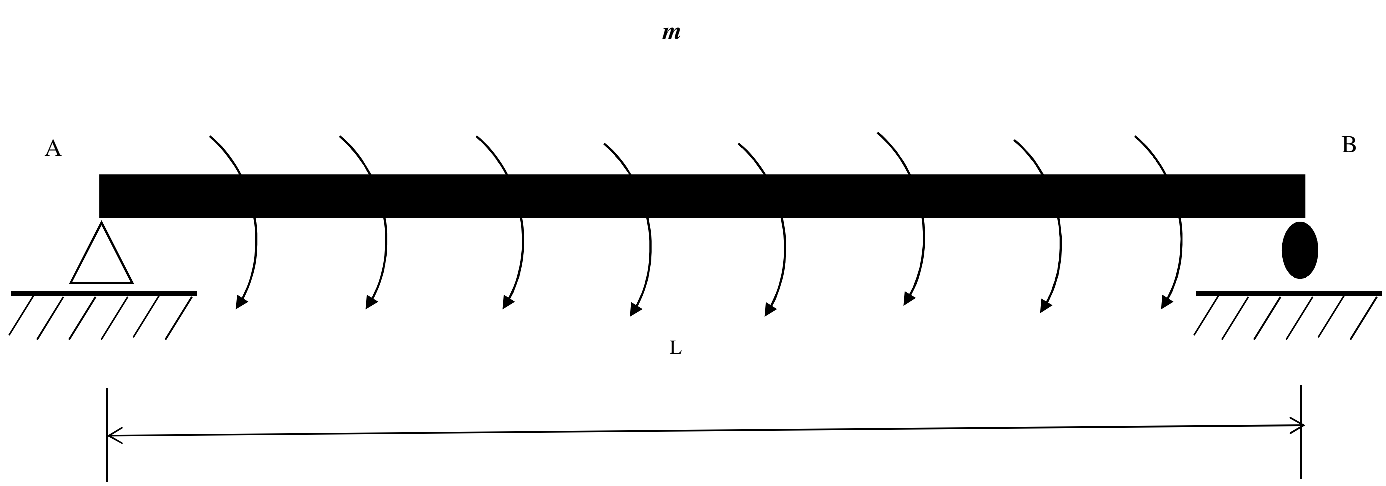
Q. No. 2 For the plane truss shown in figure below, find out the forces in members in *FE*, *FC*, and *BC* by considering all members as pin connected using method of sections.



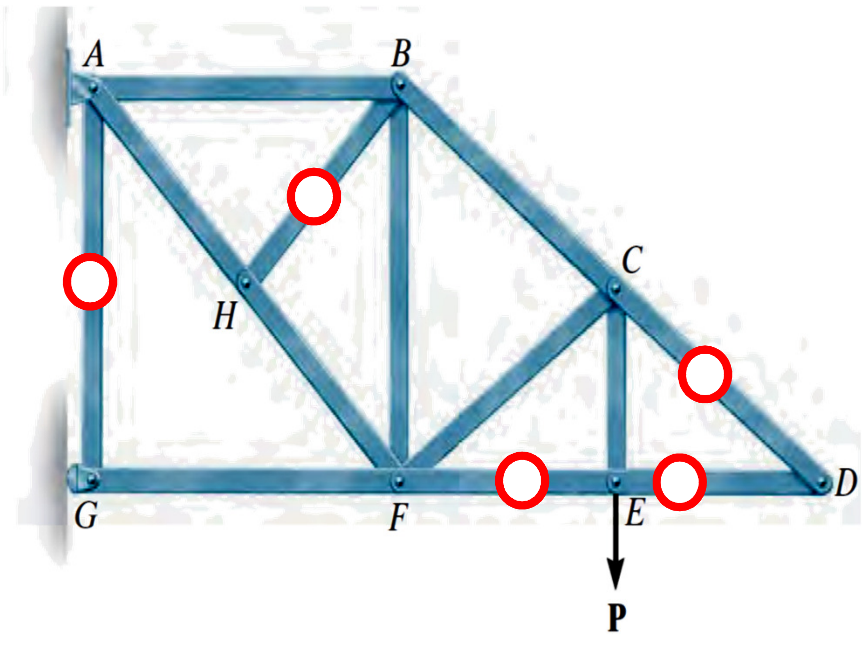
Q. No. 3 The two forces acting on the handles of the pipe wrenches constitute a couple **M**. Express the couple as a vector.



Q. No. 4 The beam is subjected to uniformly distributed moment m (moment/length) and is shown in Figure 2. Draw the shear force and bending moment diagrams for the beam.



Solution of Q. No. 1



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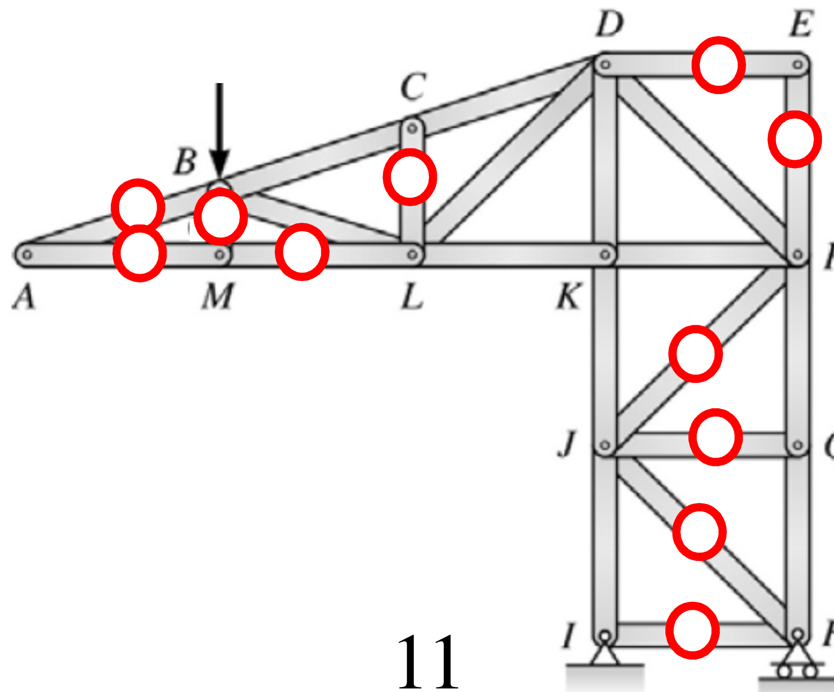

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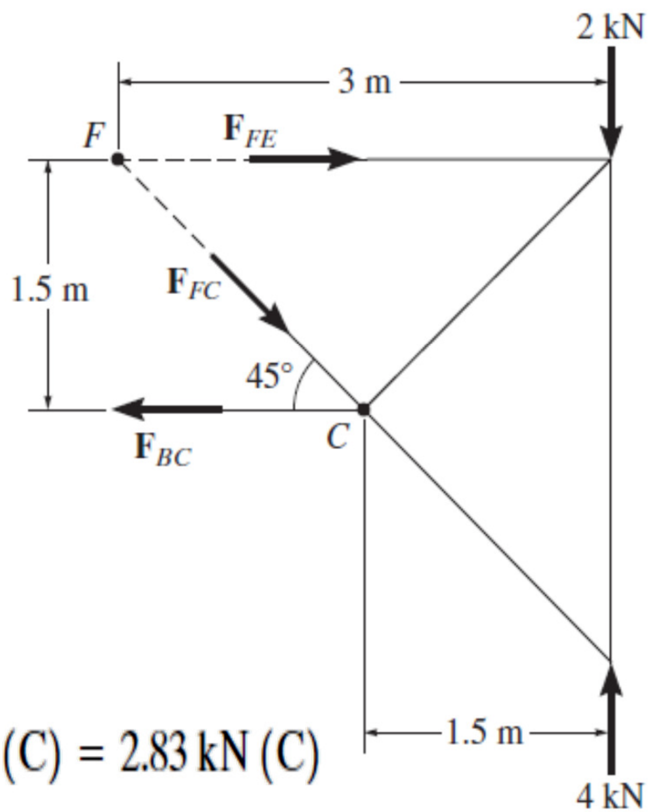
Solution of Q. No. 2



For the entire truss

$\circlearrowright + \Sigma M_A = 0;$

$N_D(6) - 2(6) - 4(3) = 0 \quad N_D = 4.00 \text{ kN}$

Consider the Right Segment:

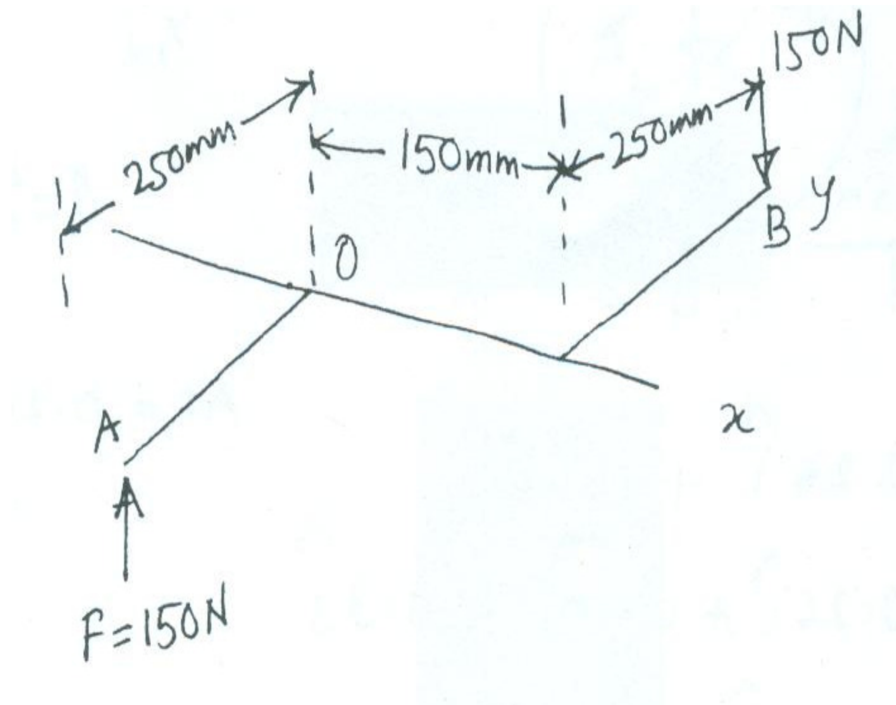
$+\uparrow \Sigma F_y = 0; \qquad 4.00 - 2 - F_{FC} \sin 45^\circ = 0 \quad F_{FC} = 2.828 \text{ kN (C)} = 2.83 \text{ kN (C)}$

$\circlearrowright + \Sigma M_F = 0; \qquad 4.00(3) - 2(3) - F_{BC}(1.5) = 0 \quad F_{BC} = 4.00 \text{ kN (T)}$

$\circlearrowright + \Sigma M_C = 0; \qquad 4.00(1.5) - 2(1.5) - F_{FE}(1.5) = 0 \quad F_{FE} = 2.00 \text{ kN (C)}$

