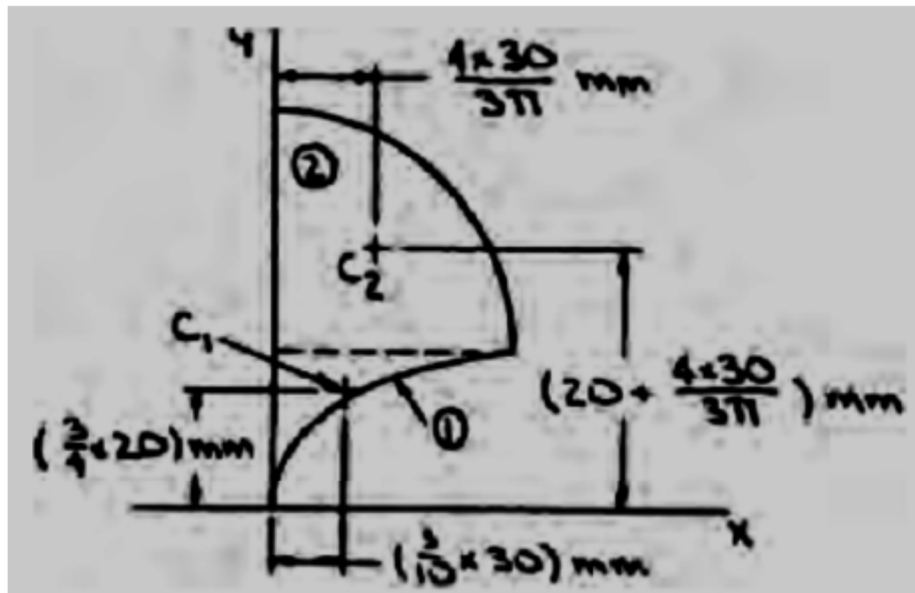


Solution of Q. No. 1



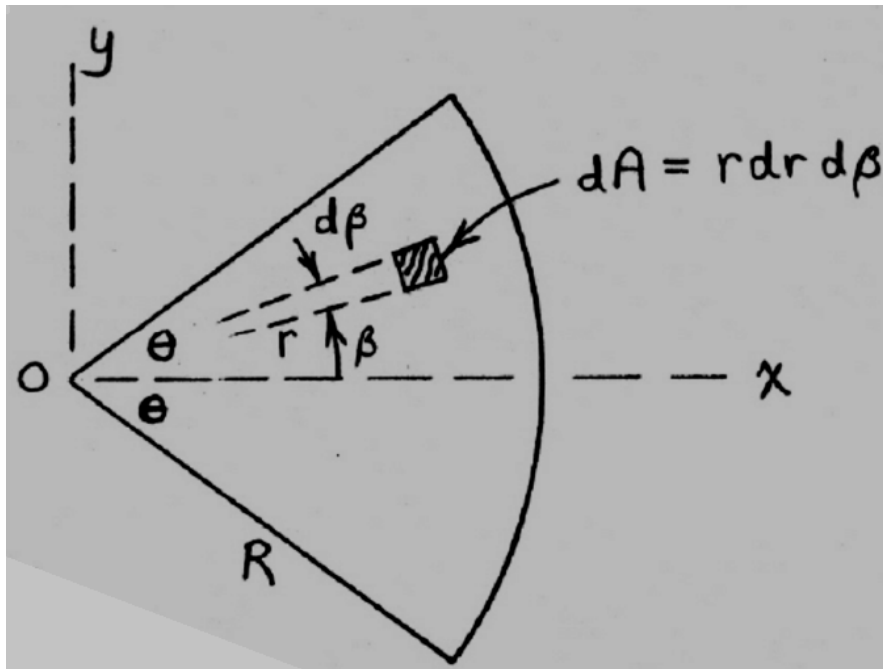
| | $A \text{ mm}^2$ | $\bar{x} \text{ mm}$ | $\bar{y} \text{ mm}$ | $\bar{x}A \text{ mm}^3$ | $\bar{y}A \text{ mm}^3$ |
|----------|---|----------------------|----------------------|-------------------------|-------------------------|
| 1 | $\frac{1}{3} \times 30 \times 20 = 200$ | 9 | 15 | 1800 | 3000 |
| 2 | $\frac{\pi}{4} (30)^2 = 706.86$ | 12.7324 | 32.7324 | 9000 | 23137 |
| Σ | 906.86 | | | 10800 | 26137 |

Then

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{10800}{906.86} \qquad \bar{X} = 11.91 \text{ mm}$$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{26137}{906.86} \qquad \bar{Y} = 28.8 \text{ mm}$$

