Q1 Solution

$\delta U = 0$

$P\delta x - k(2l - 2l\,cos\theta)\delta y = 0$

$x = l\,sin\theta, \quad \delta x = l\,cos\theta\;\delta\theta$

$y = 2l - 2l\,cos\theta\;(measured\;from\;wheel\;position\;when\;spring\;is\;unstretched)$

$\delta y = 2l\,sin\theta\;\delta\theta$

$So\;P(l\,cos\theta\;\delta\theta) - k(2l - 2l\,cos\theta)(2l\,sin\theta\;\delta\theta) = 0$

$$P = \frac{4kl\,(sin\theta - sin\theta\,cos\theta)}{cos\theta}$$

$or\;P = 4kl(tan\theta - sin\theta)$

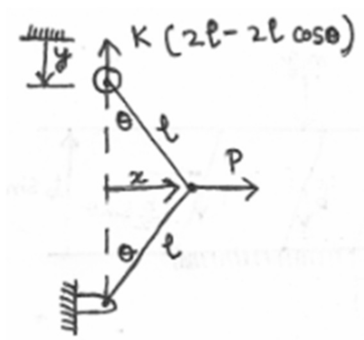


Fig 1

Q2 Solution

$$x = spring\ compression = \frac{h}{cos\theta} - h = h(sec\theta - 1)$$

$$\delta U = \delta V_e;\ \ P\delta s = \delta\left(\frac{1}{2}kx^2\right) = kx\ \delta x$$

$$s = h\ tan\theta\ so\ that$$

$$P\delta(h\ tan\theta) = kh^2(sec\theta - 1)sec\theta\ tan\theta\ \delta\theta$$

$$Ph\ sec^2\theta\ \delta\theta = kh^2(sec\theta - 1)sec^2\theta\ sin\theta\ \delta\theta$$

$$P = kh(sec\theta - 1)sin\theta$$

$$or\ P = kh\ tan\theta(1 - cos\theta)$$

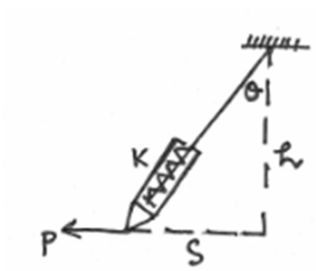


Fig 2

Q3 Solution

$\delta U = 0: 2C\ \delta(a cos\theta) + mg\delta(2a\ sin\theta) = 0$

$-2Ca\ sin\theta\ \delta\theta + 2mga\ cos\theta\ \delta\theta = 0$

$C = mg\ cot\theta$

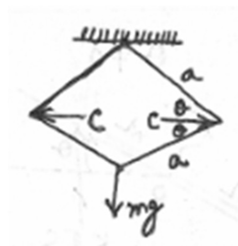


Fig 3

Q4 Solution

$\tan \emptyset = 0.30, \quad \emptyset = 16.70^o$

$\tan \propto = \tan 5^o = 0.0875$

$\sum F_x = 0:\ P = R \sin(\emptyset + \propto)$

$\sum F_y = 0:\ W = R \cos(\emptyset + \propto)$

$\tan(\emptyset + \propto) = \frac{P}{W}$

$work\ input : P\delta x$

$Work\ output : W\delta y \qquad where \quad \delta y = \delta x\ tan\alpha$

$$e = \frac{W\ \delta y}{P\ \delta x} = \frac{\tan \alpha}{\tan(\emptyset + \alpha)} = \frac{\tan 5^o}{\tan(16.70^o + 5^o)} = 0.220$$

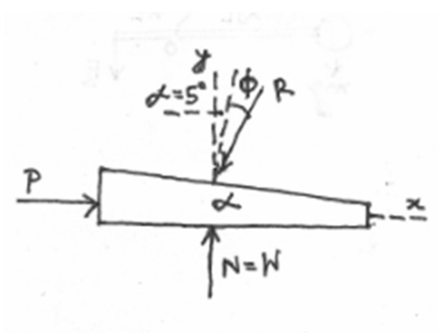
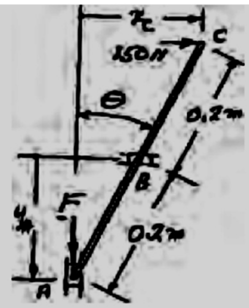


Fig 4

Q5 Solution



$$x_C = (0.4\text{ m})\sin\theta$$
$$\delta x_C = 0.4\cos\theta\delta\theta$$
$$y_A = (0.2\text{ m})\cos\theta$$
$$\delta y_A = -0.2\sin\theta\delta\theta$$

Spring:

Unstretched length $= 0.2$ m

$$F = k(0.2\text{ m} - y_A) = k(0.2 - 0.2\cos\theta)$$
$$= (300\text{ N/m})(0.2)(1-\cos\theta)$$
$$F = 600(1-\cos\theta)$$

Virtual Work: $\delta U = 0: \quad (150\text{ N})\delta x_C + F\delta y_A = 0$

$$150(0.4\cos\theta\delta\theta) + 600(1-\cos\theta)(-0.2\sin\theta\delta\theta) = 0$$

$$\frac{150(0.4)}{600(0.2)} = \frac{1}{2}: \quad \frac{1}{2} = (1-\cos\theta)\tan\theta$$

Solve by trial and error:

$\theta = 52.2^0$

Q6 Solution

$V_g = 2mgl\cos\frac{\beta}{2} + K\beta, \quad \frac{dV}{d\beta} = 0 \quad for\ \beta = 0$