Solution of Q. No. 3

Taking O as origin



$$\vec{r}_A = -0.25\hat{j}, \vec{r}_B = 0.15\hat{i} + 0.25\hat{j}$$

$$\vec{r}_{BA} = -0.25\hat{j} - 0.15\hat{i} - 0.25\hat{j}$$

$$\Rightarrow \vec{r}_{BA} = -0.15\hat{i} - 0.5\hat{j}$$

$$\vec{C} = \vec{r}_{BA} \times \vec{F} = (-0.15\hat{i} - 0.5\hat{j}) \times 150\hat{k}$$

$$\Rightarrow \vec{C} = (22.5\hat{j} - 75\hat{i})Nm$$

Solution of Q. No. 4

$\sum F_x = 0 \rightarrow A_x = 0$

$\sum M_A = 0 \rightarrow m\mathrm{L} - B_y L = 0$

$B_y = m$

$\sum M_B = 0 \rightarrow m\mathrm{L} + A_y L = 0$

$A_y = - m$

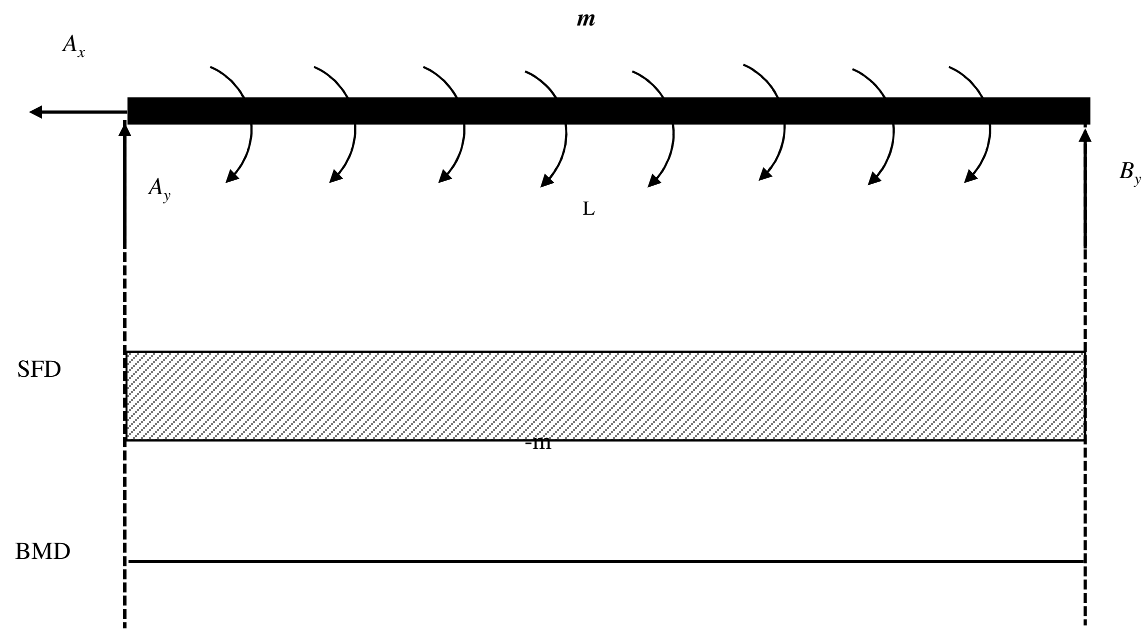
Shear force at any section = - m

Bending moment at a distance x from A:

$M_x = A_y\, x + m\, x$

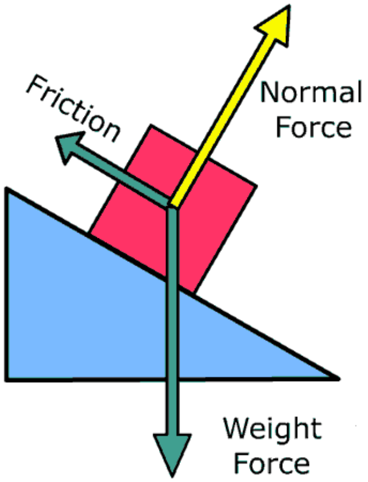
Since $A_y = - m \rightarrow M_x = 0$

Bending moment at any section = 0

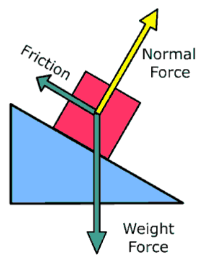


Friction

The effective design of a brake system, such as the one for this bicycle, requires an efficient capacity for the mechanism to resist frictional forces. In this chapter, we will study the nature of friction and show how these forces are considered in engineering analysis and design.



Friction



Usual Assumption till now:

Forces of action and reaction between contacting surfaces act normal to the surface

→valid for interaction between smooth surfaces

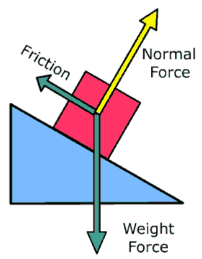
→in many cases ability of contacting surfaces to support tangential forces is very important (Ex: Figure above)

Frictional Forces

Tangential forces generated between contacting surfaces

- occur in the interaction between all real surfaces
- always act in a direction opposite to the direction of motion

Friction



Frictional forces are Not Desired in some cases:

- Bearings, power screws, gears, flow of fluids in pipes, propulsion of aircraft and missiles through the atmosphere, etc.
  - Friction often results in a loss of energy, which is dissipated in the form of heat
  - Friction causes Wear

Frictional forces are Desired in some cases:

- Brakes, clutches, belt drives, wedges
- walking depends on friction between the shoe and the ground

Ideal Machine/Process: Friction small enough to be neglected

Real Machine/Process: Friction must be taken into account

Types of Friction

Dry Friction (Coulomb Friction)

occurs between unlubricated surfaces of two solids

Effects of dry friction acting on exterior surfaces of rigid bodies → ME101

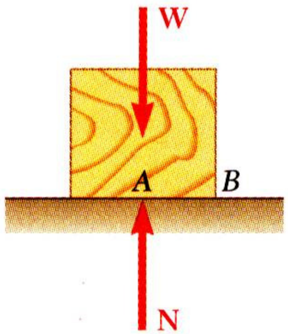
Fluid Friction

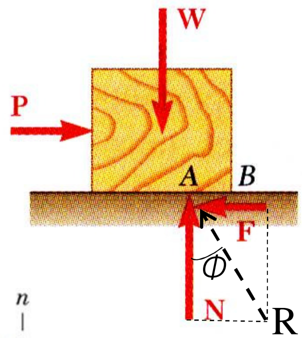
occurs when adjacent layers in a fluid (liquid or gas) move at a different velocities. Fluid friction also depends on viscosity of the fluid. → Fluid Mechanics

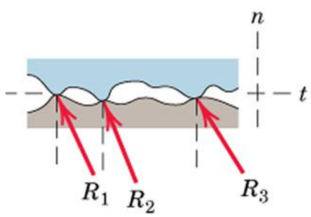
Internal Friction

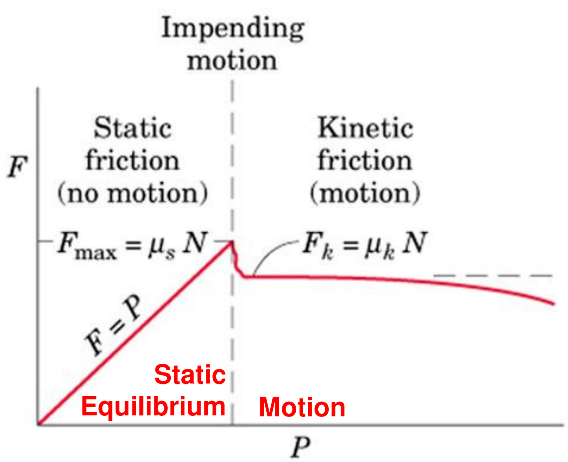
occurs in all solid materials subjected to cyclic loading, especially in those materials, which have low limits of elasticity → Material Science

Mechanism of Dry Friction









- Block of weight W placed on horizontal surface. Forces acting on block are its weight and reaction of surface N.
- Small horizontal force P applied to block. For block to remain stationary, in equilibrium, a horizontal component F of the surface reaction is required. *F is a Static-Friction force.*
- As P increases, static-friction force F increases as well until it reaches a maximum value F_m.

$$F_m = \mu_s N$$

- Further increase in P causes the block to begin to move as F drops to a smaller *Kinetic-Friction force F_k*.

$$F_k = \mu_k N$$

μ_s is the Coefficient of Static Friction
μ_k is the Coefficient of Kinetic Friction

Mechanism of Dry Friction

Table 8.1. Approximate Values of Coefficient of Static Friction for Dry Surfaces

Metal on metal	0.15–0.60
Metal on wood	0.20–0.60
Metal on stone	0.30–0.70
Metal on leather	0.30–0.60
Wood on wood	0.25–0.50
Wood on leather	0.25–0.50
Stone on stone	0.40–0.70
Earth on earth	0.20–1.00
Rubber on concrete	0.60–0.90

A friction coefficient reflects roughness, which is a geometric property of surfaces

- Maximum static-friction force:

$$F_m = \mu_s N$$

- Kinetic-friction force:

$$F_k = \mu_k N$$

$$\mu_k \cong 0.75\mu_s$$

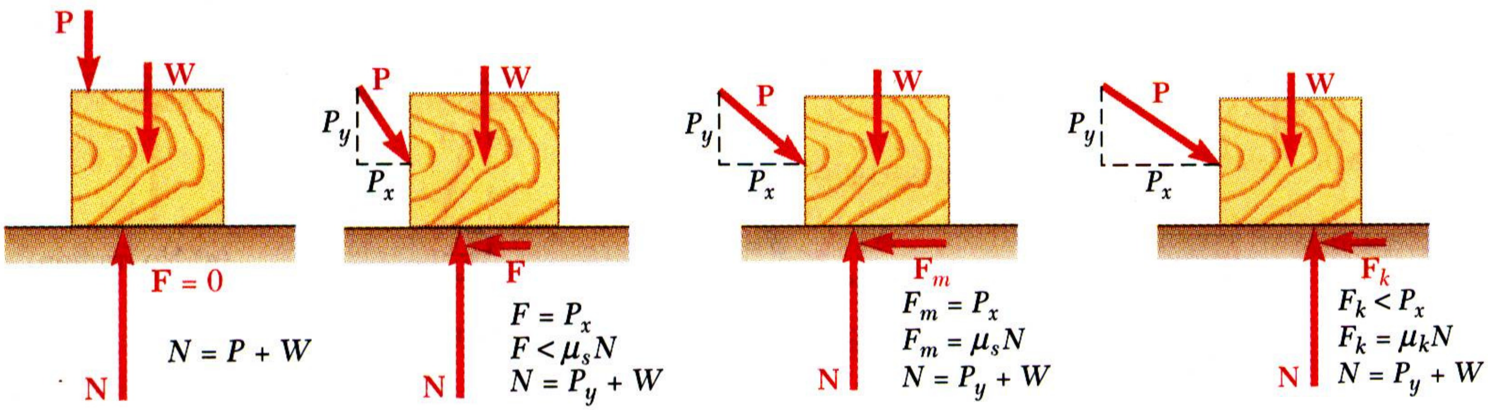
- Maximum static-friction force and kinetic-friction force are:
  - proportional to normal force
  - dependent on type and condition of contact surfaces
  - independent of contact area

When the surfaces are in relative motion, the contacts are more nearly along the tops of the humps, and the t-components of the *R*'s are smaller than when the surfaces are at rest relative to one another

→ Force necessary to maintain motion is generally less than that required to start the block when the surface irregularities are more nearly in mesh → $F_m > F_k$

Mechanism of Dry Friction

- Four situations can occur when a rigid body is in contact with a horizontal surface:



- No friction, ($P_x = 0$)

Equations of Equilibrium Valid

- No motion, ($P_x < F_m$)

Equations of Equilibrium Valid

- Motion impending, ($P_x = F_m$)

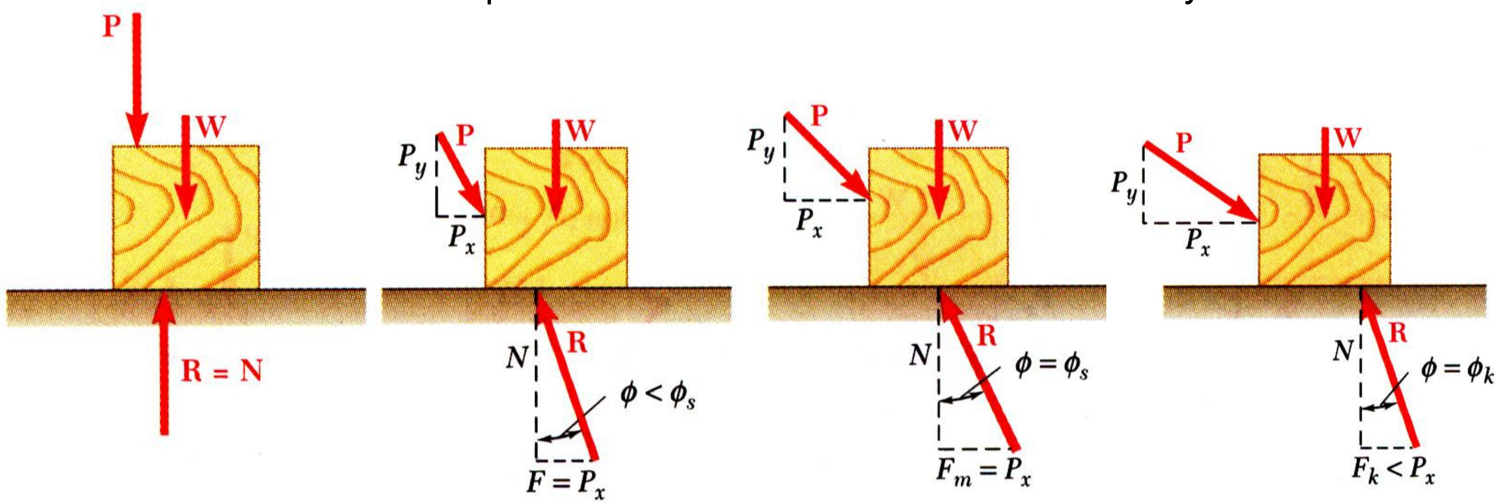
Equations of Equilibrium Valid

- Motion, ($P_x > F_m$)