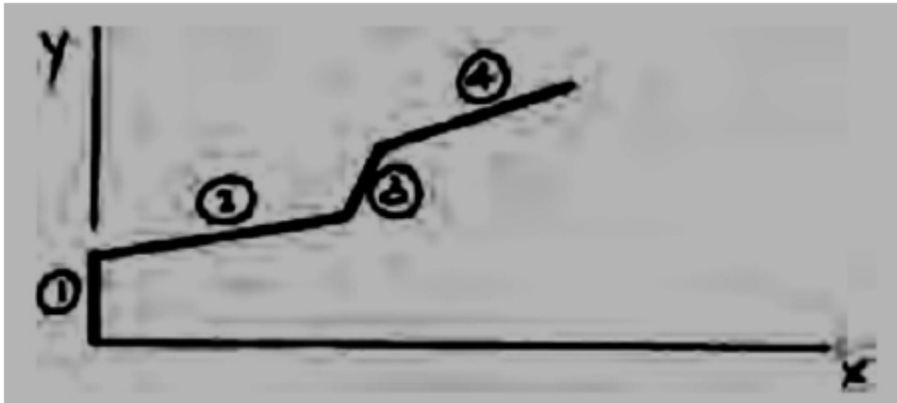
Solution of Q. No. 2



$$I_x = \int y^2 dA = \int_{-\theta}^{\theta} \int_0^R (r \sin \beta)^2 r dr d\beta$$
$$= \frac{R^4}{4} \left(\frac{\beta}{2} - \frac{\sin 2\beta}{4} \right)_{-\theta}^{\theta} = \frac{R^4}{4} \left(\theta - \frac{1}{2} \sin 2\theta \right)$$

$$I_y = \int x^2 dA = \int_{-\theta}^{\theta} \int_0^R (r \cos \beta)^2 r dr d\beta$$
$$= \frac{R^4}{4} \left(\frac{\beta}{2} + \frac{\sin 2\beta}{4} \right)_{-\theta}^{\theta} = \frac{R^4}{4} \left(\theta + \frac{1}{2} \sin 2\theta \right)$$

Solution of Q. No. 3



The mass of the lamp shade is given by

$$m = \rho V = \rho A t$$

Where A is the surface area and t is the thickness of the shade. The area can be generated by rotating the line shown about the x axis. Applying the first theorem of Pappus Guldinus we have

$$\begin{aligned} A &= 2\pi \bar{y}L = 2\pi \sum \bar{y}L \\ &= 2\pi (\bar{y}_1 L_1 + \bar{y}_2 L_2 + \bar{y}_3 L_3 + \bar{y}_4 L_4) \end{aligned}$$

$$\begin{aligned} \text{or } A &= 2\pi \left[\frac{13}{2} \times 13 + \left(\frac{13+16}{2} \right) \times \sqrt{32^2 + 3^2} + \left(\frac{16+28}{2} \right) \times \sqrt{8^2 + 12^2} + \left(\frac{28+33}{2} \right) \times \sqrt{28^2 + 5^2} \right] \\ &= 2\pi (84.5 + 466.03 + 317.29 + 867.51) \\ &= 10903.4 \text{ mm}^2 \end{aligned}$$

Then

$$m = \rho A t$$

$$\text{or} \quad = \left(2800 \frac{\text{kg}}{\text{m}^3} \right) \times (10.9034 \times 10^{-3} \text{ m}^2) \times (0.001 \text{ m})$$

$$\text{or} \quad m = 0.0305 \text{ kg}$$

Solution of Q. No. 4

First note that the required surface area A can be generated by rotating the parabolic cross section through π radians about the Y axis. Applying the first theorem of Pappus-Guldinus we have

$$A = \pi \bar{x} L$$

Now at $x=100$ mm, $y=250$ mm

$$250 = k 100^2 \quad \text{or} \quad k = 0.025 \text{ mm}^{-1}$$

And $x_{EL} = x$

$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

Where

$$\frac{dy}{dx} = 2kx$$

Then

$$dL = \sqrt{1 + 4k^2 x^2} dx$$

We have

$$xL = \int x_{EL} dL$$

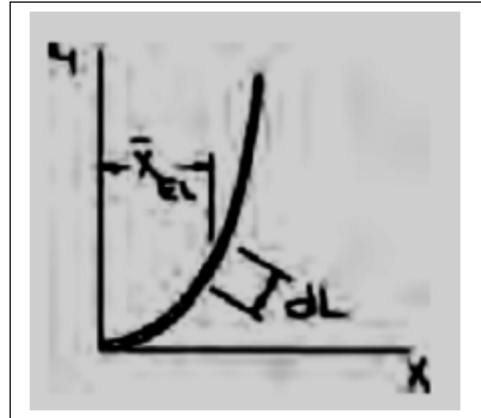
$$= \int_0^{100} x (\sqrt{1 + 4k^2 x^2}) dx$$