$\frac{d^2V}{d\beta^2} = -\frac{1}{2}mgl\ cos\frac{\beta}{2} + K$

$= -\frac{1}{2}mgl + K\ for\ \beta = 0$

$= (+)\ stable\ if\ K > \frac{1}{2}mgl$

$Thus\ K_{min} = \frac{1}{2}mgl$

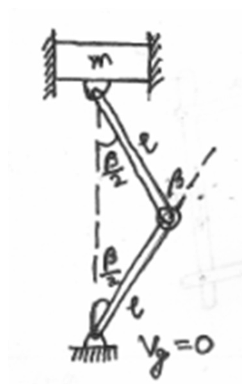


Fig 6

Q7 Solution

$m_1 = 80\ kg \quad m_2 = 10\ kg \quad b = 600\ mm$

$$\delta U = 0:\ -F\delta(b\ cos\theta) - m_1 g\delta(b\ sin\theta) - 3m_2 g\delta\left(\frac{b}{2}\ sin\theta\right) = 0$$

$$Fb\ sin\theta\ \delta\theta - m_1 gb\ cos\theta\ \delta\theta + \frac{3}{2}m_2 gb\ cos\theta\ \delta\theta$$

$$F = g\ cot\theta\left(m_1 + \frac{3}{2}m_2\right) = 9.81\left(80 + \frac{3}{2}\times 10\right)\ cot\theta = 932\ cot\theta\ N$$

Solution by force and moment equilibrium would require dismemberment with four FBD's and eventual elimination of unwanted forces and dimensions.

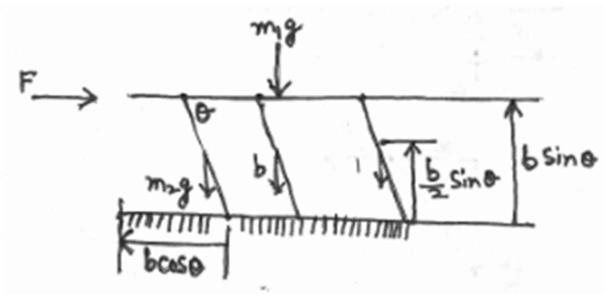


Fig 7

Q8 Solution

$\delta U = 0;$

$P\delta(2a\ sin\theta) + mg\ \delta(2a\ cos\theta) + 3mg\delta(a\ cos\theta) = 0$

$2Pa\ cos\theta\ \delta\theta = 5mga\ sin\theta\ \delta\theta$

$P = \frac{5}{2}mg\ tan\theta$

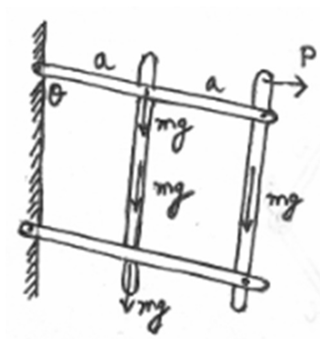


Fig 8

Q9 Solution

$\delta V_e = kx\,\delta x \;\; where\; x = s - b = \sqrt{b^2 + u^2} - b$

$\delta x = \dfrac{u\,\delta x}{\sqrt{b^2 + u^2}}$

$\delta U' = \delta V_e;\;\; P\delta u = k\left(\sqrt{b^2 + u^2} - b\right)\dfrac{u\delta u}{\sqrt{b^2 + u^2}}$

$P = \left(1 - \dfrac{b}{\sqrt{b^2 + u^2}}\right) ku$

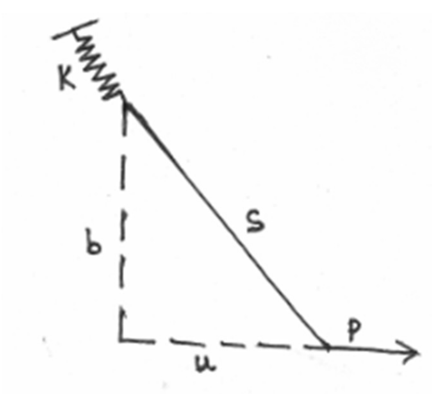


Fig 9

Q10 Solution

$FBD\ @\ \theta = 0: \quad \sum M_o = 0:$

$$mg(1-n)L - F_s(nL) = 0$$

$$F_s = \frac{mg(1-n)}{n} = k\delta_{st}$$

$$\delta_{st} = \frac{mg(1-n)}{nk}$$

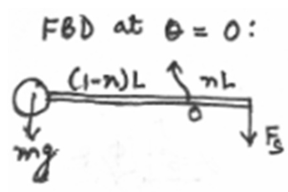


Fig 10a

$\theta \neq 0:$

$$V = V_g + V_c$$

$$= -mg[(1-n)L\,sin\theta] + \frac{1}{2k}\left[\frac{mg(1-n)}{kn} + nL\,sin\theta\right]^2$$

$$= \frac{m^2g^2(1-n)^2}{2kn^2} + \frac{n^2L^2k\,sin^2\theta}{2}$$

$$\frac{dV}{d\theta} = n^2L^2k\,sin\theta\,cos\theta = 0\ at\ \theta = 0$$

$$\frac{d^2V}{d\theta^2} = n^2L^2k(cos^2\theta - sin^2\theta) > 0\ at\ \theta = 0\ for\ all\ 0 < n < 1$$

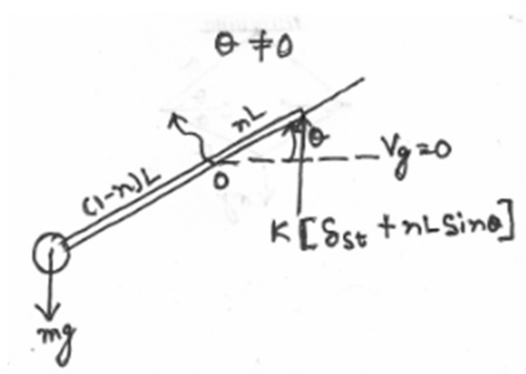


Fig 10b