Equations of Equilibrium Not Valid

Mechanism of Dry Friction

Sometimes convenient to replace normal force N & friction force F by their resultant $\boldsymbol{R}$:



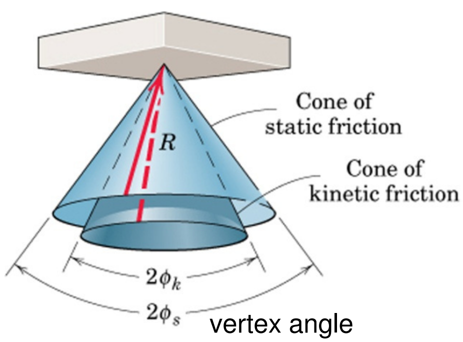
- No friction
- No motion
- Motion impending
- Motion

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan\phi_s = \mu_s$$

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\tan\phi_k = \mu_k$$

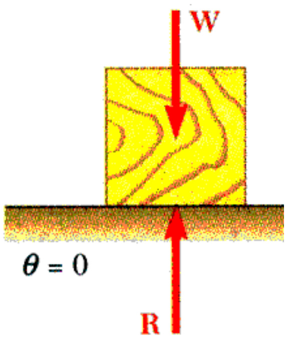


Friction Angles

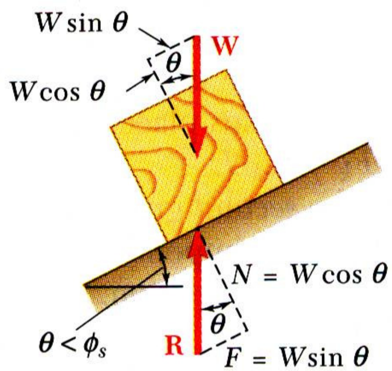
ϕ_s = angle of static friction, ϕ_k = angle of kinetic friction

Mechanism of Dry Friction

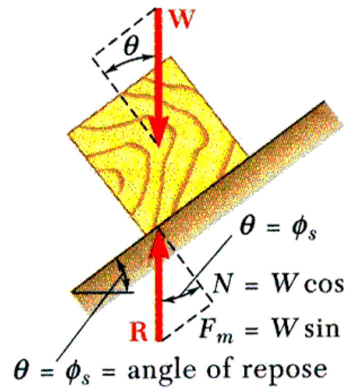
- Consider block of weight W resting on board with variable inclination angle θ.



- No friction

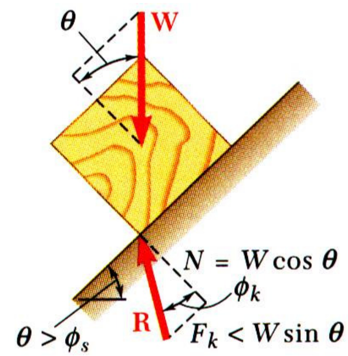


- No motion



- Motion impending

Angle of Repose = Angle of Static Friction



- Motion

The reaction R is not vertical anymore, and the forces acting on the block are unbalanced

Dry Friction

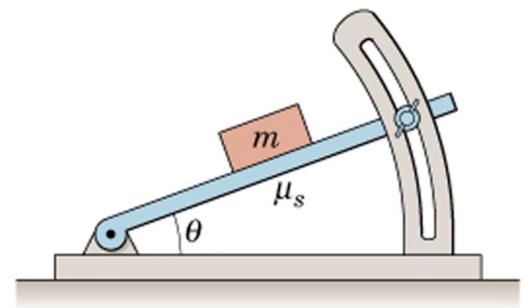
Example

Determine the maximum angle Θ

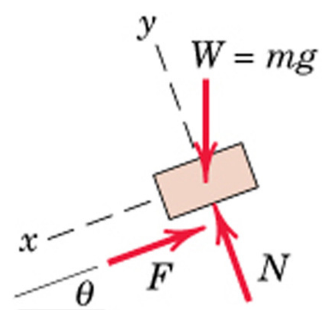
before the block begins to slip.

μ_s = Coefficient of static friction between

the block and the inclined surface



Solution: Draw the FBD of the block



$$[\Sigma F_x = 0] \qquad mg \sin\theta - F = 0 \qquad F = mg\sin\theta$$

$$[\Sigma F_y = 0] \qquad -mg\cos\theta + N = 0 \qquad N = mg\cos\theta$$

$$F/N = \tan\theta$$

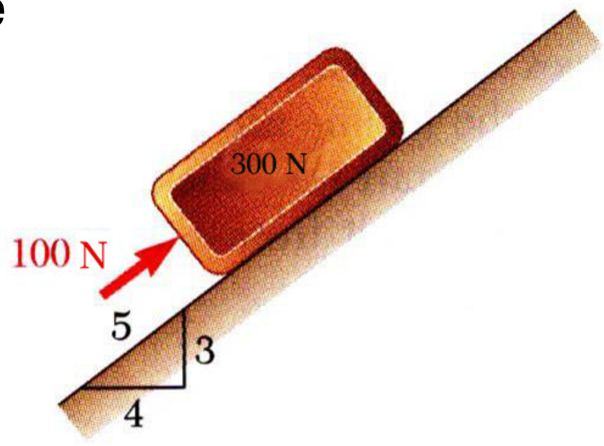
Max angle occurs when $\boldsymbol{F} = \boldsymbol{F}_{max} = \boldsymbol{\mu}_s \boldsymbol{N}$

Therefore, for impending motion: $\mu_s = \tan\theta_{max}$ or $\theta_{max} = \tan^{-1}\mu_s$

The maximum value of Θ is known as *Angle of Repose*

Dry Friction

Example

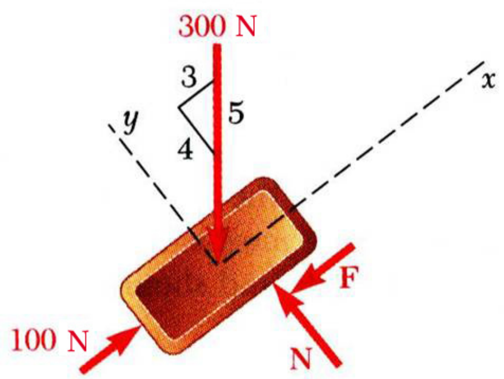


A 100 N force acts as shown on a 300 N block placed on an inclined plane. The coefficients of friction between the block and plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine whether the block is in equilibrium and find the value of the friction force.

<u>SOLUTION</u>:

- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.
- Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.
- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

Dry Friction



SOLUTION:

- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.

$$\sum F_x = 0: \qquad 100\text{ N} - \tfrac{3}{5}(300\text{ N}) - F = 0$$

$$F = -80\text{ N} \qquad \rightarrow F \text{ acting upwards}$$

$$\sum F_y = 0: \qquad N - \tfrac{4}{5}(300\text{ N}) = 0$$

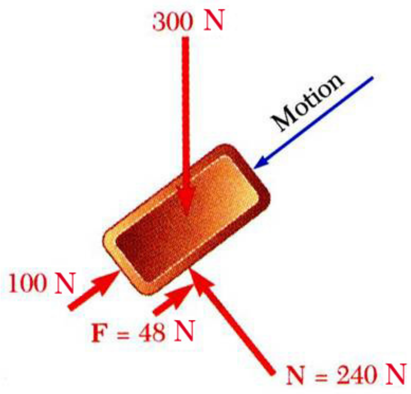
$$N = 240\text{ N}$$

- Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.

$$F_m = \mu_s N \qquad F_m = 0.25(240\text{ N}) = 60\text{ N}$$

The block will slide down the plane along F.

Dry Friction



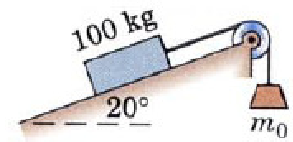
- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

$$F_{actual} = F_k = \mu_k \mathrm{N}$$
$$= 0.20(240\ \mathrm{N})$$

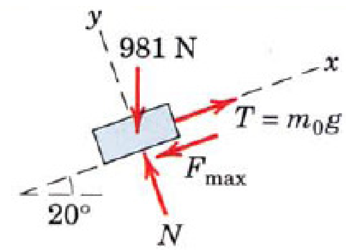
$$\boxed{F_{actual} = 48\ \mathrm{N}}$$

Sample Problem 6/2

Determine the range of values which the mass m_0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.



CASE I



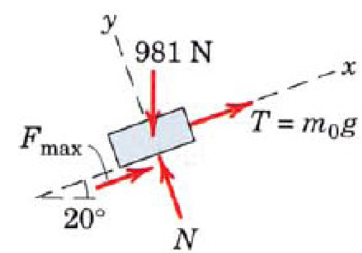
Case I

$[\Sigma F_y = 0]$ $\quad N - 981 \cos 20° = 0 \qquad N = 922$ N

$[F_{max} = \mu_s N]$ $\quad F_{max} = 0.30(922) = 277$ N

$[\Sigma F_x = 0]$ $\quad m_0(9.81) - 277 - 981 \sin 20° = 0 \quad m_0 = 62.4$ kg

CASE II

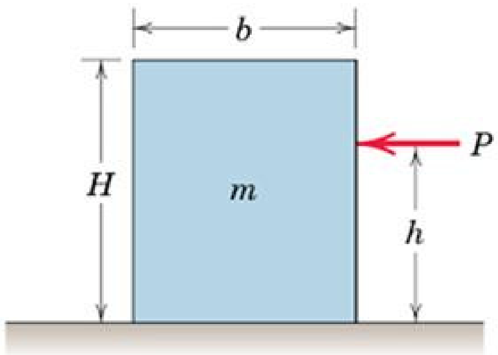


Case II

$[\Sigma F_x = 0]$ $\quad m_0(9.81) + 277 - 981 \sin 20° = 0 \quad m_0 = 6.01$ kg

Dry Friction

Example

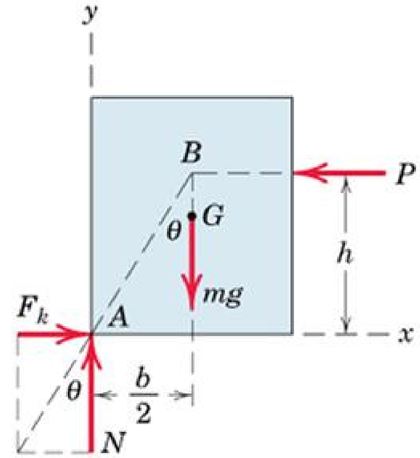


The block moves with constant velocity under the action of *P*. μ_k is the Coefficient of Kinetic Friction. Determine:

(a) Maximum value of *h* such that the block slides without tipping over

(b) Location of a point C on the bottom face of the block through which resultant of the friction and normal forces must pass if *h*=*H*/2

Solution: (a) FBD for the block on the verge of tipping:



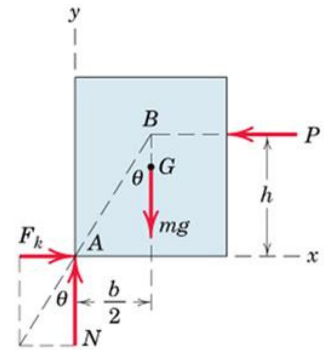
The resultant of F_k and *N* passes through point B through which P must also pass, since three coplanar forces in equilibrium are concurrent.