$$xL = \left[\frac{1}{3} \frac{1}{4k^2} (1 + 4k^2 x^2)^{3/2} \right]_0^{100}$$

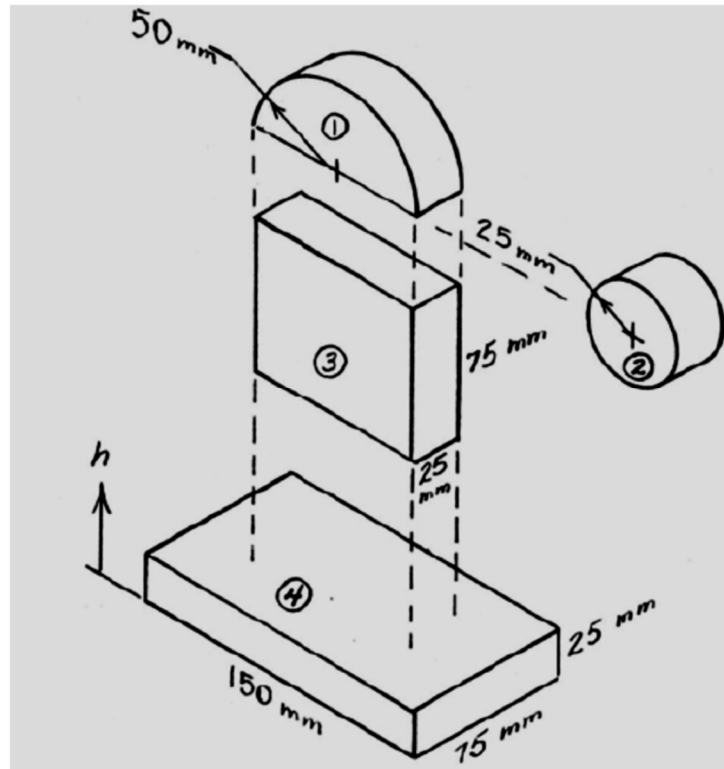
$$= \frac{1}{12 (0.025)^2} \left[\{1 + 4(0.025)^2 (100)^2\}^{3/2} - (1)^{3/2} \right]$$

$$= 17543.3 \text{ mm}^2$$

Finally, $A = \pi (17543.3 \text{ mm}^2) \quad \text{or} \quad A = 55.1 \times 10^3 \text{ mm}^2$



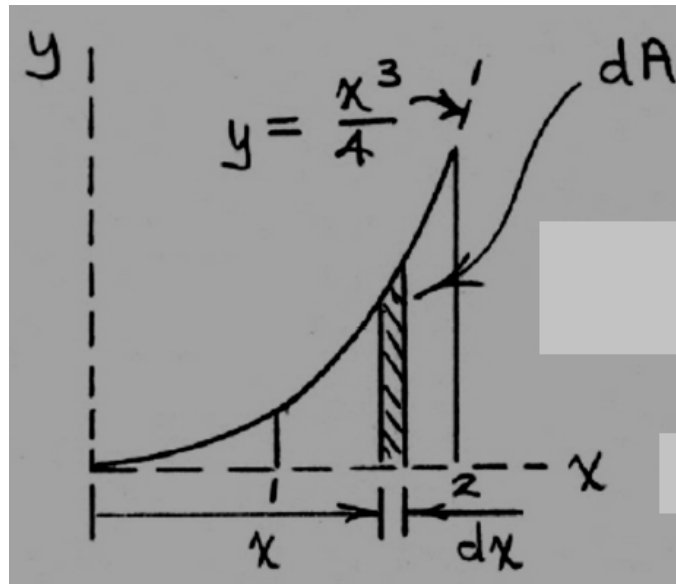
Solution of Q. No. 5



| Part | $V \text{ mm}^3$ | $\bar{h} \text{ mm}$ | $V\bar{h} \text{ mm}^4$ |
|---------------|------------------|----------------------|-------------------------|
| 1 | 98200 | 121.2 | 11.90×10^6 |
| 2 | -49100 | 100 | -4.91×10^6 |
| 3 | 187500 | 62.5 | 11.72×10^6 |
| 4 | 281000 | 12.5 | 3.52×10^6 |
| Totals | 518000 | | 22.2×10^6 |

$$\bar{H} = \frac{\sum V\bar{h}}{\sum V} = \frac{22.2 \times 10^6}{518000} = 42.9 \text{ mm}$$

Solution of Q. No. 6



$$dA = y dx = \frac{x^3}{4} dx$$

$$A = \int dA = \int_1^2 \frac{x^3}{4} dx = \frac{1}{4} \left(\frac{x^4}{4} \right)_1^2 = \frac{15}{16}$$

$$dI_y = x^2 dA = \frac{x^5}{4} dx$$

$$I_y = \int dI_y = \int_1^2 \frac{x^5}{4} dx = \frac{1}{4} \left(\frac{x^6}{6} \right)_1^2 = \frac{63}{24}$$

$$dI_x = \frac{1}{3} y^3 dx = \frac{x^9}{192} dx$$

$$I_x = \int dI_x = \int_1^2 \frac{x^9}{192} dx = \frac{1}{192} \left(\frac{x^{10}}{10} \right)_1^2 = \frac{1023}{1920}$$

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{63/24}{15/16}} = \sqrt{14/5} = 1.673$$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{1023/1920}{15/16}} = 0.754$$