Friction Force:

$\boldsymbol{F_k} = \boldsymbol{\mu_k N}$ since slipping occurs

$\Theta = \tan^{-1}\boldsymbol{\mu_k}$

Dry Friction



Solution (a) Apply Equilibrium Conditions (constant velocity!)

$$[\Sigma F_y = 0] \qquad N - mg = 0 \qquad N = mg$$

$$[\Sigma F_x = 0] \qquad F_k - P = 0 \qquad P = F_k = \mu_k N = \mu_k mg$$

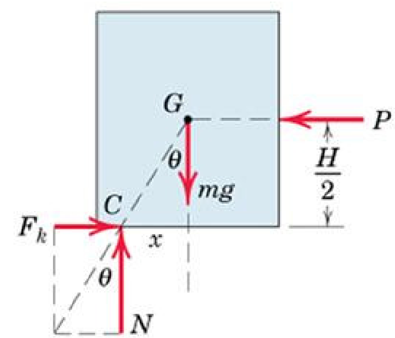
$$[\Sigma M_A = 0] \qquad Ph - mg\frac{b}{2} = 0 \qquad h = \frac{mgb}{2P} = \frac{mgb}{2\mu_k mg} = \frac{b}{2\mu_k}$$

Alternatively, we can directly write from the geometry of the FBD:

$$\tan\theta = \mu_k = \frac{b/2}{h} \qquad h = \frac{b}{2\mu_k}$$

If *h* were greater than this value, moment equilibrium at A would not be satisfied and the block would tip over.

Solution (b) Draw FBD



$\Theta = \tan^{-1}\mu_k$ since the block is slipping. From geometry of FBD:

$$\frac{x}{H/2} = \tan\theta = \mu_k \qquad \text{so} \qquad x = \mu_k H/2$$

Alternatively use equilibrium equations

Applications of Friction in Machines

Wedges

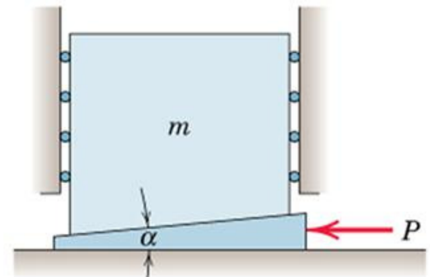
- Simple machines used to raise heavy loads.
- Force required to lift block is significantly less than block weight.
- Friction prevents wedge from sliding out.
- Want to find minimum force *P* to raise block.

FBDs:
Reactions are inclined at an angle Φ from their respective normals and are in the direction opposite to the motion.

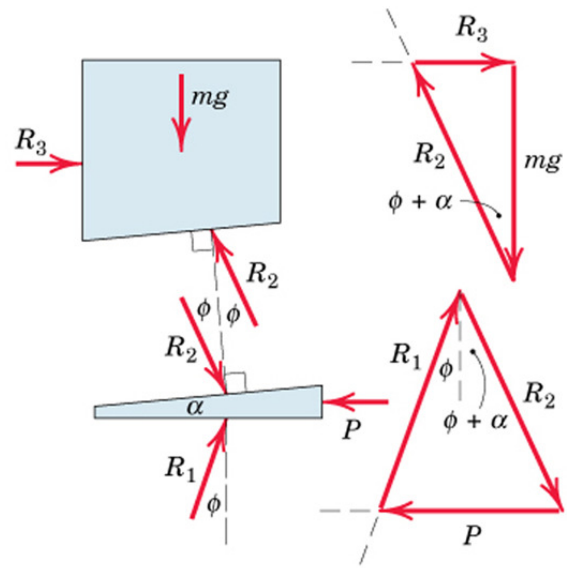
Force vectors acting on each body can also be shown.

R_2 is first found from upper diagram since mg is known.

Then P can be found out from the lower diagram since R_2 is known.



Coefficient of Friction for each pair of surfaces $\mu = \tan\phi$ (Static/Kinetic)



Forces to raise load

Applications of Friction in Machines: Wedges

P is removed and wedge remains in place

→ Equilibrium of wedge requires that the equal reactions R_1 and R_2 be collinear

→ In the figure, wedge angle α is taken to be less than ϕ

Impending slippage at the upper surface

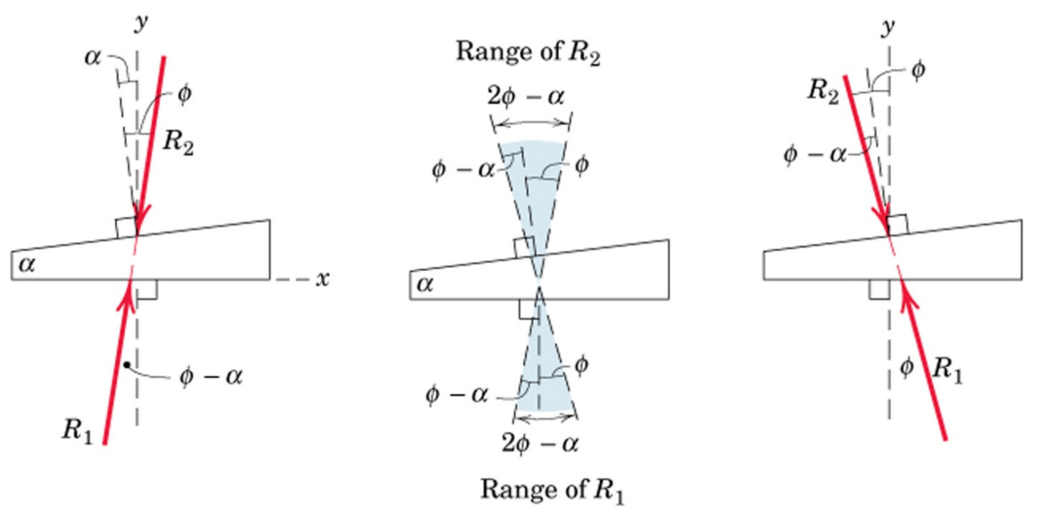
Impending slippage at the lower surface

Slippage must occur at both surfaces simultaneously

In order for the wedge to slide out of its space

→ Else, the wedge is Self-Locking

Range of angular positions of R_1 and R_2 for which the wedge will remain in place is shown in figure (b)



(a) Slipping impending at upper surface

(b) Range of $R_1 = R_2$ for no slip

(c) Slipping impending at lower surface

Simultaneous slippage is not possible if $\alpha < 2\phi$

Applications of Friction in Machines: Wedges

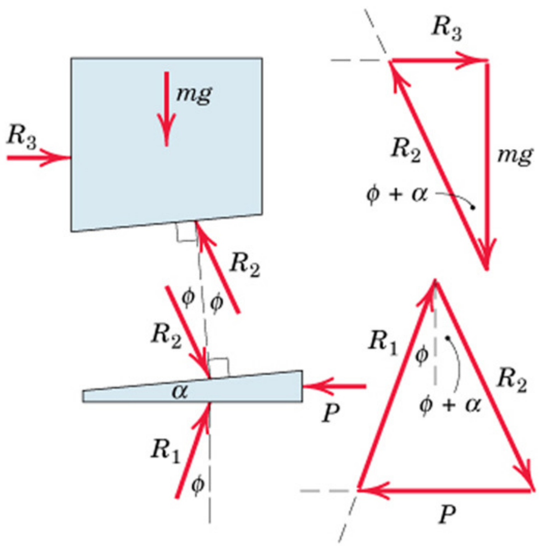
A pull P is required on the wedge for withdrawal of the wedge

→ **The reactions R_1 and R_2 must act on the opposite sides of their normal from those when the wedge was inserted**

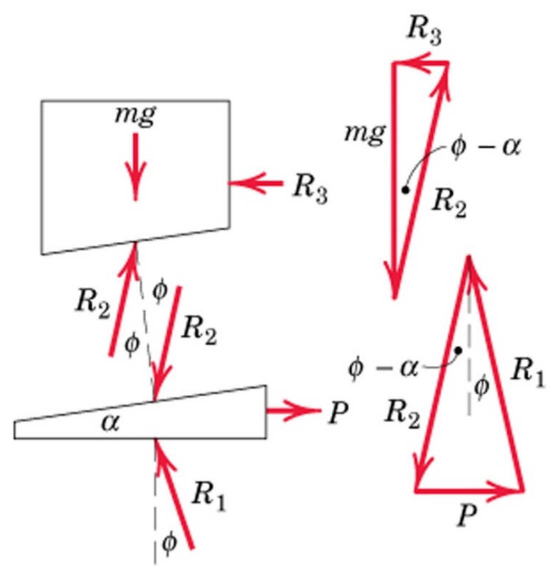
→ Solution by drawing FBDs and vector polygons

→ Graphical solution

→ Algebraic solutions from trigonometry



Forces to raise load



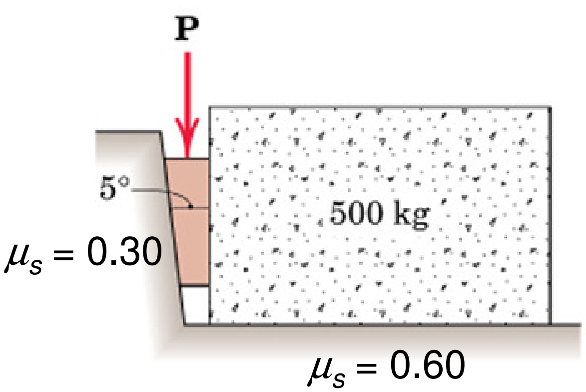
Forces to lower load

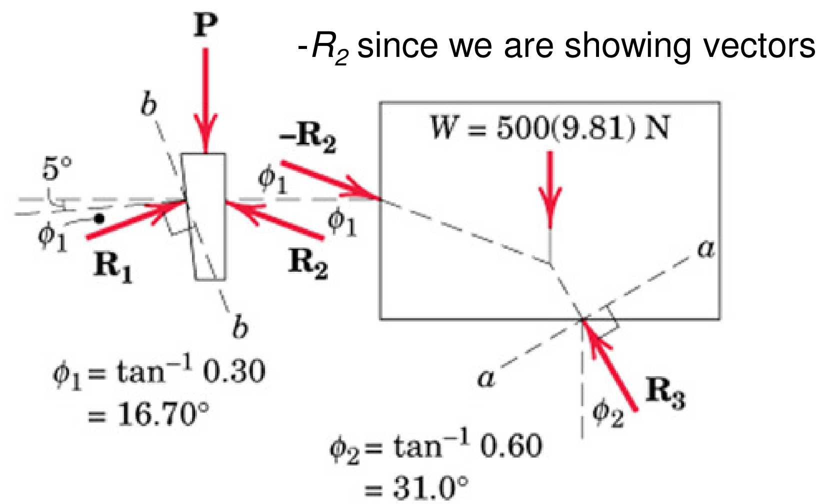
Applications of Friction in Machines

Example: Wedge

Coefficient of Static Friction for both pairs of wedge = 0.3
Coefficient of Static Friction between block and horizontal surface = 0.6
Find the least P required to move the block

Solution: Draw FBDs





Solution: $W = 500x9.81 = 4905$ N

Three ways to solve

Method 1:

Equilibrium of FBD of the Block

$\sum F_X = 0$

$R_2 \cos \phi_1 = R_3 \sin \phi_2 \rightarrow R_2 = 0.538 R_3$

$\sum F_Y = 0$

$4905 + R_2 \sin \phi_1 = R_3 \cos \phi_2 \rightarrow R_3 = 6970$ N

$\rightarrow R_2 = 3750$ N

Equilibrium of FBD of the Wedge

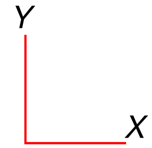
$\sum F_X = 0$

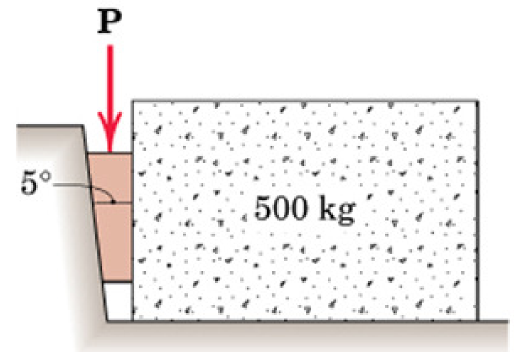
$R_2 \cos \phi_1 = R_1 \cos(\phi_1 + 5) \rightarrow R_1 = 3871$ N

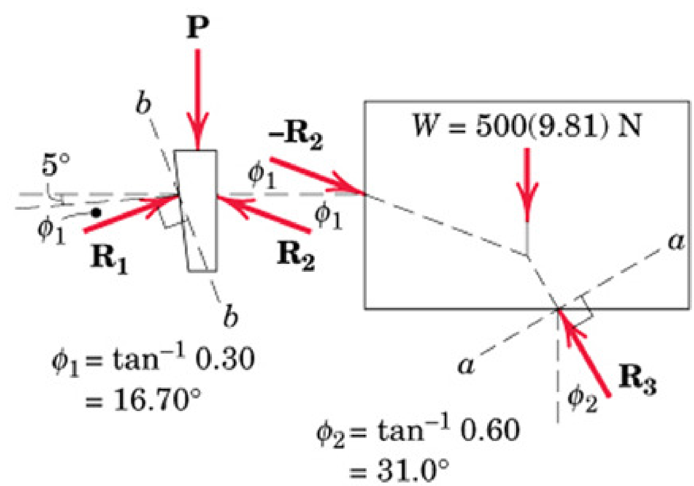
$\sum F_Y = 0$

$R_1 \sin(\phi_1 + 5) + R_2 \sin \phi_1 = P$

$\rightarrow P = 2500$ N







Solution:

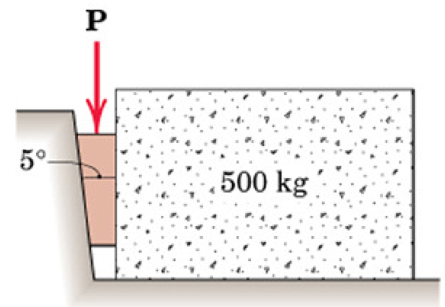
Method 2:
Using Equilibrium equations along reference axes *a-a* and *b-b*
→ No need to solve simultaneous equations
Angle between R_2 and *a-a* axis = 16.70+31.0 = 47.7°

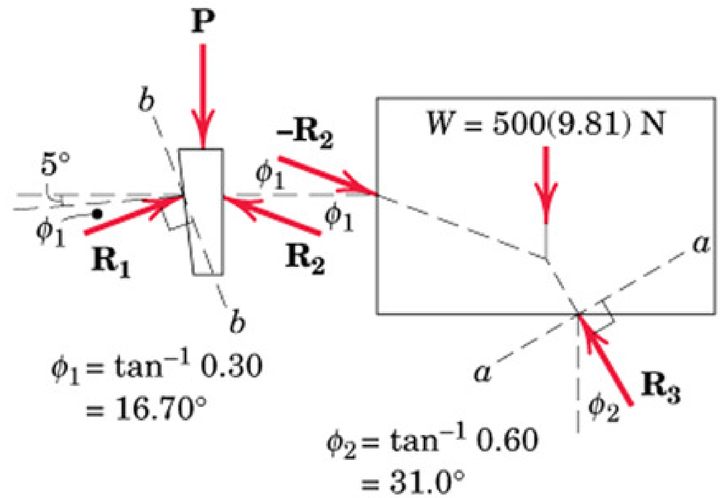
Equilibrium of Block:

$$[\Sigma F_a = 0] \quad 500(9.81)\sin 31.0° - R_2 \cos 47.7° = 0$$
$$R_2 = 3750 \text{ N}$$

Equilibrium of Wedge:
Angle between R_2 and *b-b* axis = 90-($2\Phi_1$+5) = 51.6°
Angle between *P* and *b-b* axis = Φ_1+5 = 21.7°

$$[\Sigma F_b = 0] \quad 3750 \cos 51.6° - P \cos 21.7° = 0$$
$$P = 2500 \text{ N}$$





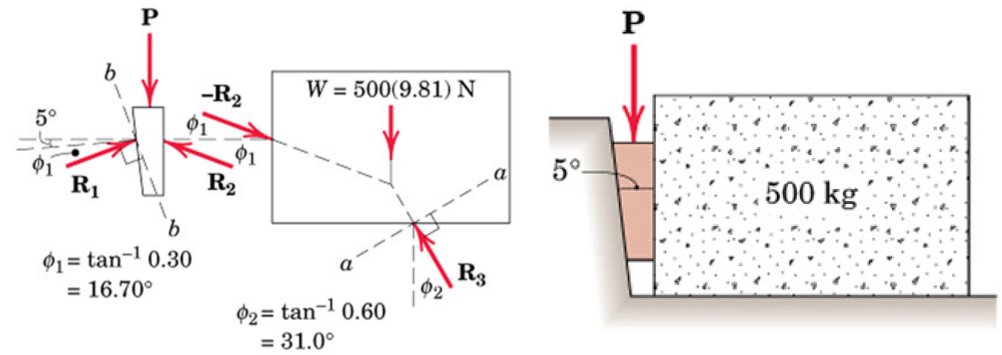
Applications of Friction in Machines

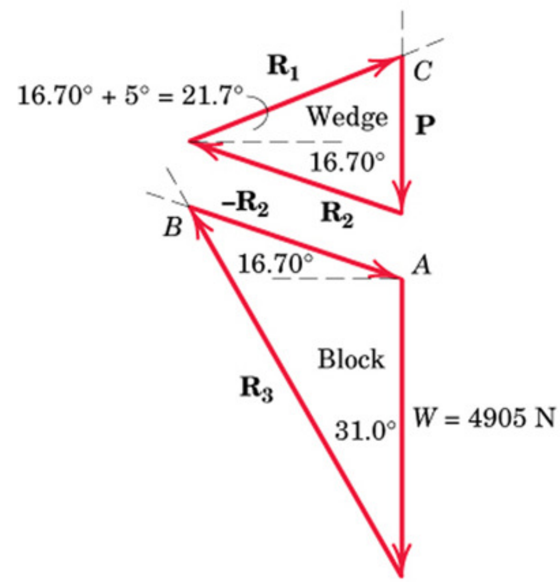
Solution:

Method 3:
Graphical solution using vector polygons

Starting with equilibrium of the block:
W is known, and directions of R_2 and R_3 are known
→ Magnitudes of R_2 and R_3 can be determined graphically

Similarly, construct vector polygon for the wedge from known magnitude of R_2, and known directions of R_2, R_1, *and P.*
→ Find out the magnitude of *P* graphically

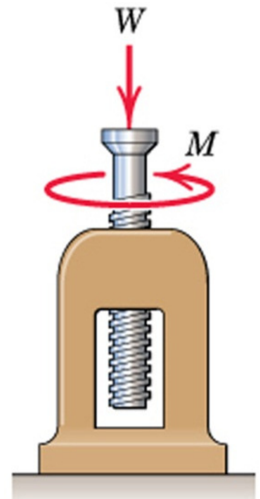


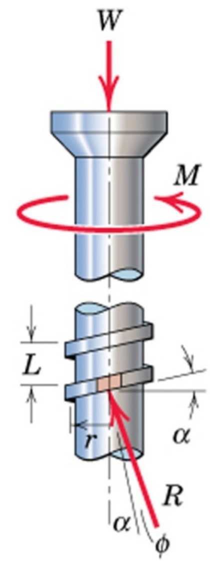


Applications of Friction in Machines

Square Threaded Screws

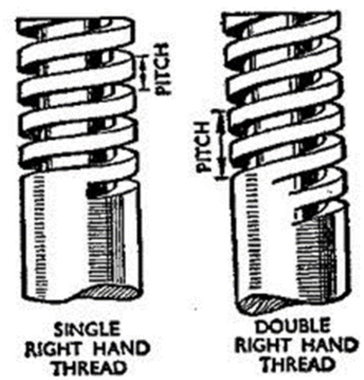
- Used for fastening and for transmitting power or motion
- Square threads are more efficient
- Friction developed in the threads largely determines the action of the screw





FBD of the Screw: R exerted by the thread of the jack frame on a small portion of the screw thread is shown

Lead = L = advancement per revolution
L = Pitch – for single threaded screw
L = 2xPitch – for double threaded screw (twice advancement per revolution)
Pitch = axial distance between adjacent threads on a helix or screw
Mean Radius = r ; α = Helix Angle



Similar reactions exist on all segments of the screw threads

Analysis similar to block on inclined plane since friction force does not depend on area of contact.

- Thread of base can be "unwrapped" and shown as straight line. Slope is $2\pi r$ horizontally and lead L vertically.

Applications of Friction in Machines: Screws

If M is just sufficient to turn the screw → Motion Impending
Angle of friction = ϕ (made by R with the axis normal to the thread)
→ $\tan\phi = \mu$

Moment of R @ vertical axis of screw = $R\sin(\alpha+\phi)r$
→ Total moment due to all reactions on the thread = $\sum R\sin(\alpha+\phi)r$
→ Moment Equilibrium Equation for the screw:

→ $M = [r\sin(\alpha + \phi)] \sum R$

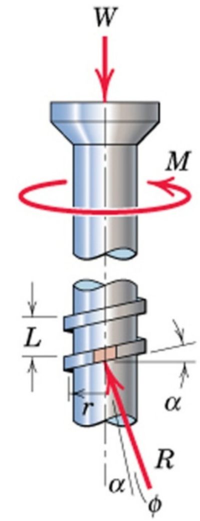
Equilibrium of forces in the axial direction: W = $\sum R\cos(\alpha + \phi)$

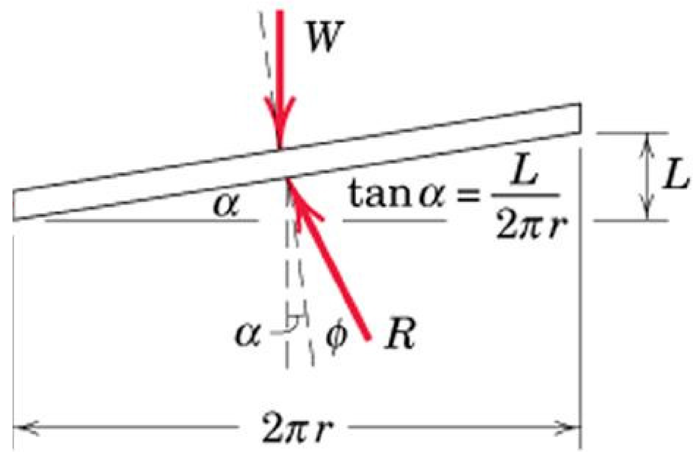
→ $W = [\cos(\alpha + \phi)] \sum R$

Finally → $\boldsymbol{M = W\,r\tan(\alpha + \phi)}$

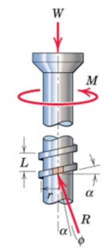
Helix angle α can be determined by unwrapping the thread of the screw for one complete turn

$\boldsymbol{\alpha = \tan^{-1}(L/2\pi r)}$

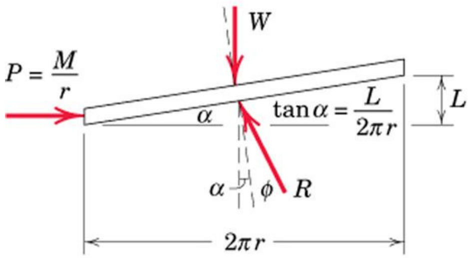




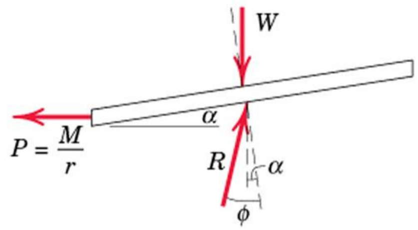
Applications of Friction in Machines: Screws



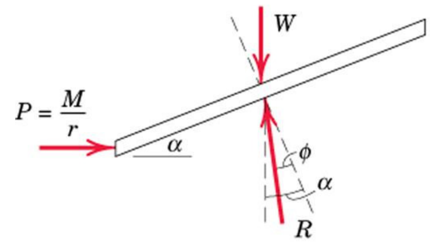
Alternatively, action of the entire screw can be simulated using unwrapped thread of the screw



To Raise Load



To Lower Load (α<ϕ)



To Lower Load (α>ϕ)

Equivalent force required to push the movable thread up the fixed incline is:
$P = M/r$

From Equilibrium:

$M = W\, r \tan(\alpha + \phi)$

If M is removed: the screw will remain in place and be self-locking provided $\alpha<\phi$ and will be on the verge of unwinding if $\alpha=\phi$

To lower the load by unwinding the screw, We must reverse the direction of M as long as $\alpha<\phi$

From Equilibrium:

$M = W\, r \tan(\phi - \alpha)$

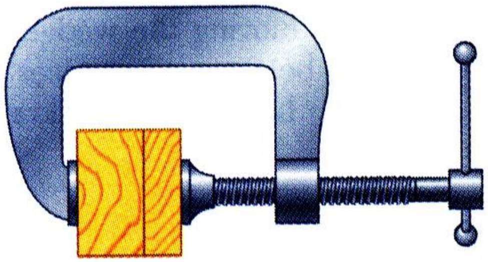
→ This is the moment required to unwind the screw

If $\alpha > \phi$, the screw will unwind by itself. Moment required to prevent unwinding:

From Equilibrium:

$M = W\, r \tan(\alpha - \phi)$

Sample Problem 8.5



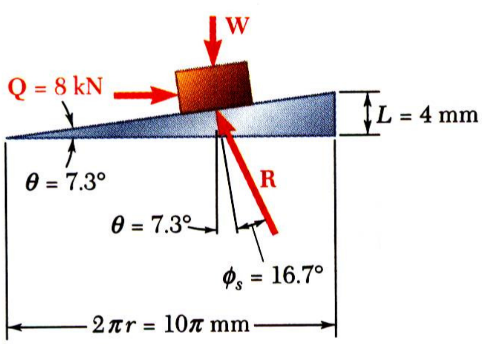
A clamp is used to hold two pieces of wood together as shown. The clamp has a double square thread of mean diameter equal to 10 mm with a pitch of 2 mm. The coefficient of friction between threads is $\mu_s = 0.30$.

If a maximum torque of 40 N*m is applied in tightening the clamp, determine (a) the force exerted on the pieces of wood, and (b) the torque required to loosen the clamp.

SOLUTION

- Calculate lead angle and pitch angle.
- Using block and plane analogy with impending motion up the plane, calculate the clamping force with a force triangle.
- With impending motion down the plane, calculate the force and torque required to loosen the clamp.

Sample Problem 8.5

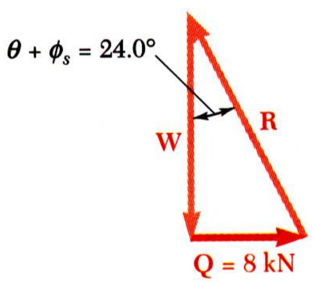


<u>SOLUTION</u>

- Calculate lead angle and pitch angle. For the double threaded screw, the lead L is equal to twice the pitch.

$$\tan\theta = \frac{L}{2\pi r} = \frac{2(2\text{ mm})}{10\pi\text{ mm}} = 0.1273 \qquad \theta = 7.3°$$

$$\tan\phi_s = \mu_s = 0.30 \qquad \phi_s = 16.7°$$

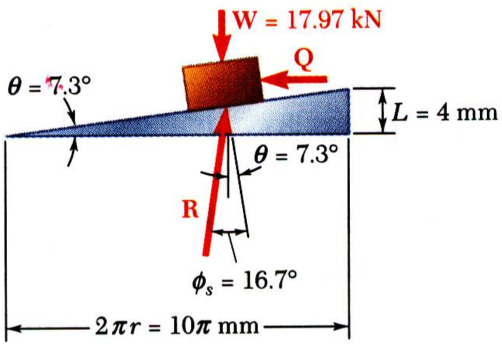


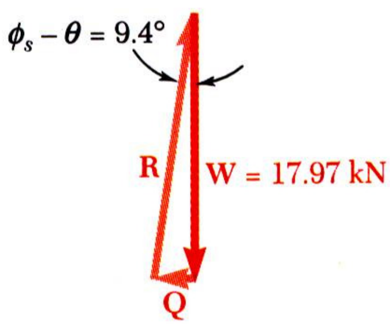
$$M = W\,r\tan(\alpha + \phi)$$

$$40 = W\frac{5}{1000}\tan(7.3+16.7)$$

$$\boxed{W = 17.97\,\text{kN}}$$

Sample Problem 8.5





- With impending motion down the plane, calculate the force and torque required to loosen the clamp.

$$M = W\,r\tan(\phi - \alpha)$$

$$M = 17.97\text{x}1000\,\frac{5}{1000}\tan(16.7\text{-}7.3)$$

$$\boxed{Torque = 14.87\ \mathrm{N\cdot m}}$$

Applications of Friction in Machines

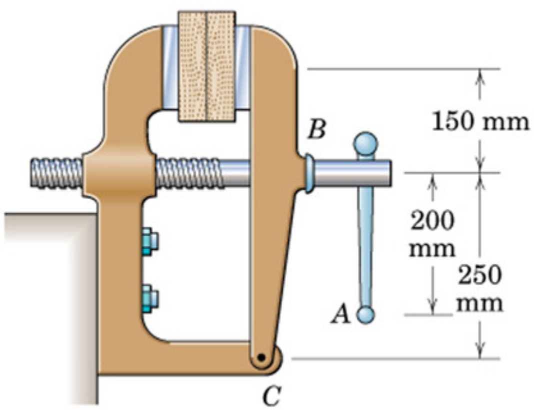
Example: Screw

Single threaded screw of the vise has a mean diameter of 25 mm and a lead of 5 mm. A 300 N pull applied normal to the handle at *A* produces a clamping force of 5 kN between the jaws of the vise. Determine:

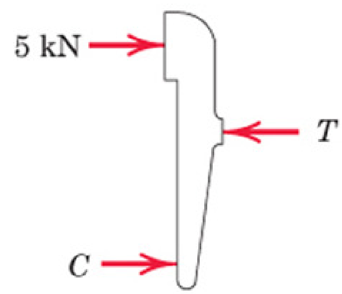
(a) Frictional moment M_B developed at *B* due to thrust of the screw against body of the jaw

(b) Force *Q* applied normal to the handle at *A* required to loosen the vise

μ_s in the threads = 0.20



Solution: Draw FBD of the jaw to find tension in the screw



$\sum M_C = 0$

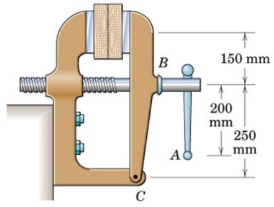
$\rightarrow T = 8$ kN

Find the helix angle α and the friction angle ϕ

$\alpha = \tan^{-1}(L/2\pi r) = 3.64°$

$\tan\phi = \mu \rightarrow \phi = 11.31°$

Applications of Friction in Machines

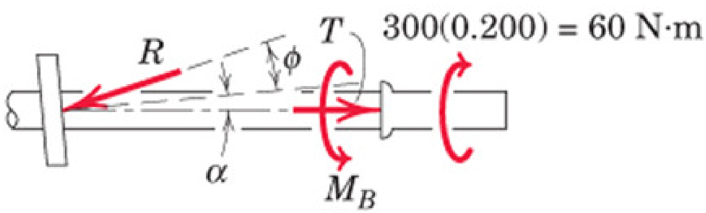


Example: Screw

Solution:

(a) To tighten the vise

Draw FBD of the screw



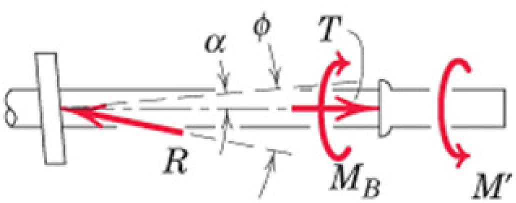
$M = T\, r \tan(\alpha + \phi)$

$60 - M_B = 8000(0.0125)\tan(3.64+11.31)$

$\boldsymbol{M_B}$ **= 33.3 Nm**

(a) To loosen the vise (on the verge of being loosened)

Draw FBD of the screw: Net moment = applied moment M' minus M_B



$M = T\, r \tan(\phi - \alpha)$

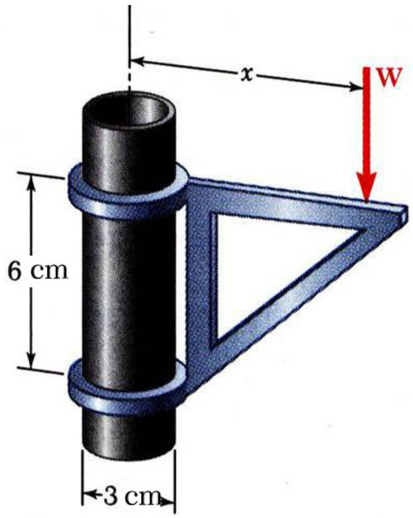
$M' - 33.3 = 8000(0.0125)\tan(11.31-3.64)$

$M' = 46.8$ Nm

***Q* = *M'*/*d* = 46.8/0.2 = 234 N**

Dry Friction

<u>Example</u>

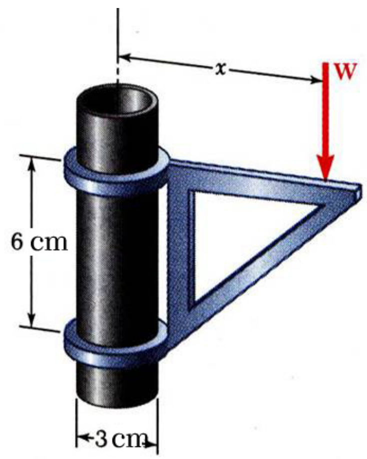


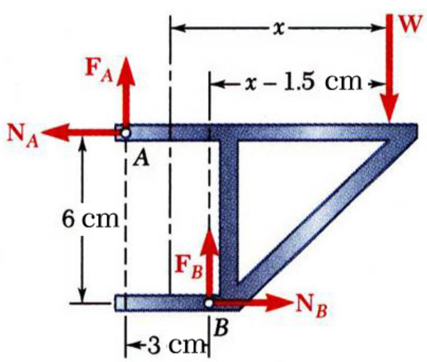
The moveable bracket shown may be placed at any height on the 3-cm diameter pipe. If the coefficient of friction between the pipe and bracket is 0.25, determine the minimum distance *x* at which the load can be supported. Neglect the weight of the bracket.

<u>SOLUTION</u>:

- When *W* is placed at minimum *x*, the bracket is about to slip and friction forces in upper and lower collars are at maximum value.
- Apply conditions for static equilibrium to find minimum *x*.

Dry Friction





<u>SOLUTION</u>:

- When W is placed at minimum x, the bracket is about to slip and friction forces in upper and lower collars are at maximum value.

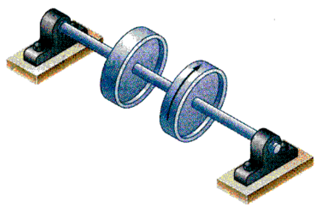
$$F_A = \mu_s N_A = 0.25 N_A$$
$$F_B = \mu_s N_B = 0.25 N_B$$

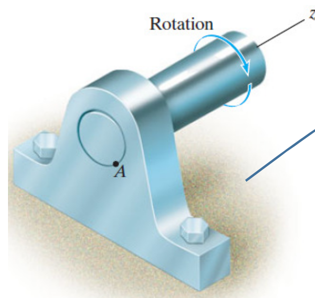
- Apply conditions for static equilibrium to find minimum x.

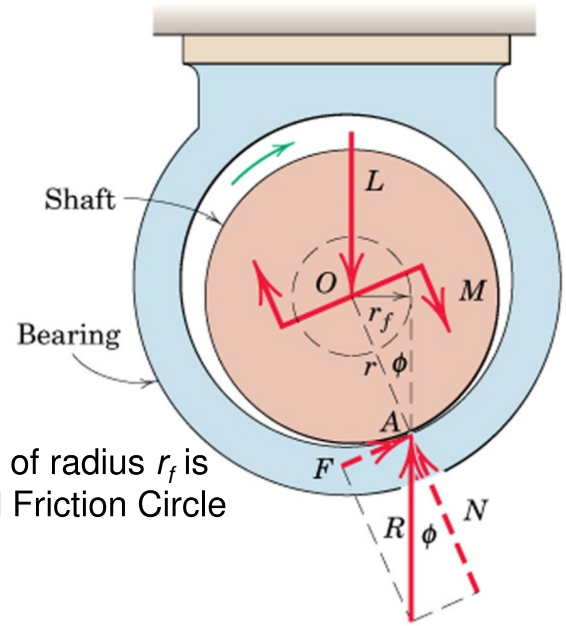
$$\sum F_x = 0: \quad N_B - N_A = 0 \qquad\qquad N_B = N_A$$
$$\sum F_y = 0: \quad F_A + F_B - W = 0$$
$$0.25 N_A + 0.25 N_B - W = 0$$
$$0.5 N_A = W \qquad\qquad N_A = N_B = 2W$$
$$\sum M_B = 0: N_A(6\text{ cm}) - F_A(3\text{ cm}) - W(x - 1.5\text{ cm}) = 0$$
$$6N_A - 3(0.25N_A) - W(x - 1.5) = 0$$
$$6(2W) - 0.75(2W) - W(x - 1.5) = 0$$
$$\boxed{x = 12\text{ cm}}$$

Applications of Friction in Machines

Journal Bearings (Axle Friction)







Circle of radius r_f is called Friction Circle

- Journal bearings provide lateral support to rotating shafts.
- Lateral load acting on the shaft is *L*.
- Thrust bearings provide axial support to rotating shafts.
- Frictional resistance of fully lubricated bearings depends on clearances, speed and lubricant viscosity.
- Partially lubricated axles and bearings can be assumed to be in direct contact along a straight line

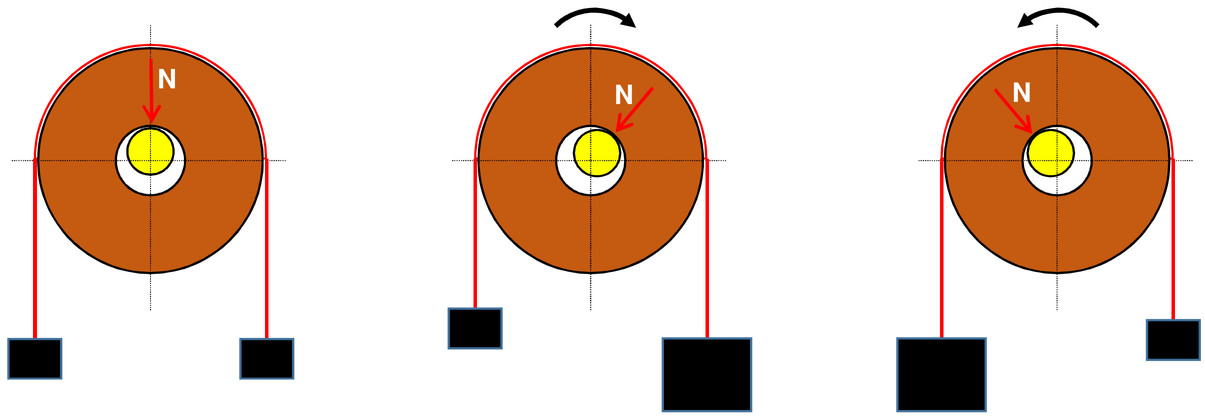
Applications of Friction in Machines

Journal Bearings (Axle Friction)

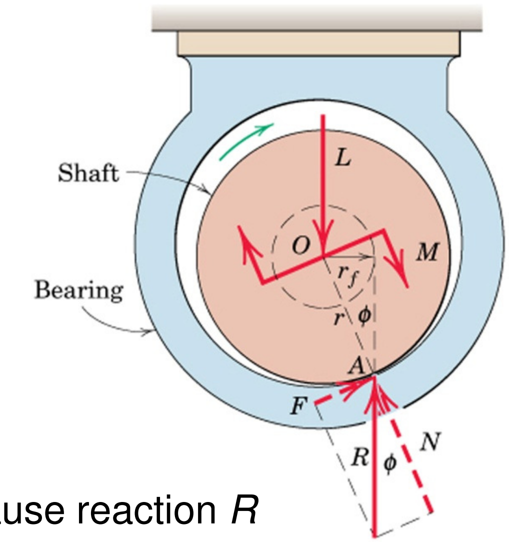
Exaggerated Figures show point of application of the normal reactions.

The frictional force will act normal to N and opposing the motion.

Resultant of frictional and normal force will act at an angle ϕ from N



Applications of Friction in Machines



Journal Bearings (Axle Friction)

Consider a dry or partially lubricated Journal Bearing

- with contact with near contact bet^n shaft and bearing
- As the shaft begins to turn in the direction shown,
  it will roll up the inner surface of bearing until it slips at A
- Shaft will remain in a more or less fixed position during rotation
- Torque *M* required to maintain rotation, and the radial load *L* on the shaft will cause reaction *R* at the contact point *A*.
- For vertical equilibrium, *R* must be equal to *L* but will not be collinear
- *R* will be tangent to a small circle of radius r_f called the *friction circle*

$\sum M_A = 0 \rightarrow M = Lr_f = Lr \sin\phi$

For a small coefficient of friction, ϕ is small → $\sin\phi \approx \tan\phi$

→ $M = \mu Lr$ (since $\mu = \tan\phi$) → Use equilibrium equations to solve a problem

→ Moment that must be applied to the shaft to overcome friction for a dry or partially lubricated journal bearing

Applications of Friction in Machines

Thrust Bearings (Disk Friction)

- Thrust bearings provide axial support to rotating shafts.
- Axial load acting on the shaft is *P*.
- Friction between circular surfaces under distributed normal pressure (Ex: clutch plates, disc brakes)

Consider two flat circular discs whose shafts are mounted in bearings: they can be brought under contact under *P*

Max torque that the clutch can transmit =

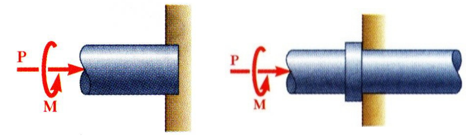
M required to slip one disc against the other

p is the normal pressure at any location between the plates

→ Frictional force acting on an elemental area = $\mu p dA$; $dA = r\, dr\, d\Theta$

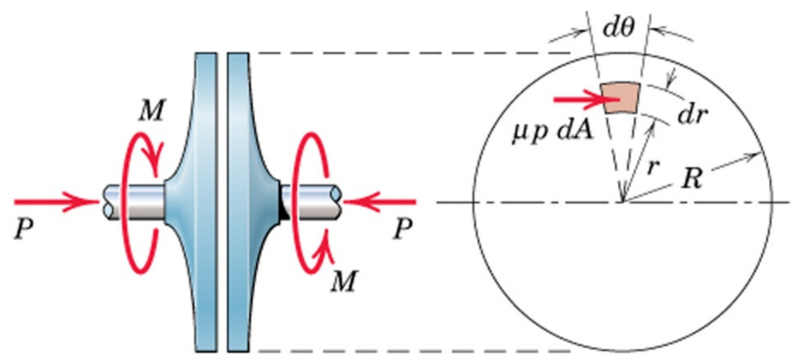
Moment of this elemental frictional force about the shaft axis = $\mu p r dA$

Total $M = \int \mu p r dA$ over the area of disc

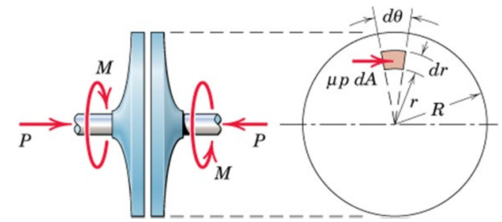


End/Pivot Bearing Collar Bearing

Sanding Machine



Applications of Friction in Machines



Thrust Bearings (Disk Friction)

Assuming that μ and p are uniform over the entire surface → $P = \pi R^2 p$

→ Substituting the constant p in $M = \int \mu p r dA$ →

$$M = \frac{\mu P}{\pi R^2} \int_0^{2\pi} \int_0^{R} r^2 dr\, d\theta$$

→ ➔ M $M = \frac{2}{3} \mu P R$ of moment reqd for impending rotation of shaft

≈ moment due to frictional force ***μp*** acting a distance ⅔ ***R*** from shaft center

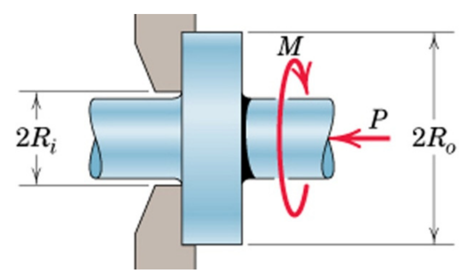
Frictional moment for worn-in plates is only about ¾ of that for the new surfaces

→ M for worn-in plates = ½(μPR)

If the friction discs are rings (Ex: Collar bearings) with outside and inside radius as R_o and R_i, respectively (limits of integration R_o and R_i) → $P = \pi(R_o^2 - R_i^2)p$

→ The frictional torque:

$$M = \frac{\mu P}{\pi\left(R_o^2 - R_i^2\right)} \int_0^{2\pi} \int_{R_i}^{R_o} r^2 dr\, d\theta$$



→

$$M = \frac{2}{3} \mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$$

Frictional moment for worn-in plates → $M = \frac{1}{2} \mu P(R_o + R_i)$

Applications of Friction in Machines

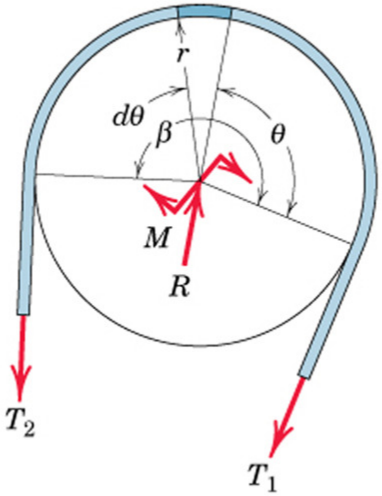
Belt Friction

Impending slippage of flexible cables, belts, ropes over sheaves, wheels, drums

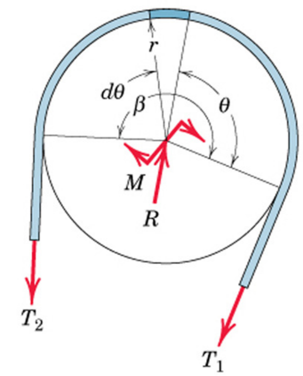
→ It is necessary to estimate the frictional forces developed between the belt and its contacting surface.

Consider a drum subjected to two belt tensions (T_1 and T_2)
M is the torque necessary to prevent rotation of the drum
R is the bearing reaction
r is the radius of the drum
β is the total contact angle between belt and surface
(β in radians)

$T_2 > T_1$ since M is clockwise



Applications of Friction in Machines



Belt Friction: Relate T_1 and T_2 when belt is about to slide to left

Draw FBD of an element of the belt of length $r\,d\vartheta$

Frictional force for impending motion = $\mu\, dN$

Equilibrium in the t-direction:

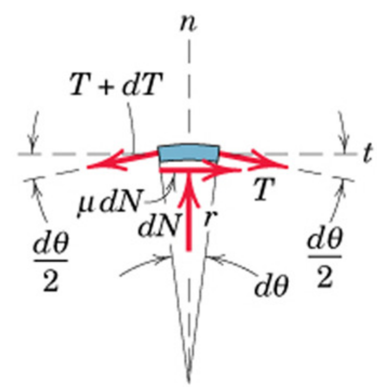
$$T \cos\frac{d\theta}{2} + \mu\, dN = (T + dT)\cos\frac{d\theta}{2}$$

→ $\mu dN = dT$ (cosine of a differential quantity is unity in the limit)

Equilibrium in the n-direction:

$$dN = (T + dT)\sin\frac{d\theta}{2} + T \sin\frac{d\theta}{2}$$

→ $dN = 2Td\theta/2 = Td\theta$ (sine of a differential in the limit equals the angle, and product of two differentials can be neglected)



Combining two equations:

$$\frac{dT}{T} = \mu\, d\theta$$

Integrating between corresponding limits:

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\beta} \mu\, d\theta$$

→ $\ln\frac{T_2}{T_1} = \mu\beta$ ($T_2>T_1$; $\boldsymbol{T_2 = T_1\, e^{\mu\beta}}$ lians)

- Rope wrapped around a drum n times → $\beta = 2\pi n$ radians
- r not present in the above eqn → eqn valid for non-circular sections as well
- In belt drives, belt and pulley rotate at constant speed → the eqn describes condition of impending slippage.

Applications of Friction in Machines

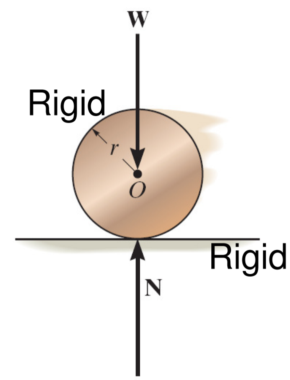
Wheel Friction or Rolling Resistance

Resistance of a wheel to roll over a surface is caused by deformation between two materials of contact.

→ This resistance is not due to tangential frictional forces

→ Entirely different phenomenon from that of dry friction

If a rigid cylinder rolls at constant velocity along a rigid surface, the normal force exerted by the surface on the cylinder acts at the tangent point of contact → No Rolling Resistance

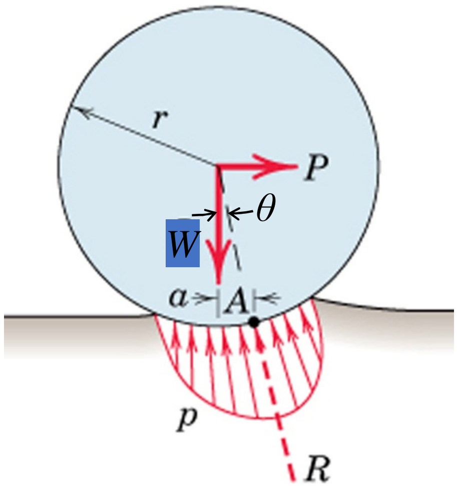


Steel is very stiff
→ Low Rolling Resistance

Significant Rolling Resistance between rubber tyre and tar road

Large Rolling Resistance due to wet field

Applications of Friction in Machines



Wheel Friction or Rolling Resistance

Actually materials are not rigid → deformation occurs
→ reaction of surface on the cylinder consists of a distribution of normal pressure.

Consider a wheel under action of a load *W* on axle and a force *P* applied at its center to produce rolling
→ Deformation of wheel and supporting surface
→ Resultant *R* of the distribution of normal pressure must pass through wheel center for the wheel to be in equilibrium (i.e., rolling at a constant speed)
→ *R* acts at point *A* on right of wheel center for rightwards motion

Force *P* reqd to maintain rolling at constant speed can be appx estimated as:

$\sum M_A = 0 \rightarrow Wa = Pr\cos\theta$ ($\cos\theta \approx 1 \leftarrow$ deformations are very small compared to r)

→ $$P = \frac{a}{r}W = \mu_r W$$ Coefficient of Rolling Resistance

- μ_r is the ratio of resisting force to the normal force → analogous to μ_s or μ_k
- No slippage or impending slippage in interpretation of μ_r

Applications of Friction in Machines

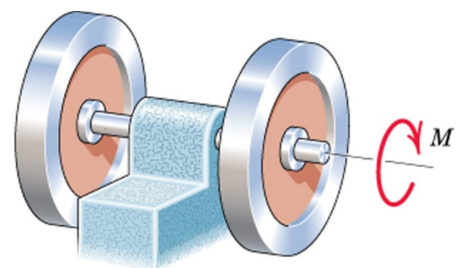
Examples: Journal Bearings

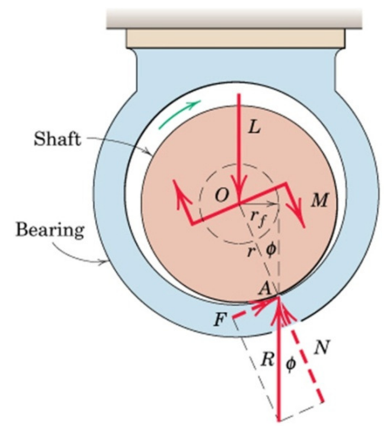
Two flywheels (each of mass 40 kg and diameter 40 mm) are mounted on a shaft, which is supported by a journal bearing. M = 3 Nm couple is reqd on the shaft to maintain rotation of the flywheels and shaft at a constant low speed.
Determine: (a) coeff of friction in the bearing, and (b) radius r_f of the friction circle.

Solution: Draw the FBD of the shaft and the bearing

(a) Moment equilibrium at O
$M = Rr_f = Rr\sin\phi$
M = 3 Nm, R = 2x40x9.81 = 784.8 N, r = 0.020 m
→ $\sin\phi$ = 0.1911 → $\phi = 11.02^{o}$

(b) $r_f = r\sin\phi$ = 3.82 mm





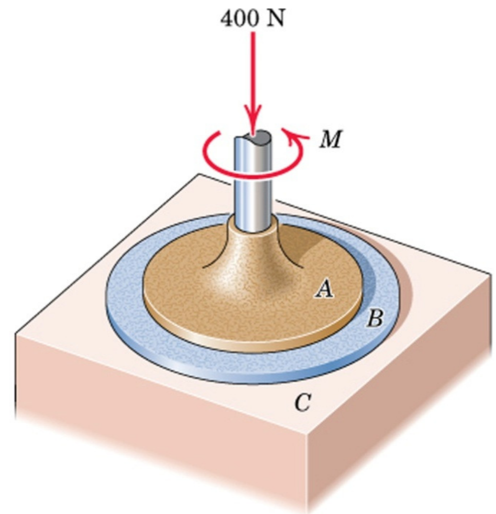
Applications of Friction in Machines

Examples: Disk Friction

Circular disk *A* (225 mm dia) is placed on top of disk *B* (300 mm dia) and is subjected to a compressive force of 400 N. Pressure under each disk is constant over its surface. Coeff of friction betn *A* and *B* = 0.4. Determine:

(a) the couple *M* which will cause *A* to slip on *B*.

(b) Min coeff of friction *μ* between *B* and supporting surface *C* which will prevent *B* from rotating.



Solution: $$M = \frac{2}{3}\mu P R$$

(a) Impending slip between *A* and *B*:

μ=0.4, P=400 N, R=225/2 mm

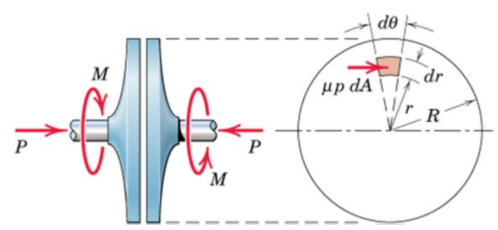
M = 2/3 x 0.4 x 400 x 0.225/2 → M = 12 Nm

(b) Impending slip between *B* and *C* :

Slip between *A* and *B* → M = 12 Nm

μ=? P=400 N, R=300/2 mm

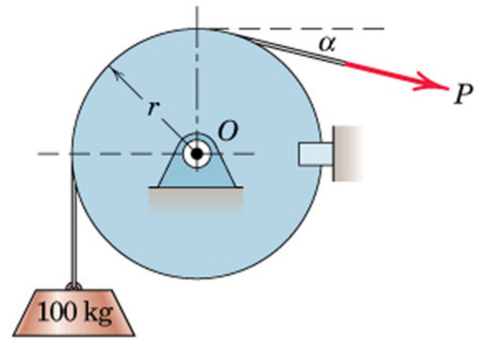
12 = 2/3 x μ x 400 x 0.300/2 → μ = 0.3



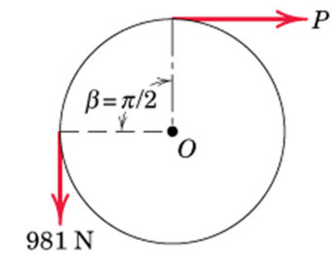
Applications of Friction in Machines

Examples: Belt Friction

A force P is reqd to be applied on a flexible cable that supports 100 kg load using a **fixed** circular drum.
μ between cable and drum = 0.3
(a) For $\alpha = 0$, determine the max and min P in order not to raise or lower the load
(b) For $P = 500$ N, find the min α before the load begins to slip

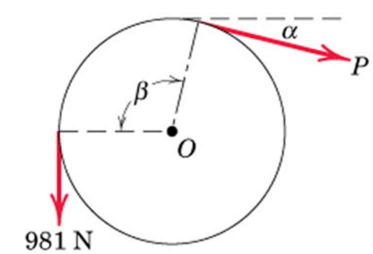


Solution: Impending slippage of the cable over the fixed drum is given by: $T_2 = T_1 e^{\mu\beta}$
Draw the FBD for each case



(a) α = 0

(a) $\mu = 0.3$, $\alpha = 0$, $\beta = \pi/2$ rad
For impending upward motion of the load: $T_2 = P_{max}$; $T_1 = 981$ N
$P_{max}/981 = e^{0.3(\pi/2)} \rightarrow P_{max} = 1572$ N
For impending downward motion: $T_2 = 981$ N; $T_1 = P_{min}$
$981/P_{min} = e^{0.3(\pi/2)} \rightarrow P_{min} = 612$ N



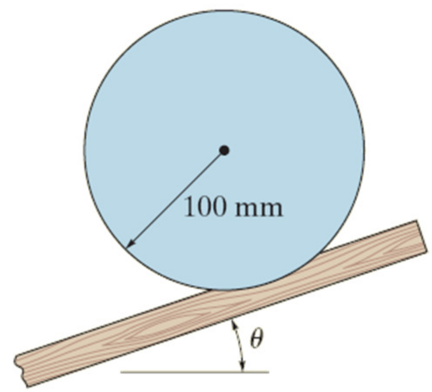
(b) P = 500 N

(b) $\mu = 0.3$, $\alpha = ?$, $\beta = \pi/2+\alpha$ rad, $T_2 = 981$ N; $T_1 = 500$ N
$981/500 = e^{0.3\beta} \rightarrow 0.3\beta = \ln(981/500) \rightarrow \beta = 2.25$ rad
$\rightarrow \beta = 2.25\text{x}(360/2\pi) = 128.7^o$
$\rightarrow \alpha = 128.7 - 90 = 38.7^o$

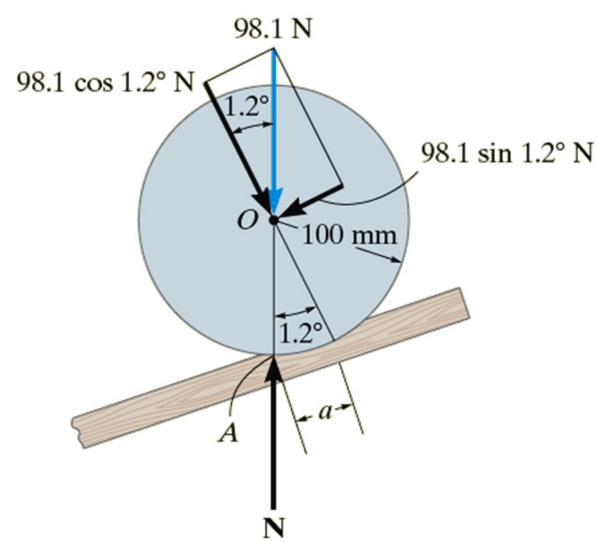
Applications of Friction in Machines

Examples: Rolling Resistance

A 10 kg steel wheel (radius = 100 mm) rests on an inclined plane made of wood. At θ=1.2°, the wheel begins to roll-down the incline with constant velocity.
Determine the coefficient of rolling resistance.



Solution: When the wheel has impending motion, the normal reaction *N* acts at point *A* defined by the dimension *a*.
Draw the FBD for the wheel:
r = 100 mm, 10 kg = 98.1 N



Using simplified equation directly:

$$P = \frac{a}{r} W = \mu_r W$$

Here P = 98.1(sin1.2) = 2.05 N
W = 98.1(cos1.2) = 98.08 N
→ Coeff of Rolling Resistance $\mu_r = 0.0209$

Alternatively, $\sum M_A = 0$
→ 98.1(sin1.2)(r appx) = 98.1(cos1.2)a
(since $r$cos1.2 = rx0.9998 ≈ r)
→ $a/r = \mu_r = 0.0209$